Neoclassical Growth in an Interdependent World*

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We generalize the closed-economy neoclassical growth model (CNGM) to allow for costly goods trade and capital flows with imperfect substitutability between countries. We show that our framework rationalizes the observed gravity equations for trade and capital holdings. We find that goods and capital market integration interact in non-trivial ways. Opening the CNGM to *only* goods trade or *only* capital flows *increases* the speed of convergence to steady-state. In contrast, opening the CNGM to *both* goods trade and capital flows *decreases* this speed of convergence. Our framework is well suited for analysing counterfactual policies that affect bilateral integration in both goods and capital markets (e.g., U.S.-China decoupling). We show that the counterfactual effects of changes in goods market integration depend heavily on levels of capital market integration (and vice versa).

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1 Introduction

The textbook neoclassical growth model remains a key benchmark for thinking about cross-country income dynamics. In the closed-economy version of this model, each country converges along a transition path to its own steady-state level of income per capita, as determined by its production technology and preference parameters. In open economy versions of this model, strong assumptions are typically made about substitutability in goods and capital markets. Often goods are assumed to be homogeneous across countries, or trade between countries is assumed to be costless, whereas conventional quantitative trade models feature both imperfect substitutability across countries and trade frictions. Similarly, capital is often assumed to be homogeneous, which with competitive markets implies perfectly elastic flows of capital between countries to arbitrage away differences in rates of return.

We generalize the closed-economy neoclassical growth model (CNGM) to allow for costly trade and capital flows with imperfect substitutability between countries. We simultaneously model international goods trade, international capital allocations at a point in time, and intertemporal savings decisions over time, and hence the current account. We show that our framework rationalizes key features of the observed data, sheds new light on the determinants of income convergence, and is well suited for understanding the counterfactual impact of policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling).

In goods markets, our model rationalizes the well-known empirical finding that international trade flows are closely approximated by a gravity equation, such that bilateral trade flows are increasing in measures of importer and exporter size, and decreasing in measures of bilateral trade frictions. In capital markets, our model generates a similar gravity equation for bilateral capital holdings, which we show again provides a close approximation to the data. More broadly, our framework yields deterministic predictions for both gross and net capital holdings, explains why gross capital holdings are substantially larger than net positions, and can generate net capital flows from capital-scarce to capital-abundant countries.

Incorporating imperfect substitutability in goods and capital markets has important implications for the speed of income convergence to steady-state. We consider a setting in which a representative agent in each country decides how much of a final consumption good to consume and invest. In this setting, the speed of convergence to steady-state depends on how the real return to investment varies with the initial level of wealth, where this real return equals the nominal return deflated by the consumption price index. In the CNGM, this real return is monotonically decreasing in the initial level of capital, because of diminishing marginal physical productivity of capital in the production technology. Furthermore, initial wealth is equal to the initial capital stock, because of autarky in capital markets. Therefore, higher initial wealth necessarily translates into a lower real return to investment, at a rate that is fully determined by the labor share in the case of a Cobb-Douglas production technology.

Opening the CNGM to *only* free trade in goods *raises* the speed of convergence to steady-state for all countries. With capital autarky, each country's wealth equals its capital stock. With free trade, the consumption price index takes the same value across all countries. But the price of the good produced by each country in general differs from the consumption price index, because countries' goods are imperfect substitutes. In the resulting environment, domestic wealth accumulation expands a country's capital stock, which leads to higher output of its good, and hence a decline in the price of this good relative to the consumption index. As a result, domestic wealth accumulation not only leads to diminishing marginal physical productivity as in the closed economy, but also leads to a fall in the price of the domestic good, thereby implying a larger decline in the real return to investment, and faster convergence to steady-state.

Opening the CNGM to *only* free capital flows also *raises* the speed of convergence to steady-state for all countries. With trade autarky, the consumption price index in each country equals the price of its domestic good. With free capital flows, capital reallocates between countries to equalize the nominal return to investment, but the real return to investment can differ across countries, because of variation in the consumption price index. Wealth accumulation expands a country's investments at home and abroad, which increases its income and hence its expenditure on domestic goods, thereby bidding up domestic factor prices and the domestic consumption price index. This higher domestic consumption price index reduces the real return to investment, thereby again implying faster convergence to steady-state.

In contrast, opening the CNGM to *both* free trade and free capital *slows* the speed of convergence to steady-state for all countries. With both free trade and free capital flows, the real return to investment is equalized across countries, and is therefore uncorrelated with countries' initial levels of wealth. All countries accumulate wealth at the same rate, as determined by this common real return to investment, and initial differences in wealth persist forever, as the world economy gradually converges to steady-state. More generally, outside these limiting cases, goods and capital market integration interact, such that further trade or capital market integration can either *raise* or *reduce* the speed of convergence, depending on the initial level of trade and capital market integration.

We use our framework to evaluate the impact of counterfactual policies that inherently involve disintegration in both goods and capital markets, such as a decoupling of China and the United States. In the neoclassical growth model with open trade but capital autarky, the conventional static welfare gains from trade integration are magnified by dynamic welfare gains from capital accumulation. The fall in the consumption goods price index from reductions in goods trade frictions raises the real return to investment in each country, which increases the rate of growth along the transition path, and the level of income per capita in steady-state. In the neoclassical growth model with open trade and capital flows, the static and dynamic effects of trade integration are considerably more subtle. Reductions in trade frictions lead to a reallocation of capital across countries, which affects income and consumption goods price indexes, and hence the static welfare gains from trade in goods. This change in consumption goods price indexes in turn feeds back to influence the real return to investment and the dynamic welfare gains from capital accumulation along the transition path to steady-state. Similarly, the static and dynamic effects of capital market frictions depend heavily on goods market openness, highlighting the importance of jointly modelling these two dimensions of international integration.

Our framework accommodates a number of existing models as special cases. As frictions in both goods and capital markets become prohibitive, we obtain the conventional CNGM. As goods and capital both become perfectly substitutable, we obtain the limiting case of an open-economy neoclassical growth model, with perfectly elastic flows of capital between countries that arbitrage away differences in rates of return. As the representative agent becomes infinitely impatient and the labor share converges to one, we obtain a standard quantitative model of international trade, in which trade is balanced in each country. Outside of these special cases, we demonstrate rich interactions between countries in goods and capital markets at a point in time, and intertemporal substitution between consumption and saving over time. These interactions shape both worldwide growth rates along the transition path and steady-state levels of income per capita.

We show how to quantify our model using readily-available data from national accounts, bilateral trade in goods, and bilateral capital holdings. We suppose that we observe the world economy somewhere along a transition path to an unobserved initial steady-state with time-invariant fundamentals. Given these observed data, we show how to undertake dynamic exact-hat algebra counterfactuals, given only the observed endogenous variables in the data. We also show how to invert the non-linear model to recover the fundamentals that rationalize these observed data as an equilibrium: goods productivity; investment productivity; trade frictions; and capital market frictions. By conditioning on the observed data, we are able to undertake this model inversion without making any assumptions about where the economy is relative to the initial steady-state, or about agents' expectations about future fundamentals. Linearizing the model around the initial steady-state, we derive a closed-form solution for the economy's transition path in response to shocks to fundamentals. We use this linearization to provide an analytical characterization of the determinants of the speed of convergence to steady-state and to exactly decompose economic growth in each country into the contributions of initial conditions and shocks to domestic and foreign fundamentals.

Our paper is related to a number of different strands of research. First, we connect with the large literature in macroeconomics on the CNGM following Ramsey (1928), Solow (1956) and Swan (1956). This CNGM's prediction of conditional convergence in income per capita finds strong empirical support in the cross-country growth literature following Barro (1991) and Mankiw et al. (1992). One quantitative challenge is that empirical estimates of income convergence imply lengthy transitions to steady-state. In specifications with an endogenous savings rate, King and Rebelo (1993) argues that such lengthy transitions require implausibly low intertemporal elasticities of substitution. In response, a number of studies have explored extensions that generate slower convergence, including installation costs in Rappaport (2006), financial frictions in Barro et al. (1995) and multiple sectors in Buera et al. (2021).

A small number of papers have developed versions of the neoclassical growth model with open goods markets, while maintaining the assumption of autarky in capital markets. Ventura (1997) combines the neoclassical growth model with the factor price equalization theorem of the Heckscher-Ohlin model to rationalize both conditional convergence and episodes of rapid growth by developing countries. Cuñat and Maffezzoli (2004) allow for complete specialization and the resulting departures from factor price equalization. Acemoglu and Ventura (2002) show that specialization and trade can generate a stable world income distribution through terms of trade effects, even without diminishing returns in production. Relative to these studies, we generalize the neoclassical growth model to introduce open capital markets with imperfect substitutability, while also allowing for trade in differentiated goods and trade costs, so as to match the observed gravity equation relationships for goods trade and capital flows.

Second, our work is related to research in international trade. We consider the class of constant elasticity trade models, which includes differentiation by country of origin (Armington 1969), Ricardian technology differences (Eaton and Kortum 2002) and horizontally-differentiated firm varieties and increasing returns to scale (Krugman 1980 and Melitz 2003 with a Pareto distribution), as examined in Arkolakis et al. (2012). A key implication of these models is that bilateral trade exhibits a gravity equation, as highlighted in Anderson and van Wincoop (2003) and Head and Mayer (2014). Manipulating the conditions for general equilibrium in these static international trade models, Kleinman et al. (2020) derive sufficient conditions for the impact of foreign productivity shocks on domestic welfare. Kleinman et al. (2023) introduce capital accumulation into a dynamic model of migration within countries. But capital markets are assumed to be autarkic in each location and a separation is assumed between workers (who live hand to mouth) and capitalists (who can save) in order to tractably model migration.

Much of the quantitative international trade literature assumes exogenous trade imbalances, although Ju et al. (2014), Reyes-Heroles (2016) Eaton et al. (2016) and Ravikumar et al. (2019) endogenize these imbalances following the intertemporal approach of Obstfeld and Rogoff (1996). A related line of research examines the relationship between trade and growth through capital accumulation, including Anderson et al. (2015), Alvarez (2017) and Mutreja et al. (2018). Compared to these studies, we simultaneously model imperfect substitutability and frictions in goods and capital markets at a point in time and consumption-savings decisions over time.

Third, our analysis relates to several lines of research in international finance and macroeconomics. A first group of studies examines the origins of global imbalances, the exorbitant privilege of the United States, and the reasons why capital does not flow from from rich to poor countries, including Lucas (1990), Jin (2012), Gourinchas and Rey (2007), Gourinchas and Jeanne (2006, 2013), Maggiori et al. (2020), Auclert et al. (2020), Coppola et al. (2021), Davis et al. (2021), and Atkeson et al. (2022). A second series of studies examines imperfect substitutability in capital markets, including Koijen and Yogo (2019, 2020), Auclert et al. (2022) and Maggiori (2021). A third line of work evaluates the international propagation of shocks through goods and capital markets, including Backus et al. (1992), Kose et al. (2003) and Huo et al. (2019). A fourth vein of research explores home bias and the international diversification of risk, including Cole and Obstfeld (1991), Obstfeld (1994), Martin and Rey (2004, 2006), Mendoza et al. (2009), Pellegrino et al. (2021), Jiang et al. (2022), Chau (2022), Hu (2022) and Kucheryavyy (2022). A fifth body of papers provides evidence that the gravity equation provides a good approximation to international capital flows, as in Portes and Rey (2005). Relative to this research, we incorporate costly trade and capital flows with imperfect substitutability into the CNGM, and study the implications for growth along the transition path and steady-state income per capita.

The remainder of the paper is structured as follows. Section 2 develops our theoretical framework. Section 3 introduces our data and undertakes our quantitative analysis. Section 4 summarizes our conclusions.

2 Theoretical Framework

We consider an economy that consists of many countries indexed by $n \in \{1, \dots, N\}$. Time is discrete and indexed by $t \in \{1, \dots, \infty\}$. Each country supplies a differentiated good that is produced using labor and capital under constant returns to scale. Markets are perfectly competitive. The representative agent in country n is endowed with a mass ℓ_n of labor.

At the beginning of each period t, this representative agent inherits a stock of wealth (a_{nt}) that can be accumulated using the local consumption good. This stock of wealth (a_{nt}) is the aggregation of the wealth invested in each country (a_{nit}). These investments are subject to idiosyncratic productivity shocks and capital market frictions. At the beginning of period t, wealth is allocated across countries. At the beginning of period $t + 1$, investment returns are realized, depreciation occurs, and wealth is again allocated across countries. We assume that agents have perfect foresight for all aggregate variables.

Throughout the paper, we use bold math font to denote a vector (lowercase letters) or matrix (uppercase letters). We summarize the main features of the model's economic environment in Table 1 below. The derivations for all expressions and results in this section are reported in the Online Appendix.

2.1 Intertemporal Problem

The representative consumer in each country chooses current consumption and saving to maximize her intertemporal utility. We assume that intertemporal utility takes the constant relative risk aversion (CRRA) form:

$$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s} \left(\prod_{u=0}^{s} \phi_{nt+u} \right) \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}, \tag{1}$$

where β is the discount rate; ϕ_{nt+s} is a discount factor shock at time $t+s$, which introduces a wedge into the Euler equation, and helps to match the fluctuations in current account imbalances observed in the data; c_{nt} is a consumption index that depends on the consumption of the goods produced by each country; and ψ is the intertemporal elasticity of substitution.

The representative consumer's period-by-period budget constraint requires that the value of consumption in period t plus the value of period $t+1$ wealth is equal to income from period t wealth net of depreciation plus labor income:

$$\text{s.t. } p_{nt} c_{nt} + p_{nt} \sum_{i=1}^{N} a_{nit+1} = \left(p_{nt} \left(1-\delta\right) + v_{nt} \right) \sum_{i=1}^{N} a_{nit} + w_{nt} \ell_n, \tag{2}$$

where p_{nt} is the price index dual to the consumption index; δ is the rate of depreciation; v_{nt} is the realized return to investment from source country n; in equilibrium, this realized return is the same across all host countries i ($v_{nit} = v_{nt}$ for all i); and w_{nt} is the wage.

Given an investment of a unit of the consumption bundle at the beginning of period $t-1$, the representative consumer receives $(1-\delta)$ units of the consumption bundle back at the beginning of period t and a return from the investment of v_{nt} units of the numeraire. Therefore, the gross nominal return from the investment made at the beginning of period $t-1$ is:

$$\mathcal{R}_{nt}^{nom} = \frac{p_{nt} \left(1-\delta\right) + v_{nt}}{p_{nt-1}}. \tag{3}$$

Dividing by the rate of inflation, the gross real return to the investment is:

$$\mathcal{R}_{nt} = \frac{\mathcal{R}_{nt}^{nom}}{p_{nt}/p_{nt-1}} = 1 - \delta + \frac{v_{nt}}{p_{nt}}. \tag{4}$$

Denoting country n's total wealth at period t by $a_{nt} \equiv \sum_{i=1}^{N} a_{nit}$, we can re-write the period-by-period budget constraint (2) as:

$$c_{nt} + a_{nt+1} = \mathcal{R}_{nt} a_{nt} + \frac{w_{nt} \ell_n}{p_{nt}}. \tag{5}$$

Table 1: Economic Environment

Production	
Production technology	$y_{it} = z_{it}\left(\frac{\ell_{it}}{\mu}\right)^{\mu}\left(\frac{k_{it}}{1-\mu}\right)^{1-\mu}$
Bilateral trade frictions	$\tau_{nit} \geq 1$
Intertemporal Preferences and Capital Accumulation	
Utility function	$u_{nt} = \sum_{s=0}^{\infty} \beta^{t+s}\left(\prod_{u=0}^{s} \phi_{nt+u}\right) \frac{c_{nt+s}^{1-1/\psi}}{1-1/\psi}$
Budget constraint	$p_{nt}c_{nt} + p_{nt}a_{nt+1} = \left(p_{nt}\left(1-\delta\right) + v_{nt}\right)a_{nt} + w_{nt}\ell_{n}$
Wealth	$a_{nt} = \sum_{i=1}^{N} a_{nit}$
Investment return	$v_{nt} = \gamma\left[\sum_{h=1}^{N}\left(\eta_{ht}r_{ht}/\kappa_{nht}\right)^{\epsilon}\right]^{\frac{1}{\epsilon}}$
Capital market frictions	$\kappa_{nit} \geq 1$
Intratemporal Preferences	
Consumption index	$c_{nt} = \left[\sum_{i=1}^{N}\left(c_{nit}\right)^{\frac{\theta}{\theta+1}}\right]^{\frac{\theta+1}{\theta}}$
Goods Market Clearing	
Goods market clearing	$y_{it} = \sum_{n=1}^{N} c_{nit} + \sum_{n=1}^{N} g_{nit}$

Note: Preferences, production technology and resource constraints; $\theta = \sigma - 1 > 0$ is the trade elasticity, as determined by the elasticity of substitution (σ); ϵ is the capital elasticity; g_{nit} denotes the use for investment in country n of the consumption good produced by country i at time t; and all other variables are defined in the main text.

Using this representation, the consumer's problem can be solved in two stages. First, she chooses how much to consume and save. Second, she chooses how much of her wealth to allocate to each country. From equations (1) and (5), the first of these two decisions for consumption-saving takes the same form as in Angeletos (2007). Therefore, optimal consumption and saving are linear functions of current period wealth:

$$c_{nt} = \varsigma_{nt}\left(\mathcal{R}_{nt}a_{nt} + \frac{w_{nt}\ell_{n}}{p_{nt}} + h_{nt}\right), \tag{6}$$

where $h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s}\ell_{nt+s}/p_{nt+s}}{\prod_{u=1}^{s}\mathcal{R}_{nt+u}}$ is the present discounted value of labor income measured in consumption units, and the saving rate $(1-\varsigma_{nt})$ is defined recursively as:

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi}\phi_{nt+1}^{\psi}\mathcal{R}_{nt+1}^{\psi-1}\varsigma_{nt+1}^{-1}, \tag{7}$$

as shown in Online Appendix C.

2.2 Intratemporal Wealth Allocation

We now turn to the second wealth allocation decision. We assume that each unit of investment from source country n is subject to an idiosyncratic productivity shock for each of the possible host countries i to which it can be allocated (φ_{nit}). Investments also face capital market frictions, such that $\kappa_{nit} \geq 1$ units of assets from source country n must be invested in host country i in order for one unit to available for production, where $\kappa_{nnt} = 1$ and $\kappa_{nit} > 1$ for $n \neq i$.

Therefore, each unit of wealth allocated from source n to host i becomes $\varphi_{nit}/\kappa_{nit}$ efficiency units that can be used for production, where each efficiency unit earns a rental rate r_{it}. The realized rate of return in country n from investing one unit of wealth in country i is thus $\varphi_{nit} r_{it}/\kappa_{nit}$. We assume that these idiosyncratic shocks to the productivity of capital are drawn independently across source and host countries from the following Fréchet distribution:

$$F_{nit}(\varphi) = e^{-(\varphi/\eta_{it})^{-\epsilon}}, \qquad \eta_{it} > 0, \qquad \epsilon > 1, \tag{8}$$

where the scale parameter (η_{it}) determines the average productivity of investments in host country i, which can depend for example on host country institutions, such as the protection of property rights. The shape parameter (ϵ) controls the dispersion of idiosyncratic productivity shocks, and regulates the sensitivity of wealth allocations to rates of return relative to idiosyncratic productivity shocks.

The first key implication of our extreme value specification for these idiosyncratic productivity shocks is that the share of wealth from source country n that is invested in host country i satisfies the following gravity equation:

$$b_{nit} = \frac{a_{nit}}{a_{nt}} = \frac{(\eta_{it} r_{it}/\kappa_{nit})^{\epsilon}}{\sum_{h=1}^{N} (\eta_{ht} r_{ht}/\kappa_{nht})^{\epsilon}}. \tag{9}$$

Therefore, bilateral capital holdings (b_{nit}) are decreasing in bilateral capital frictions (κ_{nit}). But these bilateral capital holdings (b_{nit}) also depend on capital frictions with other locations ("multilateral resistance"), as captured by the denominator. We refer to ϵ as the capital elasticity, because it controls the elasticity of capital holdings (b_{nit}) to rental rates (r_{it}), and plays a similar role in capital markets as the trade elasticity (θ) in goods markets.

This specification rationalizes a number of the observed features of international capital holdings that are discussed in Obstfeld and Rogoff (2000). First, it is consistent with empirical findings that international capital holdings are well approximated by a gravity equation (e.g., Portes and Rey 2005), if bilateral capital frictions (κ_{nit}) are increasing in the bilateral distance between countries. Second, it is in line with empirical findings of home bias in international capital allocations (e.g., French and Poterba 1991), because managing capital is more costly abroad than at home ($\kappa_{nit} > \kappa_{nnt}$ for $n \neq i$). Third, it generates gross capital holdings that are substantially larger than net capital holdings, because each source country holds a positive amount of capital in each host country for positive and finite values of capital productivity (η_{it}) and capital frictions (κ_{nit}). Fourth, it provides a natural explanation for empirical findings of limited capital flows from rich to poor countries (e.g., Lucas 1990), because capital is imperfectly substitutable across locations, and even if poor countries offer higher rental rates (higher r_{it}), they can have lower capital productivity (lower η_{it}) or higher capital frictions (higher κ_{nit}).

The second key implication of our extreme value specification for idiosyncratic productivity shocks is that the expected return to investment from source country n is the same across all host countries i and given by:

$$v_{nit} = v_{nt} = \gamma \left[\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht} \right)^{\epsilon} \right]^{\frac{1}{\epsilon}}, \qquad \gamma \equiv \Gamma \left(\frac{\epsilon - 1}{\epsilon} \right), \tag{10}$$

where $\Gamma(\cdot)$ is the Gamma function.

Intuitively, host countries can differ in terms of the rental rate for capital (r_{it}). But host countries with higher rental rates for capital (r_{it}) attract investments with lower realizations for idiosyncratic productivity (φ_{nit}), such that the expected return conditional on investing in a host country is the same across all possible host countries i for a given source country n ($v_{nit} = v_{nt}$ for all i). With a continuous measure of units of wealth, this common expected return across host countries for a given source country equals the realized return. This expected return to investment can differ across source countries ($v_{nt} \neq v_{it}$), if some source countries have lower capital market frictions to host countries than others ($\kappa_{nht} \neq \kappa_{iht}$). Consequentially, the real return to investment (R_{nt}) can differ across countries along the transition path, though we show below that in steady-state, it is equalized across countries ($R^*_{nt} = R^*_{it}$), where we denote the steady-state value of variables with an asterisk.

Finally, we can solve explicitly for the average productivity of investments from source n in host i conditional on investment occurring ($\overline{\varphi}_{nit}$), which is monotonically decreasing in the share of wealth from source country n invested in host country i (b_{nit}):

$$\overline{\varphi}_{nit} = \gamma \eta_{it} b_{nit}^{-\frac{1}{\epsilon}}. \tag{11}$$

Therefore, the third implication of this specification is that it micro founds a downward-sloping marginal efficiency of investment schedule, as in Keynes (1935). Each source country n experiences diminishing marginal returns from allocating a larger share of its investments to a given host country i (larger b_{nit}), where the rate of these diminishing returns is determined by the dispersion of idiosyncratic productivity shocks (ϵ).

Using the above expression for the average productivity of investment from equation (11), payments for capital used in production can be either written in terms of productivity-adjusted capital (k_{it}) or in terms of unadjusted units of wealth (a_{nit}):

$$r_{it} k_{it} = \sum_{n=1}^{N} v_{nt} a_{nit}, \qquad k_{it} = \sum_{n=1}^{N} \overline{\varphi}_{nit} a_{nit}. \tag{12}$$

2.3 Consumption, Production and Trade

The consumption index (c_{nt}) takes the same form as in the Armington model of international trade and is defined over consumption of the varieties produced by each country i (c_{nit}):

$$c_{nt} = \left[\sum_{i=1}^{N} (c_{nit})^{\frac{\theta}{\theta+1}} \right]^{\frac{\theta+1}{\theta}}, \qquad \theta = \sigma - 1, \qquad \sigma > 1, \tag{13}$$

where $\theta = \sigma - 1$ is the trade elasticity and $\sigma > 1$ is the elasticity of substitution between varieties.

Using the properties of CES demand, the share of importer n's expenditure on exporter i takes the conventional form:

$$s_{nit} = \frac{p_{nit}^{-\theta}}{\sum_{h=1}^{N} p_{nht}^{-\theta}}. \tag{14}$$

Each country's variety is produced using labor and capital according to a constant returns to scale Cobb-Douglas production technology. Production occurs under conditions of perfect competition. Varieties can be traded between locations subject to iceberg variable trade costs, where $\tau_{nit} \geq 1$ units of a variety must be shipped from country i in order for one unit to arrive in country n, where $\tau_{nnt} = 1$ and $\tau_{nit} > 1$ for $n \neq i$.

Profit maximization and zero profits imply that the price to a consumer in country n of sourcing the variety supplied by country i is given by:

$$p_{nit} = \frac{\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i}}{z_{it}}, \qquad 0 < \mu_i < 1, \tag{15}$$

where w_{it} is the wage; r_{it} corresponds to the rental rate per effective unit of capital; and z_{it} denotes country productivity.

Substituting the equilibrium pricing rule (15) into the CES expenditure share (14), the model also rationalizes empirical findings that bilateral international trade is well approximated by a gravity equation. Therefore, bilateral trade flows are decreasing in bilateral trade frictions, and increasing in measures of multilateral resistance.

The price index (p_{nt}) dual to the consumption index (c_{nt}) is given by:

$$p_{nt} = \left[\sum_{i=1}^{N} p_{nit}^{-\theta} \right]^{-\frac{1}{\theta}}. \tag{16}$$

Applying Shephard's Lemma to the unit cost function, total payments for the capital used in country i are proportional to the total wagebill in that country:

$$\sum_{n=1}^{N} v_{nt} a_{nit} = r_{it} k_{it} = \frac{1-\mu_i}{\mu_i} w_{it} \ell_{it}. \tag{17}$$

2.4 Market Clearing

Goods market clearing requires that payments to the factors of production used in a country equal expenditure on the goods produced by it:

$$\left(w_{it}\ell_{it} + \sum_{h=1}^{N} v_{hit}a_{hit} \right) = \sum_{n=1}^{N} s_{nit} \left[p_{nt}c_{nt} + p_{nt}a_{nt+1} - p_{nt} \left(1 - \delta \right) a_{nt} \right], \tag{18}$$

where the term inside the square brackets on the right-hand side is total expenditure on the consumption good in market n at time t for both consumption and net investment.

Using the period-by-period budget constraint (2) and our expression for factor payments in equation (17) above, we can rewrite this equality between income and expenditure as follows:

$$w_{it}\ell_{it} = \mu_i \sum_{n=1}^{N} s_{nit} \left[v_{nt}a_{nt} + w_{nt}\ell_n \right]. \tag{19}$$

We choose world GDP as our numeraire, such that:

$$\begin{aligned} 1 &= \sum_{i=1}^{N} \left(w_{it}\ell_i + \sum_{n=1}^{N} v_{nit}a_{nit} \right), \\ &= \sum_{i=1}^{N} \frac{1}{\mu_i} w_{it}\ell_i. \end{aligned} \tag{20}$$

2.5 Balance of Payments

We now use our framework to illustrate the conventional balance of payments accounting identities. The financial account (FA_{it}) is defined as the increase in foreign assets in country i minus the increase in country i's assets abroad:

$$FA_{it} = \underbrace{\left(\sum_{n=1}^{N} p_{nt}a_{nit+1} - \sum_{n=1}^{N} p_{nt-1}a_{nit} \right)}_{\text{Increase in foreign assets in country } i} - \underbrace{\left(p_{it}a_{it+1} - p_{it-1}a_{it} \right)}_{\text{Increase in country } i\text{'s assets abroad}}. \tag{21}$$

Trade balance (TB_{it}) corresponds to the difference between the value of goods produced in a country and the value of goods used in that country:

$$TB_{it} = \underbrace{w_{it}\ell_i + \sum_{n=1}^{N} v_{nt}a_{nit}}_{\text{Value of goods produced}} - \underbrace{\left(p_{it}c_{it} + \sum_{n=1}^{N} p_{nt}a_{nit+1} - (1-\delta) \sum_{n=1}^{N} p_{nt}a_{nit} \right)}_{\text{Value of goods used in the country}}. \tag{22}$$

Net investment income (NII_{it}) is the difference between income receipts from assets owned by country i minus income payments on foreign-owned assets used in country i:

$$\text{NII}_{it} = \underbrace{\left(\mathcal{R}_{it}^{Nom} - 1\right) p_{it-1} a_{it}}_{\text{Income receipts from assets owned}} - \underbrace{\sum_{n=1}^{N} \left(\mathcal{R}_{nt}^{Nom} - 1\right) p_{nt-1} a_{nit}}_{\text{Income payments to foreign-owned assets}} . \tag{23}$$

Combining these definitions in equations (21)-(23), we confirm that the conventional balance of payments accounting identity holds:

$$CA_{it} = TB_{it} + NII_{it} = -FA_{it}. \tag{24}$$

2.6 General Equilibrium

Given the wealth state variables $\{a_{nt}\}_{n=1}^{N}$, the equilibrium endogenous variables in the static trade and cross-country capital allocation bloc of the model $\{w_{nt}, r_{nt}, s_{nt}, v_{nt}, b_{nt}\}_{n=1}^{N}$ are determined as the solution to the following system of equations:

$$s_{nit} = \frac{\left(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it}\right)^{-\theta}}{\sum_{h=1}^{N} \left(\tau_{nht} w_{ht}^{\mu_h} r_{ht}^{1-\mu_h} / z_{ht}\right)^{-\theta}}, \tag{25}$$

$$w_{it}\ell_i = \mu_i \sum_{n=1}^{N} s_{nit} \left(v_{nt} a_{nt} + w_{nt}\ell_n\right), \tag{26}$$

$$b_{nit} = \frac{\left(\eta_{it} r_{it} / \kappa_{nit}\right)^{\epsilon}}{\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}}, \tag{27}$$

$$v_{nt} = \gamma \left[\sum_{h=1}^{N} \left(\eta_{ht} r_{ht} / \kappa_{nht}\right)^{\epsilon}\right]^{1/\epsilon}, \tag{28}$$

$$\sum_{n=1}^{N} v_{nt} b_{nit} a_{nt} = \frac{1-\mu_i}{\mu_i} w_{it}\ell_i, \tag{29}$$

along with the choice of numeraire:

$$\sum_{i=1}^{N} \frac{1}{\mu_i} w_{it}\ell_i = 1. \tag{30}$$

The evolution of the state variables $\{a_{nt}\}_{n=1}^{N}$ over time is determined by optimal consumption-saving decisions according the following dynamic bloc of equations:

$$a_{nt+1} = (1 - \varsigma_{nt}) \left(\mathcal{R}_{nt} a_{nt} + \frac{w_{nt}\ell_n}{p_{nt}} + h_{nt}\right) - h_{nt}, \tag{31}$$

$$h_{nt} \equiv \sum_{s=1}^{\infty} \frac{w_{nt+s}\ell_{nt+s}/p_{nt+s}}{\prod_{u=1}^{s} \mathcal{R}_{nt+u}}, \tag{32}$$

$$p_{nt} \equiv \left[\sum_{i=1}^{N} \left(\tau_{nit} w_{it}^{\mu_i} r_{it}^{1-\mu_i} / z_{it} \right)^{-\theta} \right]^{-1/\theta}, \tag{33}$$

where

$$\mathcal{R}_{nt} = 1 - \delta + v_{nt}/p_{nt}, \tag{34}$$

and ς_{nt} is defined recursively as

$$\varsigma_{nt}^{-1} = 1 + \beta^{\psi} \phi_{nt+1}^{\psi} \mathcal{R}_{nt+1}^{\psi-1} \varsigma_{nt+1}^{-1}. \tag{35}$$

2.7 Trade and Capital Share Matrices

We now introduce the trade and capital share matrices, and the labor and capital income vectors, which we use to characterize the evolution of the world income distribution over time. To reduce notational clutter, we suppress the time subscript throughout this subsection.

Let $\boldsymbol{S}$ be the $N \times N$ matrix with the ni-th element equal to importer n's expenditure on exporter i ($S_{ni} \equiv [s_{ni}]$) . Let $\boldsymbol{T}$ be the $N \times N$ matrix with the in-th element equal to the fraction of income that exporter i derives from selling to importer n ($T_{in} \equiv \frac{s_{ni}(v_n a_n + w_n \ell_n)}{\sum_{h=1}^{N} s_{hi}(v_h a_h + w_h \ell_h)}$). We refer to $\boldsymbol{S}$ as the *expenditure share* matrix and to $\boldsymbol{T}$ as the *income share* matrix. Intuitively, S_{ni} captures the importance of i as a supplier to location n, and T_{in} captures the importance of n as a buyer for country i. Note the order of subscripts: in matrix $\boldsymbol{S}$, rows are buyers and columns are suppliers, whereas in matrix $\boldsymbol{T}$, rows are suppliers and columns are buyers.

Similarly, let $\boldsymbol{B}$ be the $N \times N$ matrix with the ni-th element equal to the share of source country n's wealth allocated to host country i ($B_{ni} \equiv [b_{ni}]$). Let $\boldsymbol{X}$ be the $N \times N$ matrix with the in-th element equal to the share of capital income in host country i paid to source country n ($X_{in} \equiv \frac{v_n b_{ni} a_n}{\sum_{h=1}^{N} v_h b_{hi} a_h}$). We refer to $\boldsymbol{B}$ as the *portfolio share* matrix and to $\boldsymbol{X}$ as the *payment share* matrix. Intuitively, B_{ni} captures the importance of i as a host for capital investments from source n, and X_{in} captures the importance of n as a source of capital investments to host i. Again note the order of subscripts: in matrix $\boldsymbol{B}$, rows are sources and columns are hosts, whereas in matrix $\boldsymbol{X}$, rows are hosts and columns are sources.[1]

Finally, let $\boldsymbol{q}$ be the $N \times 1$ vector of labor income with the n-th element equal to the labor income of country n ($q_n \equiv w_n \ell_n$), and let ζ be the $N \times 1$ vector of capital income with the n-th element equal to the capital income of country n ($\zeta_n \equiv v_n a_n$).

[1]For theoretical completeness, we maintain two assumptions on these matrices, which are satisfied empirically in all years of our data. First, we assume that the $\boldsymbol{S}$ and $\boldsymbol{B}$ matrices are irreducible, such that all locations are connected directly or indirectly by trade flows and capital holdings: For any i, n, there exists k such that $\left[\boldsymbol{S}^k\right]_{in} > 0$ and $\left[\boldsymbol{B}^k\right]_{in} > 0$. Second, we assume that each location consumes a positive amount of domestic goods and allocates a positive share of capital domestically: For all i, $\boldsymbol{S}_{ii} > 0$ and $\boldsymbol{B}_{ii} > 0$.

2.8 Steady-state Equilibrium

The steady-state equilibrium of the model is characterized by time-invariant values of the state variables $\{a_n^*\}_{n=1}^N$ and the other endogenous variables of the model $\{w_n^*, r_n^*, s_{ni}^*, v_{nt}^*, b_{ni}^*\}_{n=1}^N$, given time-invariant values of country fundamentals $\{\ell_n, z_n, \eta_n\}_{n=1}^N$ and $\{\tau_{ni}, \kappa_{ni}\}_{n,i=1}^N$, where recall that we denote the steady-state values of endogenous variables by an asterisk.

Given constant population in each country (ℓ_n), diminishing marginal physical productivity of capital in the production technology implies a steady-state level of wealth (a_n^*), as in the traditional Solow-Swan Model. Unlike that Solow-Swan model, the saving rate here is endogenously determined as the solution to a forward-looking consumption-saving problem. As a result, the steady-state gross real return to investment ($\mathcal{R}_n^*$) and the steady-state saving rate (ς_n^*) are inversely related to discount factor (β):

$$\mathcal{R}_n^* = \frac{1}{\beta}, \qquad \varsigma_n^* = 1 - \beta. \tag{36}$$

This common steady-state value of the gross real return to investment (R_n^*) implies that the steady-state realized real return to investment (v_n^*/p_n^*) is the same across all countries:

$$\frac{v_n^*}{p_n^*} = \beta^{-1} - 1 + \delta. \tag{37}$$

2.9 Transition Dynamics

As in the conventional closed-economy neoclassical growth model, our open-economy framework features conditional convergence in income per capita, in the sense that each country converges to its own steady-state level of income per capita. In contrast to this conventional framework, each country's steady-state level of income per capita and its growth rate along the transition path are influenced by shocks to fundamentals in other countries around the globe, where these fundamentals comprise trade frictions (τ_{ni}), capital market frictions (κ_{ni}), goods productivity (z_i), and capital productivity (η_i).

In Subsection 2.9.1, we show that we can solve for the dynamic response of the economy to fundamental shocks in the non-linear model using dynamic exact-hat algebra techniques. In Subsection 2.9.2, we linearize the general equilibrium conditions of the model to obtain a closed-form solution for the transition path of the global economy in response to these fundamental shocks. In Subsection 2.9.3, we use this linearization to quantify the contributions of convergence and fundamental shocks to the evolution of the world income distribution. In Subsection 2.9.4, we undertake a spectral analysis to provide an analytical characterization of the speed of convergence to steady-state. Finally, in Subsection 2.9.5, we use our closed-form solution to analyze the role of goods and capital market integration in shaping the speed of convergence.

2.9.1 Dynamic Exact-Hat Algebra

We suppose that we observe the world economy somewhere along the transition path towards an unobserved steady state. Given the initial observed endogenous variables of the model, we show that we are able to solve for the economy's transition path in time differences ($\dot{x}_{it+1} = x_{it+1}/x_{it}$) for any anticipated convergent sequence of future changes in fundamentals, without having to solve for the initial level of fundamentals.

Proposition 1. ***Dynamic Exact Hat Algebra.*** *Given observed initial populations $\{\ell_{i0}\}_{i=1}^{N}$, an initial observed allocation of the economy, ($\{a_{i0}\}_{i=1}^{N}$, $\{a_{i1}\}_{i=1}^{N}$, $\{S_{ni0}\}_{n,i=1}^{N}$, $\{T_{ni0}\}_{n,i=1}^{N}$, $\{B_{ni0}\}_{n,i=1}^{N}$, $\{X_{ni0}\}_{n,i=1}^{N}$), and a convergent sequence of future changes in fundamentals under perfect foresight:*

$$\left\{ \{\dot{z}_{it}\}_{i=1}^{N}, \{\dot{\eta}_{it}\}_{i=1}^{N}, \{\dot{\tau}_{it}\}_{i,j=1}^{N}, \{\dot{\kappa}_{it}\}_{i,j=1}^{N} \right\}_{t=1}^{\infty},$$

the solution for the sequence of changes in the model's endogenous variables does not require information on the level of fundamentals:

$$\left\{ \{z_{it}\}_{i=1}^{N}, \{\eta_{it}\}_{i=1}^{N}, \{\tau_{it}\}_{i,j=1}^{N}, \{\kappa_{it}\}_{i,j=1}^{N} \right\}_{t=1}^{\infty}.$$

Proof. See Online Appendix F. □

Intuitively, we use the initial observed endogenous variables and the equilibrium conditions of the model to control for the unobserved initial level of fundamentals. Applying this proposition, we can employ dynamic exact-hat algebra methods to solve for the unobserved initial steady state in the absence of any further changes in fundamentals. We can also use this approach to solve counterfactuals for the transition path of the global economy in response to assumed sequences of future changes in fundamentals.

In addition to these dynamic exact-hat algebra results in Proposition 1, we can invert the model to solve for the unobserved changes in goods productivity, capital productivity, trade frictions and capital market frictions that are implied by the observed changes of the endogenous variables of the model under perfect foresight, as shown in Online Appendix G. Importantly, we can undertake this model inversion along the transition path without making assumptions about the precise sequence of future fundamentals, because the observed changes in wealth capture agents' expectations about this sequence of future fundamentals.

2.9.2 Linearization

To further understand the determinants of the speed of convergence to steady-state, we now linearize the model to provide an analytical characterization of the economy's transition path.

We suppose that we observe population ($\boldsymbol{\ell}$), the wealth state variable ($\boldsymbol{a_t}$) for time $t = 0$ and $t = 1$, and the trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) of the economy at time $t = 0$. The economy need not be in steady-state at $t = 0$, but we assume that it is on a convergence path toward a steady-state with constant fundamentals ($\boldsymbol{z}$, $\boldsymbol{\eta}$, $\boldsymbol{\tau}$, $\boldsymbol{\kappa}$). We refer to the steady-state implied by these initial fundamentals as the *initial steady-state.* We use a tilde above a variable to denote a log deviation from this initial steady-state (e.g., $\widetilde{a}_{it+1} = \ln a_{it+1} - \ln a_i^*$).

We begin by totally differentiating the conditions for general equilibrium around this unobserved initial steady-state, holding constant countries' labor endowments. We thus derive the following system of linear equations that fully characterizes the transition path up to first-order:

$$-\frac{1}{\theta}\widetilde{S}_{nit} = \widetilde{\tau}_{nit} + \mu_i \widetilde{w}_{it} + (1-\mu_i)\,\widetilde{r}_{it} - \widetilde{z}_{it} - \sum_h S_{nh}\left(\widetilde{\tau}_{nht} + \mu_i \widetilde{w}_{ht} + (1-\mu_i)\,\widetilde{r}_{ht} - \widetilde{z}_{ht}\right) \tag{38}$$

$$\widetilde{w}_{it} = \sum_{n=1}^{N} T_{in}\left(\widetilde{S}_{nit} + \chi_n\left(\widetilde{v}_{nt} + \widetilde{a}_{nt}\right) + (1-\chi_n)\,\widetilde{w}_{nt}\right) \tag{39}$$

$$\frac{1}{\epsilon}\widetilde{B}_{nit} = \widetilde{\eta}_{it} + \widetilde{r}_{it} - \widetilde{\kappa}_{nit} - \sum_h B_{nh}\left(\widetilde{\eta}_{ht} + \widetilde{r}_{ht} - \widetilde{\kappa}_{nht}\right) \tag{40}$$

$$\widetilde{v}_{nt} = \sum_h B_{nh}\left(\widetilde{\eta}_{ht} + \widetilde{r}_{ht} - \widetilde{\kappa}_{nht}\right) \tag{41}$$

$$\widetilde{r}_{it} + \sum_n X_{in}\left(\widetilde{\eta}_{it} + (1-1/\epsilon)\,\widetilde{B}_{nit} + \widetilde{a}_{nt} - \widetilde{\kappa}_{nit}\right) = \widetilde{w}_{it} \tag{42}$$

$$\sum_i q_i \widetilde{w}_{it} = 0 \tag{43}$$

$$\widetilde{p}_{nt} \equiv \sum_i S_{ni}\left(\widetilde{\tau}_{nit} + \mu_i \widetilde{w}_{it} + (1-\mu_i)\,\widetilde{r}_{it} - \widetilde{z}_{it}\right) \tag{44}$$

$$\widetilde{h}_{nt} = \frac{1-\beta}{\beta}\sum_{s=1}^{\infty}\beta^s\left(\widetilde{w}_{nt+s} - \widetilde{p}_{nt+s}\right) - \frac{1}{\beta}\sum_{s=1}^{\infty}\beta^s \widetilde{\mathcal{R}}_{nt+s}, \tag{45}$$

$$\widetilde{\mathcal{R}}_{nt} = (1-\beta+\beta\delta)\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right) \tag{46}$$

$$\xi_n \widetilde{a}_{nt+1} = -\frac{1-\beta}{\beta}\widetilde{\varsigma}_n + \xi_n\left(\widetilde{\mathcal{R}}_{nt} + \widetilde{a}_{nt}\right) + (1-\xi_n)(1-\beta)\left(\widetilde{w}_{nt} - \widetilde{p}_{nt} - \widetilde{h}_{nt}\right), \tag{47}$$

$$-\widetilde{\varsigma}_{nt} = \beta\left(\psi\widetilde{\phi}_{nt+1} + (\psi-1)\,\widetilde{\mathcal{R}}_{nt+1} - \widetilde{\varsigma}_{nt+1}\right), \tag{48}$$

where we define $\chi_{nt} \equiv \frac{v_{nt}a_{nt}}{w_{nt}\ell_{nt}+v_{nt}a_{nt}}$ as the capital income share and $\xi_{nt} \equiv \frac{a_{nt}}{a_{nt}+h_{nt}}$ as the share of physical capital in the sum of physical and human capital.

In this system of linear equations, there are no terms in the change in the trade and capital share matrices, because these terms are second order in the underlying Taylor-series expansion, involving interactions between the changes in fundamentals and the resulting changes in trade and capital shares. As we consider first-order changes in fundamentals, these second-order, nonlinear terms drop out of the linearization. Therefore, we can write the trade and capital shares on the right-hand side of these equations with no time subscript for first-order changes in fundamentals. In our empirical analysis below, we find similar results from our spectral analysis whether we use the observed trade and capital share matrices or the implied steady-state matrices.

We now show that this system of linearized equations can be reduced to a second-order difference equation in the wealth state variables ($\widetilde{\boldsymbol{a}}_{\boldsymbol{t}}$) and shocks to fundamentals. All other endogenous variables can be recovered as linear functions of these wealth state variables. For expositional convenience, we focus here on the simplest form of fundamental shocks, such that agents at time $t = 0$ learn about a one-time permanent shock to fundamentals from time $t = 1$ onwards. However, analogous results hold for any expected convergent sequence of future shocks to fundamentals under perfect foresight, and for the case in which agents observe an initial shock to fundamentals and form rational expectations about future shocks based on a known stochastic process for fundamentals. We define measures of incoming and outgoing shocks to trade and capital frictions, which aggregate bilateral changes across partner countries, using initial trade and capital share weights: $\widetilde{\tau}_{nt}^{in} \equiv \sum_{i=1}^{N} S_{nit}\widetilde{\tau}_{nit}$, $\widetilde{\tau}_{it}^{out} \equiv \sum_{n=1}^{N} T_{int}\widetilde{\tau}_{nit}$, $\widetilde{\kappa}_{nt}^{out} \equiv \sum_{i=1}^{N} B_{nit}\widetilde{\kappa}_{nit}$, and $\widetilde{\kappa}_{it}^{in} \equiv \sum_{n=1}^{N} X_{int}\widetilde{\kappa}_{nit}$. Using these definitions, we have the following result.

Proposition 2. ***State Variables.*** *Suppose that the economy at time $t = 0$ is on a convergence path toward an initial steady state with constant fundamentals ($\boldsymbol{z}$, $\boldsymbol{\eta}$, $\boldsymbol{\tau}$, $\boldsymbol{\kappa}$). At time $t = 0$, agents learn about one-time, permanent shocks to fundamentals ($\widetilde{\boldsymbol{f}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{bmatrix}'$) from time $t = 1$ onwards. The evolution of the economy's wealth state variables from time $t = 1$ onwards satisfies the following second-order difference equation:*

$$\boldsymbol{\Psi}\widetilde{\boldsymbol{a}}_{t+2} = \boldsymbol{\Gamma}\widetilde{\boldsymbol{a}}_{t+1} + \boldsymbol{\Theta}\widetilde{\boldsymbol{a}}_{t} + \boldsymbol{\Pi}\widetilde{\boldsymbol{f}}, \tag{49}$$

where the matrices ($\boldsymbol{\Psi}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Theta}$, $\boldsymbol{\Pi}$) are functions of the trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) and model parameters (ψ, θ, β, ϵ, μ_i), as defined in Online Appendix H.4.

Proof. See Online Appendix H.4. □

We solve this matrix system of equations using the method of undetermined coefficients following Uhlig (1999) to obtain a closed-form solution for the evolution of the state variables $\{\widetilde{\boldsymbol{a}}_{\boldsymbol{t}}\}_{t=1}^{\infty}$ in terms of an impact matrix ($\boldsymbol{Q}$), which captures the initial impact of the fundamental shocks, and a transition matrix ($\boldsymbol{P}$), which governs the updating of the state variables over time.

Proposition 3. ***Transition Matrix.*** *Suppose that the economy at time $t = 0$ is on a convergence path toward an initial steady state with constant fundamentals ($\boldsymbol{z}$, $\boldsymbol{\eta}$, $\boldsymbol{\tau}$, $\boldsymbol{\kappa}$). At time $t = 0$, agents* *learn about one-time, permanent shocks to fundamentals* ($\widetilde{\boldsymbol{f}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{bmatrix}'$) *from time* $t = 1$ *onwards. There exists a* $N \times N$ *transition matrix* ($\boldsymbol{P}$) *and a* $N \times 6N$ *impact matrix* ($\boldsymbol{R}$) *such that the second-order difference equation system in* (49) *has the closed-form solution:*

$$\widetilde{\boldsymbol{a}}_t = \boldsymbol{P}\widetilde{\boldsymbol{a}}_{t-1} + \boldsymbol{R}\widetilde{\boldsymbol{f}}. \tag{50}$$

The transition matrix $\boldsymbol{P}$ *satisfies:*

$$\boldsymbol{P} = \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^{-1},$$

where $\boldsymbol{\Lambda}$ *is a diagonal matrix of* N *stable eigenvalues* $\{\lambda_k\}_{k=1}^{N}$ *and* $\boldsymbol{U}$ *is a matrix stacking the corresponding* N *eigenvectors* $\{\boldsymbol{u}_k\}_{k=1}^{N}$. *The impact matrix* ($\boldsymbol{R}$) *is given by:*

$$\boldsymbol{R} = \left(\boldsymbol{\Psi}\boldsymbol{P} + \boldsymbol{\Psi} - \boldsymbol{\Gamma}\right)^{-1}\boldsymbol{\Pi},$$

where ($\boldsymbol{\Psi}$, $\boldsymbol{\Gamma}$, $\boldsymbol{\Theta}$, $\boldsymbol{\Pi}$) *are the matrices from the system of second-order difference equations* (49).

Proof. See Online Appendix H.5. □

The solutions for the impact and transition matrices ($\boldsymbol{R}$, $\boldsymbol{P}$) depend only on the trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) and parameters (ψ, θ, β, ϵ, μ_i). Given this closed-form solution for the transition path of the state variables $\{\widetilde{\boldsymbol{a}}_t\}_{t=1}^{\infty}$, we can recover all other endogenous variables as linear functions of these state variables, as shown in Online Appendix H.2.

2.9.3 Convergence Dynamics Versus Fundamental Shocks

Using Proposition 3, the transition path of the economy's state variables can be additively decomposed into the contributions of convergence dynamics given initial conditions and fundamental shocks. Applying equation (50) across time periods, we obtain:

$$\ln \boldsymbol{a}_t - \ln \boldsymbol{a}_{-1} = \underbrace{\sum_{s=0}^{t} \boldsymbol{P}^s \left(\ln \boldsymbol{a}_0 - \ln \boldsymbol{a}_{-1}\right)}_{\substack{\text{convergence given} \\ \text{initial fundamentals}}} + \underbrace{\sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R}\widetilde{\boldsymbol{f}}}_{\substack{\text{dynamics from} \\ \text{fundamental shocks}}} \qquad \text{for all } t \geq 1. \tag{51}$$

In the absence of shocks to fundamentals ($\widetilde{\boldsymbol{f}} = \boldsymbol{0}$), the second term on the right-hand side of equation (51) is zero. In this case, the evolution of the state variables is shaped solely by convergence dynamics given initial conditions, and converges over time to:

$$\ln \boldsymbol{a}^*_{\text{initial}} = \lim_{t\to\infty} \ln \boldsymbol{a}_t = \ln \boldsymbol{a}_{-1} + \left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1}\left(\ln \boldsymbol{a}_0 - \ln \boldsymbol{a}_{-1}\right), \tag{52}$$

where $\left(\boldsymbol{I} - \boldsymbol{P}\right)^{-1} = \sum_{s=0}^{\infty} \boldsymbol{P}^s$ is well-defined under the condition that the spectral radius of $\boldsymbol{P}$ is smaller than one.

In contrast, if the economy is initially in a steady-state at time 0, the first term on the right-hand side of equation (51) is zero. In this case, the transition path of the state variables is solely driven by the second term for fundamental shocks, and follows:

$$\widetilde{\boldsymbol{a}}_t = \ln \boldsymbol{a}_t - \ln \boldsymbol{a}_0 = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R} \widetilde{\boldsymbol{f}} = \left(\boldsymbol{I} - \boldsymbol{P}^t\right) (\boldsymbol{I} - \boldsymbol{P})^{-1} \boldsymbol{R} \widetilde{\boldsymbol{f}} \quad \text{for all } t \geq 1. \tag{53}$$

In period $t = 1$ when the shocks occur, the response of the state variables is $\widetilde{\boldsymbol{a}}_1 = \boldsymbol{R}\widetilde{\boldsymbol{f}}$. Taking the limit as $t \to \infty$ in equation (53), the comparative steady-state response is:

$$\lim_{t \to \infty} \widetilde{\boldsymbol{a}}_t = \ln \boldsymbol{a}^*_{\text{new}} - \ln \boldsymbol{a}^*_{\text{initial}} = (\boldsymbol{I} - \boldsymbol{P})^{-1} \boldsymbol{R} \widetilde{\boldsymbol{f}}. \tag{54}$$

A key implication of this additive separability in equation (51) is that we can examine the economy's dynamic response to fundamental shocks separately from its convergence towards an initial steady-state with unchanged fundamentals. Therefore, without loss of generality, we focus in the remainder of this section on an economy that is initially in steady-state.

2.9.4 Spectral Analysis of the Transition Matrix $\boldsymbol{P}$

We now provide a further analytical characterization of the economy's dynamic response to shocks using a spectral analysis of the transition matrix. We show that the speed of convergence to steady-state and the evolution of the state variables along the transition path can be written solely in terms of the eigenvalues and eigenvectors of this transition matrix.

Eigendecomposition of the Transition Matrix We begin by using the eigendecomposition of the transition matrix, $\boldsymbol{P} \equiv \boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{V}$, where $\boldsymbol{\Lambda}$ is a diagonal matrix of eigenvalues arranged in decreasing order by absolute values, and $\boldsymbol{V} = \boldsymbol{U}^{-1}$. For each eigenvalue λ_h, the h-th column of $\boldsymbol{U}$ ($\boldsymbol{u_h}$) and the h-th row of $\boldsymbol{V}$ ($\boldsymbol{v'_h}$) are the corresponding right- and left-eigenvectors of $\boldsymbol{P}$, respectively, such that

$$\lambda_h \boldsymbol{u}_h = \boldsymbol{P}\boldsymbol{u}_h, \qquad \lambda_h \boldsymbol{v}'_h = \boldsymbol{v}'_h \boldsymbol{P}.$$

That is, $\boldsymbol{u}_h$ ($\boldsymbol{v}'_h$) is the vector that, when left-multiplied (right-multiplied) by $\boldsymbol{P}$, is proportional to itself but scaled by the corresponding eigenvalue λ_h.[2] We refer to $\boldsymbol{u}_h$ simply as eigenvectors. Both $\{\boldsymbol{u}_h\}$ and $\{\boldsymbol{v}'_h\}$ are bases that span the N-dimensional vector space.

[2]Note that $\boldsymbol{P}$ need not be symmetric. This eigendecomposition exists if the transition matrix has distinct eigenvalues. We construct the right-eigenvectors such that the 2-norm of $\boldsymbol{u}_h$ is equal to 1 for all h, where note that $\boldsymbol{v}'_i \boldsymbol{u}_h = 1$ for $i = h$ and $\boldsymbol{v}'_i \boldsymbol{u}_h = 0$ otherwise.

Eigen-shock We next introduce a particular type of shock to fundamentals that proves useful for characterizing the model's transition dynamics. We define an *eigen-shock* as a shock to fundamentals ($\widetilde{\boldsymbol{f}}_{(h)}$) for which the initial impact of the shock on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$) coincides with a real eigenvector of the transition matrix ($\boldsymbol{u}_h$). Because the space of all fundamental shocks is higher dimensional than the space of the endogenous state variable (wealth), many fundamental shocks generate identical time paths of the impact on the state variables. In fact, for each fundamental shock vector $\widetilde{\boldsymbol{f}}$, there exists a productivity shock vector $\widetilde{\boldsymbol{z}}$ such the time path of the state variables are identical under both shock vectors. For expositional simplicity, we define the eigen-shocks in terms of shocks to productivity ($\widetilde{\boldsymbol{z}}$), setting all other shocks equal to zero.[3] Consequentially, the impact of the eigen-shocks $\left\{\widetilde{\boldsymbol{f}}_{(h)}\right\}_{h=1}^{N}$ form a basis that spans the N-dimensional state space. Each eigenvector of $\boldsymbol{P}$ ($\boldsymbol{u}_h$) has a corresponding eigen-shock for which $\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$.

In general, there is no reason why any vector of empirical shocks to fundamentals in each country should correspond to an eigen-shock. But we can use these eigen-shocks to characterize the impact of any empirical shock using the following two properties. First, we can solve for these eigen-shocks from the observed data, because the impact matrix ($\boldsymbol{R}$) and the transition matrix ($\boldsymbol{P}$) depend solely on our observed trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) and the structural parameters of the model $\{\psi, \theta, \beta, \epsilon, \mu_i\}$. Second, the initial and dynamic impact on the state variables from any vector of empirical shocks to fundamentals ($\widetilde{\boldsymbol{f}}$) can be equivalently expressed as a linear combination of the impact from eigen-shocks ($\widetilde{\boldsymbol{f}}_{(h)}$), where the weights or loadings in this linear combination can be recovered from a linear projection (regression) of the initial impact from the observed shocks ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) on the initial impact from the eigen-shocks ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$). Using this property, the transition path of the state variables in response to any vector of empirical shocks to fundamentals can be expressed solely in terms of the eigenvalues and eigenvectors of the transition matrix, as summarized in the following proposition.

Proposition 4. ***Spectral Analysis.*** *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to fundamentals ($\widetilde{\boldsymbol{f}} \equiv \begin{bmatrix} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{bmatrix}'$) from time $t = 1$ onwards. The transition path of the state variables ($\boldsymbol{a_t}$) can be written as a linear combination of the eigenvalues (λ_h) and eigenvectors ($\boldsymbol{u}_h$) of the transition matrix:*

$$\widetilde{\boldsymbol{a}}_t = \sum_{s=0}^{t-1} \boldsymbol{P}^s \boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{h=1}^{2N} \frac{1-\lambda_h^t}{1-\lambda_h} \boldsymbol{u}_h \boldsymbol{v}_h' \boldsymbol{R}\widetilde{\boldsymbol{f}} = \sum_{h=2}^{2N} \frac{1-\lambda_h^t}{1-\lambda_h} \boldsymbol{u}_h \varrho_h, \tag{55}$$

[3] Recall from Proposition 2 that the dimension of the state space is N, whereas the shocks to fundamentals are of higher dimension, since $\widetilde{\boldsymbol{f}}$ includes shocks to the goods and capital productivities ($\widetilde{\boldsymbol{z}}$, $\widetilde{\boldsymbol{\eta}}$) and aggregations ($\widetilde{\boldsymbol{\kappa}}^{in}, \widetilde{\boldsymbol{\kappa}}^{out}, \widetilde{\boldsymbol{\tau}}^{in}, \widetilde{\boldsymbol{\tau}}^{out}$) of the bilateral shocks to trade and capital frictions. Therefore, defining our eigen-shocks in terms of shocks to productivity ($\widetilde{\boldsymbol{z}}$) ensures that each eigenvector is associated with a unique eigen-shock (up to scale).

where the weights in this linear combination (ϱ_h) can be recovered as the coefficients in a linear projection (regression) of the initial impact from the observed shocks ($\boldsymbol{R}\widetilde{\boldsymbol{f}}$) on the initial impact from the eigen-shocks ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$).

Proof. The proposition follows from the eigendecomposition of the transition matrix: $\boldsymbol{P} \equiv \boldsymbol{U\Lambda V}$, as shown in Online Appendix H.6. $\square$

Another important property of an eigen-shock is that the speed of convergence to steady-state, as measured by the half-life of convergence to steady-state, depends solely on the associated eigenvalue of the transition matrix, as summarized in the following proposition.

Proposition 5. Speed of Convergence. *Consider an economy that is initially in steady-state at time $t = 0$ when agents learn about one-time, permanent shocks to fundamentals ($\widetilde{\boldsymbol{f}} \equiv \left[\begin{array}{cccccc} \widetilde{\boldsymbol{z}} & \widetilde{\boldsymbol{\eta}} & \widetilde{\boldsymbol{\kappa}}^{in} & \widetilde{\boldsymbol{\kappa}}^{out} & \widetilde{\boldsymbol{\tau}}^{in} & \widetilde{\boldsymbol{\tau}}^{out} \end{array}\right]'$) from time $t = 1$ onwards. Suppose that these shocks are an eigen-shock ($\widetilde{\boldsymbol{f}}_{(h)}$), for which the initial impact on the state variables at time $t = 1$ coincides with a real eigenvector ($\boldsymbol{u}_h$) of the transition matrix ($\boldsymbol{P}$): $\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$. The transition path of the state variables ($\boldsymbol{a_t}$) in response to such an eigen-shock ($\widetilde{\boldsymbol{f}}_{(h)}$) is :*

$$\widetilde{\boldsymbol{a}}_t = \sum_{j=2}^{2N} \frac{1-\lambda_j^t}{1-\lambda_j} \boldsymbol{u}_j \boldsymbol{v}_j' \boldsymbol{u}_h = \frac{1-\lambda_h^t}{1-\lambda_h} \boldsymbol{u}_h \qquad \Longrightarrow \qquad \ln \boldsymbol{a}_{t+1} - \ln \boldsymbol{a}_t = \lambda_h^t \boldsymbol{u}_h,$$

and the half-life of convergence to steady-state is given by:

$$t_h^{(1/2)}\left(\widetilde{\boldsymbol{f}}\right) = -\left\lceil \frac{\ln 2}{\ln \lambda_h} \right\rceil,$$

for all state variables $h = 2, \cdots, 2N$, where $\widetilde{a}_{i\infty} = a^*_{i,new} - a^*_{i,initial}$, and $\lceil \cdot \rceil$ is the ceiling function.

Proof. The proposition follows from the eigendecomposition of the transition matrix ($\boldsymbol{P} \equiv \boldsymbol{U\Lambda V}$), for the case of an eigen-shock in which the initial impact of the shocks to fundamentals on the state variables at time $t = 1$ coincides with a real eigenvector ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$) of the transition matrix ($\boldsymbol{P}$), as shown in Online Appendix H.7. $\square$

From Proposition 5, the impact of an eigen-shock ($\widetilde{\boldsymbol{f}}_{(h)}$) on the state variables in each time period is always proportional to the corresponding eigenvector ($\boldsymbol{u}_h$), and decays exponentially at a rate determined by the associated eigenvalue (λ_h), as the economy converges to the new steady-state.[4] These eigenvalues fully summarize the economy's speed of convergence in response to

[4] In general, these eigenvectors and eigenvalues can be complex-valued. If the initial impact is the real part of a complex eigenvector $\boldsymbol{u}_h$ ($\boldsymbol{R}\widetilde{\boldsymbol{f}} = \mathrm{Re}\,(\boldsymbol{u}_h)$), then $\ln \boldsymbol{a}_{t+1} - \ln \boldsymbol{a}_t = \mathrm{Re}\,(\lambda_h^t \boldsymbol{u}_h) \neq \mathrm{Re}\,(\lambda_h) \cdot \mathrm{Re}\left(\lambda_h^{t-1} \boldsymbol{u}_h\right)$. That is, the impact no longer decays at a constant rate λ_h. Instead, the complex eigenvalues introduce oscillatory motion as the dynamical system converges to the new steady-state. In our empirical application, the imaginary components of $\boldsymbol{P}$'s eigenvalues are small, implying that oscillatory effects are small relative to the effects that decay exponentially.

eigen-shocks, even in our setting with many asymmetric countries and a rich geography of trade and capital market frictions.

In general, each eigen-shock ($\widetilde{\boldsymbol{f}}_{(h)}$) has a different speed of convergence (as captured by the associated eigenvalue λ_h), which reflects the fact that the speed of convergence to steady-state does not only depend on the structural parameters of the model (ψ, θ, β, ϵ, μ_i), but also on the incidence of the fundamental shock on the state variables in each country (as captured by $\boldsymbol{u}_h = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$). Using Proposition 4, any empirical shock ($\widetilde{\boldsymbol{f}}$) can be expressed as a linear combination of the eigen-shocks. Therefore, the speed of convergence also varies across these empirical shocks with their incidence on the state variables in each country, reflecting the extent to which they load on eigen-shocks with slow versus fast convergence.

2.9.5 Goods and Capital Market Integration and Convergence

We now use our analytical results for the economy's transition path to examine the role of goods and capital market integration in determining the speed of convergence to steady-state. To simplify the exposition, we begin by considering the special case of the model with a separation between (i) workers, who earn wage income and live hand to mouth, and (ii) capitalists, who have log utility and make forward-looking consumption-saving decisions. We later generalize our analysis to a representative agent and CRRA preferences.

In this special case of a separation between workers and capitalists, capitalists with log utility consume a constant fraction $(1-\beta)$ of their capital wealth every period. Therefore, the evolution of the log deviations in the wealth state variables from steady-state simplifies as follows:

$$\widetilde{a}_{nt+1} - \widetilde{a}_{nt} = (1-\beta+\beta\delta)\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}\right), \tag{56}$$

where the derivations for this subsection are reported in Online Appendix I.

A common measure of the speed of convergence is the slope coefficient from a regression of log changes on log initial levels of a variable, as in a conventional β-convergence regression from the growth literature. From equation (56), this measure of the speed of convergence for log deviations in wealth depends on the covariance between the log deviation in the real return to investment ($\widetilde{v}_{nt} - \widetilde{p}_{nt}$) and the log deviation in the initial level of wealth ($\widetilde{a}_{nt}$):

$$\frac{\text{Cov}\left(\widetilde{a}_{nt+1} - \widetilde{a}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = (1-\beta+\beta\delta)\frac{\text{Cov}\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)}. \tag{57}$$

These log deviations in the real return to investment ($\widetilde{v}_{nt} - \widetilde{p}_{nt}$) depend on both capital market integration (through the nominal return to investment ($\widetilde{v}_{nt}$)) and goods market integration (through the consumption price index ($\widetilde{p}_{nt}$)). The log deviations in the nominal return to investment ($\widetilde{v}_{nt}$) in turn depend on log deviations in rental rates ($\widetilde{r}_{nt}$). Using the first-order condition for cost minimization in production, and assuming a common labor share (μ) across countries and a constant labor endowment in each country (ℓ_n), we have the following relationship between log deviations in the rental rate ($\widetilde{r}_{nt}$) and log deviations in the capital stock ($\widetilde{k}_{nt}$):

$$\widetilde{r}_{nt} = \widetilde{p}_{nnt} - \mu \widetilde{k}_{nt}, \tag{58}$$

where $\widetilde{p}_{nnt}$ is the log deviation in the local price of a country's own good from steady-state (recall $\tau_{nnt} = 1$), and in general differs from the log deviation in the consumption price index ($\widetilde{p}_{nt}$) that is a CES aggregate of the goods produced by all countries.

To provide economic intuition for the impact of goods and capital market integration on the speed of convergence to steady-state, we evaluate this measure of the speed of convergence for the limiting cases of completely open and completely closed goods and capital markets.

CNGM (Trade and Capital Autarky) Under capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$), each country's wealth equals its capital stock ($\widetilde{k}_{nt} = \widetilde{a}_{nt}$), and the nominal return to investment equals the domestic rental rate ($\widetilde{v}_{nt} = \widetilde{r}_{nt}$). Under trade autarky ($\tau_{nit} \to \infty$ for $n \neq i$), the consumption price index equals the local price of a country's own good ($\widetilde{p}_{nt} = \widetilde{p}_{nnt}$). Using these results in equations (56)-(58), we find that with a Cobb-Douglas production technology and a common labor share (μ), the speed of convergence to steady-state depends solely on this labor share:

$$\frac{\text{Cov}\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\mu. \tag{59}$$

Intuitively, there is diminishing marginal physical productivity of capital in the production technology. The larger the labor share (μ), the stronger these diminishing marginal returns to capital, and the faster the rate of convergence in capital and wealth towards steady-state.

Free Trade and Capital Autarky Under capital autarky ($\kappa_{nit} \to \infty$ for $n \neq i$), each country's wealth equals its capital stock ($\widetilde{k}_{nt} = \widetilde{a}_{nt}$), and the nominal return to investment equals the domestic rental rate ($\widetilde{v}_{nt} = \widetilde{r}_{nt}$). With free trade ($\tau_{nit} = 1$ for all n, i), the consumption price index takes the same value across all countries ($\widetilde{p}_{nt} = \widetilde{p}_t$ for all n). But the local price of a country's own good can differ from the consumption price index ($\widetilde{p}_{nt} \neq \widetilde{p}_{nnt}$), because countries' goods are imperfect substitutes ($1 < \sigma < \infty$). Using these results in equations (56)-(58), free trade in goods alone implies *faster* convergence than in the CNGM:

$$\frac{\text{Cov}\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\frac{1}{\sigma}\left(1-\mu\right) - \mu. \tag{60}$$

Intuitively, with autarkic capital markets, wealth accumulation in a given country expands its capital stock, which raises output of its good. With free trade in goods, in order for consumers worldwide to demand more of this good instead of the substitutes produced by other countries, the price of this good must fall. Therefore, wealth accumulation not only leads to a decline in the marginal physical product of capital as in the closed economy (captured by μ), but also leads to a fall in the price of a country's good (with an elasticity determined by σ), which implies a larger decline in the marginal value product of capital, and faster convergence to steady-state.

Trade Autarky and Free Capital Under trade autarky ($\tau_{nit} \to \infty$ for $n \neq i$), the consumption price index equals the price of a country's domestic good ($\widetilde{p}_{nt} = \widetilde{p}_{nnt}$). Under free capital ($\kappa_{nit} = 1$ for all n, i), the nominal return to investment takes the same value across all countries ($\widetilde{v}_{nt} = \widetilde{v}_t$ for all n). But the domestic capital stock can differ from domestic wealth ($\widetilde{k}_{nt} \neq \widetilde{a}_{nt}$), and the domestic rental rate can differ from the nominal return to investment ($\widetilde{r}_{nt} \neq \widetilde{v}_{nt}$), because of imperfect substitutability of capital between countries ($1 < \epsilon < \infty$). Using these results in equations (56)-(58), free capital flows alone imply *faster* convergence than in the CNGM:

$$\frac{\text{Cov}\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = -\frac{1}{\epsilon}\left(1 - \mu\right) - \mu. \tag{61}$$

Intuitively, with free capital flows, capital reallocates across countries to equalize the nominal return to investment for a given world stock of wealth. Nevertheless, countries accumulate wealth at different rates, because of differences in the real consumption price index under trade autarky, which lead to differences in the real rate of return to investment. Under free capital flows, wealth accumulation in a given country expands investment at home and abroad, which raises the country's income from these investments. Under trade autarky, this increased country income is spent on domestic goods, which bids up the domestic factor prices, where the elasticity of the domestic rental rate with respect to this expenditure depends on ϵ. Higher domestic factor prices raise the price of the domestic good, and hence the domestic consumption price index, which reduces the real return to investment, and speeds up convergence to steady-state.

Free Trade and Free Capital Under free trade ($\tau_{nit} = 1$ for all n, i), the consumption price index takes the same value across all countries ($\widetilde{p}_{nt} = \widetilde{p}_t$ for all n). Under free capital flows ($\kappa_{nit} = 1$ for all n, i), the nominal return to investment takes the same value across all countries ($\widetilde{v}_{nt} = \widetilde{v}_t$ for all n). Using these results in equations (56)-(58), the real return to investment takes the same value across all countries ($\widetilde{v}_{nt} - \widetilde{p}_{nt} = \widetilde{v}_t - \widetilde{p}_t$ for all n), and hence is uncorrelated with the initial level of wealth in each country ($\widetilde{a}_{nt}$). Therefore, free trade and free capital together imply *slower* convergence to steady-state than in the CNGM:

$$\frac{\text{Cov}\left(\widetilde{v}_{nt} - \widetilde{p}_{nt}, \widetilde{a}_{nt}\right)}{\text{Var}\left(\widetilde{a}_{nt}\right)} = 0 \tag{62}$$

Intuitively, with free trade and free capital, movements of goods and capital between countries equalize the real return to investment for a given world stock of wealth. Wealth accumulation in each country only affects this common real return to investment ($\widetilde{v}_{nt} - \widetilde{p}_{nt} = \widetilde{v}_t - \widetilde{p}_t$) through the world stock of wealth. Therefore, each country accumulates wealth at the same rate, as determined by this common real return to investment, and initial differences in wealth persist forever, as the world economy gradually converges to the world steady-state level of wealth.

While for expositional simplicity, we have illustrated these results for the special case of a separation of workers and capitalists and logarithmic utility, analogous results hold for a representative agent and CRRA utility, as summarized in the following proposition.

Proposition 6. ***Goods and Capital Market Integration.*** *The speed of convergence to steady-state is **faster** than in the closed-economy neoclassical growth model (CNGM) with **either** (i) free trade and capital autarky **or** (ii) trade autarky and free capital. This speed of convergence is **slower** than in the CNGM with (iii) **both** free trade and free capital.*

Proof. See Online Appendix I. □

Taking these results together, with trade autarky and capital autarky in the CNGM, countries with lower initial levels of wealth have higher real returns to investments, which induces them to accumulate wealth more rapidly than countries higher initial levels of wealth, thereby bringing about a convergence in the wealth state variables. Opening *both* free trade *and* free capital equalizes the real return to investment across countries, which eliminates this force for convergence in the wealth state variables. Although we derive this theoretical result here for the limiting cases of autarky and perfect integration, we show in our quantitative analysis below that we find substantially slower rates of convergence in the wealth state variables for the values in trade and capital frictions implied by the observed data than in the conventional CNGM.

2.10 Two-Country Example

We now illustrate these general results for our baseline quantitative model using a simple two-country example. We show that reductions in goods and capital market frictions have non-monotonic effects on the speed of convergence to steady-state. We consider a world of two symmetric countries with identical fundamentals (z_i, η_i) but potentially different initial wealth (a_{i0}). We denote the steady-state trade and capital share matrices by:

$$\boldsymbol{S} = \frac{1}{2}\begin{bmatrix} 1+s & 1-s \\ 1-s & 1+s \end{bmatrix}, \qquad \boldsymbol{B} = \frac{1}{2}\begin{bmatrix} 1+b & 1-b \\ 1-b & 1+b \end{bmatrix}.$$

The parameters $s, b \in [0,1]$ are inverse measures of the degree of openness in goods and capital markets, respectively. The parameter s increases one-for-one with bilateral trade frictions ($\tau \geq$

1), while the parameter b increases one-for-one with bilateral capital frictions ($\kappa \geq 1$):

$$s = \frac{1 - \tau^{-\theta}}{1 + \tau^{-\theta}}, \qquad b = \frac{1 - \kappa^{-\epsilon}}{1 + \kappa^{-\epsilon}}.$$

When $s = 0$, the steady-state equilibrium is characterized by free trade ($\tau = 1$). In contrast when $s = 1$, the steady-state equilibrium is characterized by trade autarky ($\tau = \infty$). Analogously, $b = 0$ and $b = 1$ correspond to free capital ($\kappa = 1$) and capital autarky ($\kappa = \infty$), respectively.

Applying Proposition 3 to this simple two-country example, we can write the law of motion for wealth in the absence of shocks to fundamentals ($\widetilde{\boldsymbol{f}} = 0$) as follows:

$$\widetilde{\boldsymbol{a}}_{t+1} = \boldsymbol{P}\widetilde{\boldsymbol{a}}_t,$$

where $\boldsymbol{P}$ is the two-by-two transition matrix and the derivations for this section are reported in Online Appendix J. When the two countries are symmetric, the two eigenvectors of $\boldsymbol{P}$ are:

$$\boldsymbol{u}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \boldsymbol{u}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

with corresponding eigenvalues λ_1 and λ_2. From Proposition 5, these eigenvalues determine the speed of convergence to steady-state, such that the larger the absolute value of these eigenvalues, the longer the half-life of convergence to steady-state.

The first eigencomponent ($\boldsymbol{u_1}$) captures the state in which both countries have identical wealth. In this case, the two countries behave symmetrically as they converge at rate λ_1 towards the world steady-state. The two countries are symmetric along the entire transition path, such that the degree of home bias in goods and capital markets does not affect convergence dynamics. The global economy converges uniformly to steady-state, and no reallocation of economic activity across countries is required along the transition path. As such, the rate of convergence λ_1 is independent of s and b, and is equal to the rate of convergence in the CNGM.

In contrast, the second eigencomponent ($\boldsymbol{u_2}$) captures the state in which the two countries have asymmetric initial conditions: initial wealth is above steady-state in one country and below steady-state in the other country. Global-level aggregates are at their steady-state values, and the transition captures reallocation of economic activity across countries. As such, the rate of convergence λ_2 depends on the degrees of openness in goods and capital markets (as captured by s and b). In particular, the rate of wealth accumulation depends on the gross real return $R_{nt} = 1 - \delta + v_{nt}/p_{nt}$, which is lower for the economy with higher wealth, and provides the force for the wealth convergence. Therefore, the speed of convergence depends on how differences in wealth translate into differences in the real return to investment (v_{nt}/p_{nt}), which in turns depends on openness in goods and capital markets (as captured by s and b).

In the special case of our model with (i) a separation between workers (who live hand to mouth) and capitalists (who can save) and (ii) log utility ($\psi = 1$), we can provide a further analytical characterization of the dependence of λ_2 on s and b. In this special case, the law of motion for the wealth state variables is given by equation (56). Now consider an initial state of our symmetric two-country economy that coincides with the second eigenvector: $\widetilde{\boldsymbol{a}}_0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Given this initial state, the evolution of the wealth state variables follows $\widetilde{\boldsymbol{a}}_{t+1} = \lambda_2 \widetilde{\boldsymbol{a}}_t$, and we can write the nominal return to investment and the consumption price index as $\widetilde{\boldsymbol{v}}_t = v\widetilde{\boldsymbol{a}}_t$ and $\widetilde{\boldsymbol{p}}_t = p\widetilde{\boldsymbol{a}}_t$, where the scalar constants v and p depend on model parameters and the degrees of openness in goods and capital markets. Combining these results, the law of motion for the wealth state variables from equation (56) can be re-written as follows:

$$\widetilde{\boldsymbol{a}}_{t+1} - \widetilde{\boldsymbol{a}}_t = (1 - \beta + \beta\delta)\,(v - p)\,\widetilde{\boldsymbol{a}}_t, \tag{63}$$

where the real return to investment is lower for an economy with higher wealth, such that $v - p < 0$. Combining this law of motion (63) with $\widetilde{\boldsymbol{a}}_{t+1} = \lambda_2 \widetilde{\boldsymbol{a}}_t$, we obtain the following closed-form solution for the rate of convergence to steady-state (λ_2):

$$\lambda_2 = 1 + (1 - \beta + \beta\delta)\,(v - p)\,, \tag{64}$$

where a larger difference in the real return to investment ($v - p$ more negative) implies faster convergence to steady-state.

In Online Appendix J, we show that reductions in trade and capital frictions in general have ambiguous effects on this rate of convergence to steady-state λ_2. First, we consider the case of different levels of opennes in goods and capital markets. Under trade autarky, a reduction in capital frictions speeds up convergence. Similarly, under capital autarky, a reduction in trade costs also speeds up convergence:

$$\left.\frac{\partial \lambda_2}{\partial s}\right|_{b=1,s<1} > 0, \qquad \left.\frac{\partial \lambda_2}{\partial b}\right|_{s=1,b<1} > 0.$$

Second, we consider the case of the same level of openness in goods and capital markets ($b = s$), in which case a reduction in either trade or capital frictions reduces the speed of convergence:

$$\left.\frac{\partial \lambda_2}{\partial s}, \frac{\partial \lambda_2}{\partial b}\right|_{b=s} < 0.$$

As we reduce this common level of trade and capital frictions towards the limiting case of free trade and free capital ($\tau_{ni} = 1$ and $\kappa_{ni} = 1$ for all n, i), we find that the absolute magnitude of the speed of convergence falls smoothly towards zero, confirming our general results for our baseline quantitative model above.

3 Quantitative Analysis

We now use our theoretical framework to provide quantitative evidence on the determinants of the speed of convergence to steady-state and the effects of counterfactual changes in trade and capital market frictions. In Subsection 3.1, we discuss our data sources and the parameterization of our model. In Subsection 3.2, we confirm that the gravity equation provides a good approximation to the observed data on trade in goods and capital holdings, consistent with our assumption of costly trade and capital flows with imperfect substitutability between countries. In Subsection 3.3, we provide evidence on the speed of convergence to steady-state, and the role of goods and capital market integration in shaping this speed of convergence. In Subsection 3.4, we report our main counterfactuals for a decoupling of China and the United States.

3.1 Data and Parameterization

We quantify our model using readily-available data on national accounts, bilateral international trade, and bilateral international capital holdings on 47 countries from 2001-2019.[5]

Penn World Tables We use the national accounts data from the Penn World Tables (PWT) to measure gross domestic product (GDP), capital stock, labor compensation and population. We set the labor share for each country in the model (μ_{nt}) equal to labor compensation as a share of GDP in the PWT data. We equate the capital stock in the model (k_{nt}) with the capital stock in the PWT data. We set the labor endowment in the model (ℓ_{nt}) equal to population in the PWT data. We set the wage in the model (w_{it}) equal to labor compensation divided by population in the PWT data. We recover the rental rate in the model (r_{it}) using GDP, labor compensation, and the capital stock in the PWT.

International Trade We use data on bilateral trade (E_{nit}) between countries from the United Nations COMTRADE database.[6] Following Feenstra et al. (2005), we use the trade flows reported by the importer whenever they are available, but use the corresponding exporter's report if the importer report is unavailable for a country pair. We measure expenditure on domestic goods (E_{nnt}) as GDP minus total exports. Using the resulting bilateral expenditures (E_{nit}), we construct the expenditure share of each importer on each exporter ($S_{nit} = E_{nit}/\sum_{h=1}^{N} E_{nht}$), and the income share of each exporter from each importer ($T_{int} = S_{nit}E_{nt}/\sum_{h=1}^{N} S_{hit}E_{ht}$, where $E_{nt} = \sum_{i=1}^{N} E_{nit}$).

We augment these trade data with information on the bilateral distance between countries from the GEODIST dataset from CEPII.[7] We use the bilateral distances between countries' largest

[5]See Online Appendix L for further details on the data sources and definitions.

[6]See https://comtradeplus.un.org/.

[7]See http://www.cepii.fr/CEPII/en/bdd_modele/bdd_modele_item.asp?id=6.

cities, weighted by the population of those cities, as our distance measure.

International Capital Holdings We combine a number of different sources of data on international capital holdings to construct as comprehensive a measure of bilateral portfolio shares as possible. Our main data source is the Coordinated Portfolio Investment Survey (CPIS), which reports international holdings of portfolio investment assets in the form of equity and investment fund shares, long-term debt securities, and short-term debt securities (Josyula, 2018).[8] We augment the CPIS data with information on the total amount of outstanding equity and debt claims from the Organizational for Economic Cooperation and Development (OECD), Bank for International Settlements (BIS), and World Bank.

We adjust the raw data on equity and debt holdings to nationality-based accounting following Global Capital Allocation Project (Maggiori et al. 2020 and Coppola et al. 2021).[9] We thus obtain a matrix of gross bilateral capital holdings (H_{nit}). We rescale the rows and columns of this matrix, such that total inward and outward holdings of capital (including own holdings) are consistent with our national accounts data, as required for internal consistency in the model.

Using the resulting matrix of gross bilateral capital holdings between all pairs of countries (H_{nit}), we construct the portfolio share of each source n allocated to each host i ($B_{nit} = H_{nit} / \sum_{h=1}^{N} H_{nht}$), and the payment share from each source i to each host n ($X_{nit} = H_{nit} / \sum_{h=1}^{N} H_{hit}$). The international capital holdings data only enter our quantitative analysis through the matrices of portfolio shares ($\boldsymbol{B}$) and payment shares ($\boldsymbol{X}$). To the extent that our observed bilateral capital holdings (H_{nit}) do not perfectly capture the true value of these holdings, we implicitly assume that the unobserved holdings have the same portfolio shares ($\boldsymbol{B}$) and payment shares ($\boldsymbol{X}$) as the observed holdings.

Parameterization To quantify the implications of introducing costly trade and capital flows with imperfect substitutability, we assume standard parameter values from the existing empirical literature. We assume a discount factor equal to $\beta = 0.95$; an intertemporal elasticity of substitution of $\psi = 0.2$; a depreciation rate of $\delta = 0.05$; and we set the labor share (μ_{it}) equal to its empirical value in the data for each country, as discussed above. We assume a trade elasticity of $\theta = 5$, which lies in the center of the range from 2-12 considered in Eaton and Kortum (2002), and is the baseline value used in Costinot and Rodríguez-Clare (2014). We assume a capital elasticity of $\epsilon = 3.15$, which lies within the range of values estimated in Koijen and Yogo (2020).

[8] See https://datahelp.imf.org/knowledgebase/articles/505725-what-is-the-coordinated-portfolio-investment-surve.
[9] See https://www.globalcapitalallocation.com/.

3.2 Gravity in Trade and Capital Holdings

We now confirm that the gravity equation provides a good approximation to our observed data on trade in goods and capital holdings. We estimate the following gravity equation specification between countries for a single year:

$$Y_{ni} = \vartheta_i^O \vartheta_n^D \text{dist}_{ni}^{\delta} u_{ni}, \tag{65}$$

where Y_{ni} is either expenditure of importer n on exporter i to importer n (E_{ni}) or the capital holdings of source n in host i (H_{ni}); ϑ_i^O is an origin fixed effect; ϑ_n^D is a destination fixed effect; dist_{ni} is bilateral distance; and u_{ni} is a stochastic error. We report two-way clustered standard errors by origin and destination.

Table 2: Gravity Equation Regressions

	(1)	(2)	(3)	(4)
	Log Trade 2012	Trade 2012	Log Capital 2012	Capital 2012
Log Distance	-1.053	-0.876	-1.426	-0.930
	(0.0844)	(0.0664)	(0.137)	(0.132)
Estimation	OLS	PPML	OLS	PPML
Origin FEs	Yes	Yes	Yes	Yes
Destination FEs	Yes	Yes	Yes	Yes
Observations	2103	2112	2112	2112
R-squared	0.849		0.827	
Pseudo R-squared		0.897		0.859

Note: Cross-section of origin and destination countries in 2012; all columns include origin and destination fixed effects (FEs); Columns (1)-(2) show results for bilateral trade; Columns (3)-(4) report results for bilateral capital holdings; Columns (1) and (3) estimated in logs using ordinary least squares (OLS); Columns (2) and (4) estimated using the Poisson Pseudo Maximum Likelihood (PPML) estimator; standard errors two-way clustered by origin and destination.

In Column (1) of Table 2, we report the results of taking logs in equation (65) and estimating this gravity equation for international trade using ordinary least squares (OLS) with origin and destination fixed effects. In line with existing evidence, we find a negative and highly significant relationship between bilateral trade and distance, with an elasticity of around minus one, and a regression R-squared of close to 85 percent. We next show that these findings are not sensitive to the dropping of zeros when we take logs. In Column (2), we demonstrate the same pattern of results if we estimate this gravity equation in levels using the Poisson Pseudo Maximum Likelihood (PPML) estimator, as in Santos Silva and Tenreyro (2006) and Head and Mayer (2014). Again we find a negative and highly statistically significant coefficient on bilateral distance that is only marginally smaller than that in Column (1).

In Column (3), we estimate this same specification for international capital holdings. Although capital holdings are not subject to transportation costs in the way that goods flows are, we again find a negative and highly statistically significant coefficient on distance, and a regression R-squared of around 80 percent. Indeed, the estimated elasticity for capital holdings is if anything larger in absolute magnitude than for goods trade. In Column (4), we show that we find the same pattern of results using the Poisson Pseudo Maximum Likelihood (PPML) estimator.

While Table 2 provides overall evidence on the explanatory power of the gravity equation specification for trade and capital holdings, it does not reveal the relative importance of bilateral distance and the fixed effects for this explanatory power. To separate out the contribution from bilateral distance, we use the Frisch-Waugh-Lovell Theorem. We run two separate OLS regressions of log values and log distance on origin and destination fixed effects, generate the two residuals, and then regress these two residuals on one another. As shown in Section K.1 of the Online Appendix, we find that bilateral distance has as much explanatory power for capital holdings as for trade, even after removing the origin and destination fixed effects.

Taken together, these findings support for the gravity equation predictions of our assumption of costly trade and capital flows with imperfect substitutability between countries.

3.3 Speed of Convergence

We now examine the implications of costly trade and capital flows with imperfect substitutability for the speed of convergence to steady-state. We use our closed-form solution for the economy's transition path in the linearized model to provide an analytical characterization of the determinants of this speed of convergence to steady-state.

Empirical Speeds of Convergence We begin by using our eigendecomposition in Proposition 3 to recover the eigenvectors ($\boldsymbol{u}_h$) and eigenvalues (λ_h) of the transition matrix ($\boldsymbol{P}$) that governs the updating of the wealth state variables over time. Using Proposition 5, we define an eigen-shock as a shock to fundamentals ($\widetilde{\boldsymbol{f}}_{(h)}$) for which the initial impact on the state variables ($\boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)}$) coincides with a real eigenvector of this transition matrix ($\boldsymbol{u}_h$). Each of these eigen-shocks corresponds to a different incidence of fundamental shocks on the wealth state variables ($\widetilde{\boldsymbol{a}}_t$) for each country and is characterized by a speed of convergence to steady-state that is determined by the corresponding eigenvalue (λ_h).

In Figure 1, the long-dashed blue line shows the implied half lives of convergence to steady-state for each eigen-shock using the observed trade and capital share matrices for the year 2019. The vertical axis displays the half life for each eigen-shock, while the horizontal axis sorts these eigen-shocks in terms of increasing half-lives. With open goods and capital markets, these half-lives depend on the entire network of bilateral trade and capital frictions (as captured in the observed trade and capital share matrices) and the parameters of the model.

As point of comparison, the solid red line displays half-lives of convergence to steady-state for the CNGM, which corresponds to the special case of our model with autarky in both goods and capital markets. In this special case, the half-life of convergence only varies across eigen-shocks, because of differences across countries in the labor share (μ_i). As a further benchmark, the short-dashed black line shows the common half-life of convergence for the CNGM with a common labor share ($\mu_i = \mu$), equal to the average labor share across countries.

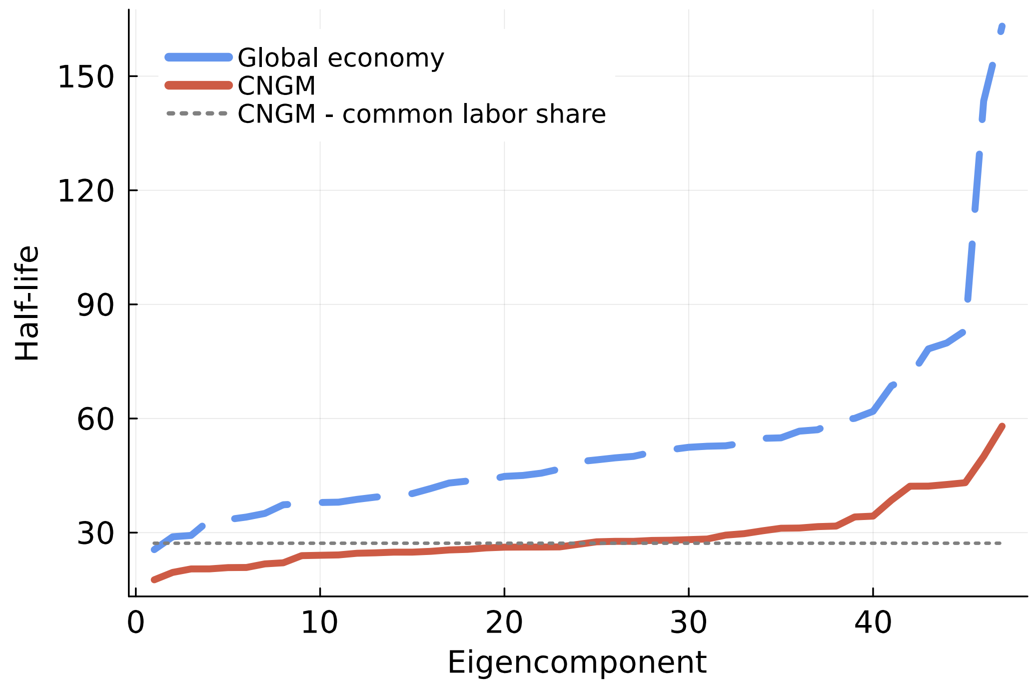
We find a substantially slower rate of convergence to steady-state in our neoclassical growth model with open goods and capital markets and imperfect substitutability than in the CNGM. This speed of convergence also displays considerable heterogeneity across the eigen-shocks, ranging from around 25 to 175 years, compared to a range from 15 to 55 years in the CNGM with country-specific labor shares. Since any vector of empirical shocks to fundamentals can be written as a linear combination of the eigen-shocks, these results imply slow rates of convergence to steady-state in response to vectors of empirical shocks.

Intuitively, open goods and capital markets promote a convergence in the real return to investment across countries, which diminishes the absolute magnitude of the covariance between the real return to investment and initial wealth across countries, and hence reduces the speed of convergence to steady-state. Therefore, our open-economy framework with imperfect substitutability provides a natural approach to addressing the concern that the speed of convergence in the CNGM is too fast relative to empirical transitions for plausible values of the intertemporal elasticity of substitution (as discussed, for example, in King and Rebelo 1993).

Comparison with Limiting Cases To provide further economic intuition, we now compare our open-economy model and the CNGM to the limiting cases of perfect integration in either goods and/or capital markets, as considered in our theoretical analysis above.

For each of our eigen-shocks h, we can compute the log deviation in the wealth state variables from steady-state ($\widetilde{\boldsymbol{a}}_{(h)} = \boldsymbol{R}\widetilde{\boldsymbol{f}}_{(h)} = \boldsymbol{u}_h$), and use the structure of the model to solve for the implied log deviation in each of the other endogenous variables from steady-state, including the real return to investment ($\widetilde{\boldsymbol{v}}_{(h)} - \widetilde{\boldsymbol{p}}_{(h)}$). In Figure 2, the solid black line shows the covariance across countries between these log deviations in the real return to investment ($\widetilde{\boldsymbol{v}}_{(h)} - \widetilde{\boldsymbol{p}}_{(h)}$) and the log deviation in the initial level of wealth ($\widetilde{\boldsymbol{a}}_{(h)}$) for each eigen-shock for 2019. In the special case of our model with (i) a separation between workers (who live hand to mouth) and capitalists (who can save) and (ii) log utility ($\psi = 1$), this covariance captures the speed of convergence to steady state, as shown theoretically in Section 2.9.5 above. The more negative this covariance between log deviations in the real return to investment ($\widetilde{\boldsymbol{v}}_{(h)} - \widetilde{\boldsymbol{p}}_{(h)}$) and the initial level of wealth ($\widetilde{\boldsymbol{a}}_{(h)}$), the faster the speed of convergence to steady-state.

Figure 1: Half Lives of Convergence to Steady-State for each Eigen-shock



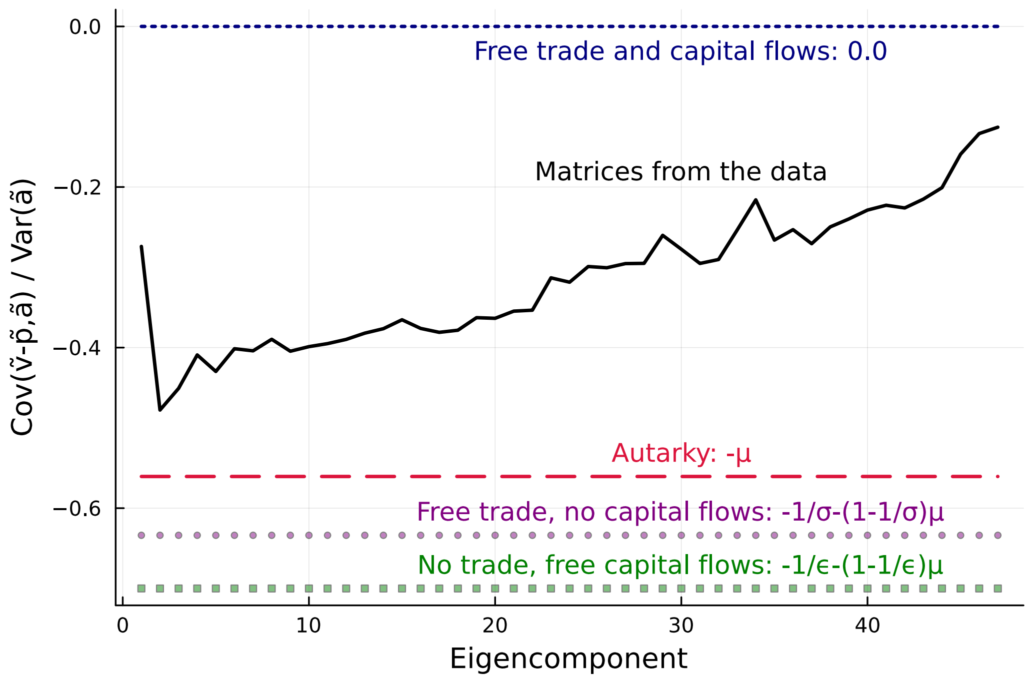
Note: Vertical axis shows half life of convergence to steady-state in years for each eigen-shock; horizontal axis shows the rank of the eigen-shocks in terms of increasing half life in years; long-dashed blue line shows these half lives for our baseline model with costly trade and capital flows and imperfect substitutability between countries for the year 2019; solid red line shows these half lives of convergence for the special case of the closed-economy neoclassical growth model (CNGM) with a country-specific labor share; short-dashed black line shows these half-lives for the special case of the CNGM with a common labor share.

As an initial point of comparison, the long-dashed red line shows the value of this covariance in the special case of the CNGM with a common labor share across countries ($\mu_i = \mu$), in which case this covariance equals minus the common labor share. Consistent with our results in Figure 1 above, we find that this covariance is substantially less negative for the observed trade and capital share matrices (black solid line) than in the CNGM (red dashed line), implying slower convergence in our open-economy model than in this special case.

As a second benchmark, the dotted purple line shows this covariance in the limiting case of free trade and capital autarky, in which case this covariance across countries is more negative than the common labor share ($-\frac{1}{\sigma}(1-\mu)-\mu$). Therefore, starting from the CNGM and opening free trade in goods *raises* the speed of convergence to steady-state. Intuitively, as a country accumulates capital, its invests domestically under autarkic capital markets and produces more output. In order to sell more in open goods markets, the country needs to lower its price relative to its competitors, which reduces the country's value marginal product of capital and real return to investment, and speeds up convergence. The elasticity of substitution in goods markets (σ) regulates the rate at which the country's price falls relative to its competitors.

As a third yardstick, the dotted green line shows this covariance in the limiting case of free

Figure 2: Covariances of Log Deviations from Steady-State in Real Returns to Investment ($\widetilde{v}_n - \widetilde{p}_n$) and Initial Wealth ($\widetilde{a}_n$)



Note: Vertical axis shows the covariance between log deviations from steady-state in the real return to investment ($\widetilde{v}_n - \widetilde{p}_n$) and initial wealth ($\widetilde{a}_n$) for each eigen-shock; horizontal axis shows the rank of the eigen-shocks in terms of increasing half life in years; solid back line shows this covariance for our baseline model with costly trade and capital flows and imperfect substitutability between countries for the year 2019; black dotted line shows this covariance for the special case of our model with free trade in goods and capital; red dashed line shows this covariance for the special case of our model with autarky in goods and capital markets and a common labor share across countries; purple circles show this covariance for the special case of our model with free trade in goods, capital autarky and a common labor share across countries; green squares show this covariance for the special case of our model with autarky in goods markets, free capital flows and a common labor share across countries.

capital and trade autarky, in which case this covariance is again more negative than the common labor share ($-\frac{1}{\epsilon}(1-\mu)-\mu$). Hence, starting from the CNGM and opening free capital flows also *raises* the speed of convergence to steady-state. Intuitively, as a country accumulates more wealth and invests worldwide in open capital markets, it generates more income. With autarkic goods markets, this increased income is spent domestically and bids up the domestic wage and rental rate, which raises the domestic consumption price index, and lowers the real return to investment, thereby again speeding up convergence. The elasticity of substitution in capital markets (ϵ) controls the rate at which the domestic rental rate rises in response to increased spending on domestic goods.

As a fourth gauge, the short-dashed blue lie shows this covariance in the limiting case of free trade and free capital, in which case this covariance is smaller in absolute value than the common labor share, and equal to zero. Therefore, starting from the CNGM and opening free trade and free capital *reduces* the speed of convergence to steady-state. Intuitively, free trade in goods and free flows of capital equalize the real return to investment across countries, breaking the link between low wealth and a high real return to investment that drives income convergence.

Comparing our empirical results for the observed trade and capital share matrices (black line) to these limiting cases, we find that the speed of convergence varies across the eigen-shocks from values close to those in the CNGM (bottom left) to values close to those under free trade and free capital (top right). Therefore, depending on extent to which empirical shocks to fundamentals load on these different eigen-shocks (i.e., depending on the extent to which their incidence falls on the state variables for different countries), the speed of convergence can range across nearly the full spectrum of values from the CNGM to perfectly integrated markets.

Speed of Convergence for Alternative Trade and Capital Frictions While we have used these comparisons with limiting cases to provide economic intuition, we now show that this relationship between the speed of convergence and goods and capital market frictions holds more generally for empirically relevant values of these frictions.

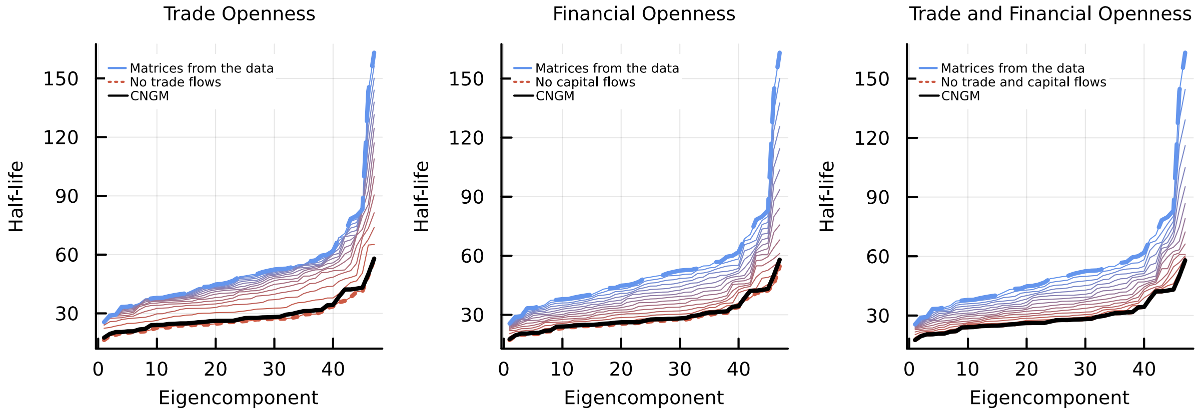
In all three panels of Figure 3, the long-dashed blue line shows the half-life of convergence to steady-state for each eigen-shock ($\boldsymbol{u}_{(h)}$) from the transition matrix ($\boldsymbol{P}$) for our benchmark year of 2019 (equivalent to the long-dashed blue line in Figure 1). In contrast, the solid black line in all three panels shows the half-life of convergence to steady-state for the special case of the closed-economy neoclassical growth model (equivalent to the solid red line in Figure 1).

In the left panel of Figure 3, we hold capital frictions (κ_{ni}) constant at their values implied by the observed capital shares matrices, and increase trade frictions (τ_{ni}) relative to those implied by the observed trade shares matrices, by taking a weighted average of the observed trade shares matrices and the identity matrix. As the weight on the identity matrix converges to zero, we obtain the observed trade shares (long-dashed blue line), while as the weight on the identity matrix converges to one, we approach trade autarky (dashed red line). Each of the thin red lines shows intermediate values for trade frictions (τ_{ni}) in between those implied by the observed data (long-dashed blue line) and autarky in goods markets (dashed red line).

We find a similar pattern of results as for the comparison with perfectly integrated markets above. We observe slower speeds of convergence for the trade and capital frictions implied by the observed data than for the CNGM, as reflected in the location of the long-dashed blue line above the solid black line. In contrast, we observe faster speeds of convergence for an economy with trade autarky and the capital frictions implied by the observed data than the CNGM, as reflected in the location of the dashed red line below the solid black line. The difference between the solid black and dashed red lines is marginal, because the observed capital share matrices have high diagonal terms, implying capital frictions relatively close to capital autarky.

In the middle panel of Figure 3, we hold trade frictions (τ_{ni}) constant at their values implied by

Figure 3: Half Lives of Convergence to Steady-State for Alternative Trade Frictions (τ_{ni}) and Capital Frictions (κ_{ni})



Note: Vertical axis shows half life of convergence to steady-state in years for each eigen-shock; horizontal axis shows the rank of the eigen-shocks in terms of increasing half life in years; long-dashed blue line shows these half lives of convergence for our baseline model with costly trade and capital flows and imperfect substitutability between countries for the year 2019; solid black line shows half lives of convergence for the closed-economy neoclassical growth model (CNGM) with a country-specific labor share; in the left panel, the thin solid red lines show half lives of convergence for higher trade frictions, and the red dashed line shows these half lives for the limiting case of trade autarky ($\tau_{ni} \to \infty$) and open capital markets; in the middle panel, the thin solid red lines show half lives for higher capital market frictions, and the red dashed line shows these half lives for the limiting case of capital market autarky ($\kappa_{ni} \to \infty$) and trade openness; in the right panel, the thin solid red lines show half lives for higher trade and capital frictions.

the observed trade shares matrices, and increase capital frictions (κ_{ni}) relative to those implied by the observed capital shares matrices, by taking a weighted average of the observed capital shares matrices and the identity matrix. As the weight on the identity matrix converges to zero, we obtain the observed capital shares (long-dashed blue line), while as the weight on the identity matrix converges to one, we approach autarky in capital markets (dashed red line). Each of the thin red lines shows intermediate values for capital frictions (κ_{ni}) in between those implied by the observed data (long-dashed blue line) and autarky in capital markets (dashed red line).

Again we find a similar pattern of results as for the comparisons with perfectly integrated markets above. We find slower speeds of convergence for the trade and capital frictions implied by the observed data than in the CNGM, as reflected in the location of the long-dashed blue line above the solid black line. In contrast, we find faster speeds of convergence for an economy with capital market autarky and the trade frictions implied by the observed data than in the CNGM, as reflected in the location of the dashed red line below the solid black line. The difference between the solid black and dashed red lines is again slight, because the observed trade matrices have high diagonal terms, implying trade frictions relatively close to trade autarky.

In the right panel of Figure 3, we increase both trade frictions (τ_{ni}) and capital frictions (κ_{ni}), by taking a weighted average of the observed trade and capital share matrices and the identity matrix. Starting from the observed trade and capital shares (long-dashed blue line), each of the thin red lines shows progressively higher values of both trade and capital frictions, until we obtain the limiting case of the CNGM (solid black line). Again our findings are in line with our earlier results. We find slower speeds of convergence for the trade and capital market frictions implied by the observed data than in the CNGM, as reflected in the location of the long-dashed blue line above the solid black line. As we increase both trade and capital frictions to prohibitive levels, we obtain the limiting case of the CNGM (solid black line), and hence there is no red dashed line in this right panel.

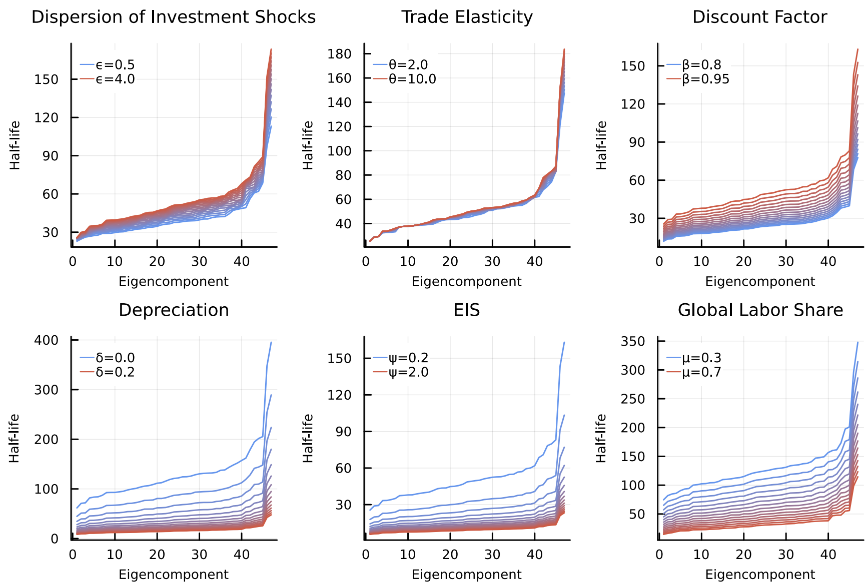
Speeds of Convergence for Alternative Parameter Values We now use our spectral analysis to evaluate the comparative statics of the speed of convergence with respect to changes in model parameters. Undertaking these comparative statics in the non-linear model is challenging, because the speed of convergence to steady-state depends on the incidence of fundamental shocks on the wealth state variables in each country. As a result, to fully characterize the impact of changes in model parameters on the speed of convergence in the non-linear model, one needs to undertake counterfactuals over infinitely many possible shocks, which is not empirically well defined.

In contrast, our spectral analysis has two key properties. First, the set of all possible fundamental shocks is spanned by the set of eigen-shocks, which is well defined. Second, we have a closed-form solution for the impact matrix ($\boldsymbol{R}$) and transition matrix ($\boldsymbol{P}$) in terms of the trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) and model parameters $\{\psi, \beta, \theta, \epsilon, \mu_i, \delta\}$. Therefore, for any alternative model parameters, we can immediately solve for the entire spectrum of eigenvalues (and corresponding half-lives) associated with the eigen-shocks using the observed data. Because the eigen-shocks span all possible fundamental shocks, understanding how parameters affect the entire spectrum of half-lives translates into an analytically sharp understanding of how convergence rates are affected by model parameters.

In Figure 4, we display the half-lives of convergence to steady-state across the entire spectrum of eigen-shocks for different values of model parameters, based on the observed trade and capital shares for the year 2019. Each panel varies the noted parameter, holding constant the other parameters at their baseline values. On the vertical axis, we display the half-life of convergence to steady-state. On the horizontal axis, we rank the eigen-shocks in terms of increasing half-lives of convergence to steady-state for our baseline parameter values.

In the top-left panel, a higher capital elasticity (ϵ) implies a longer half-life (slower convergence), because greater substitutability of capital across countries reduces the absolute value of the covariance between the real return to investment and the initial level of wealth. In the top-middle panel, a higher trade elasticity (θ) also implies a longer half-life (slower convergence), because greater substitutability of goods across countries also reduces the absolute value of the covariance between the real return to investment and the initial level of wealth. In the top-right panel, a higher discount factor (β) implies a longer half-life (slower convergence), because the representative agent has a higher saving rate, which implies a greater role for wealth accumulation, thereby magnifying the impact of fundamental shocks, and implying a longer length of time for adjustment to occur in response to these shocks.

Figure 4: Half Lives of Convergence to Steady-State for Alternative Parameter Values



Note: Half lives of convergence to steady-state for each eigen-shock for alternative parameter values for our baseline model with costly trade and capital flows and imperfect substitutability between countries for the year 2019; vertical axis shows half-life in years; horizontal axis shows the rank of the eigen-shocks in terms of increasing half lives; each panel varies the noted parameter, holding the other parameters at their baseline value; the blue and red solid lines denote the lower and upper range of the parameter values considered, respectively; each of the other lines in between varies the parameters uniformly in the stated range.

In the bottom-right panel, we solve the model for alternative values of a common labor share ($\mu_i = \mu$) across countries. A lower labor share (μ) implies a longer half-life (slower convergence), because it implies a greater role for wealth accumulation, which again magnifies the impact of fundamental shocks, and hence requires a greater length of time for adjustment to occur. In the bottom-middle panel, a lower intertemporal elasticity of substitution (ψ) implies a longer half-life (slower convergence), because consumption becomes less substitutable across time, which reduces the willingness of the representative agent to respond to investment opportunities. Finally, in the bottom-left panel, a smaller depreciation rate (δ) implies a longer half-life (slower convergence), because it takes longer for investments to depreciate, implying a longer length of time for the distribution of wealth to adjust in response to shocks.

3.4 Counterfactuals for China-U.S. Decoupling

Given that our framework incorporates trade and capital holdings that match the gravity equation relationships observed in the data, and allows for intertemporal consumption-savings decisions, it is particularly well suited for evaluating counterfactual policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling).

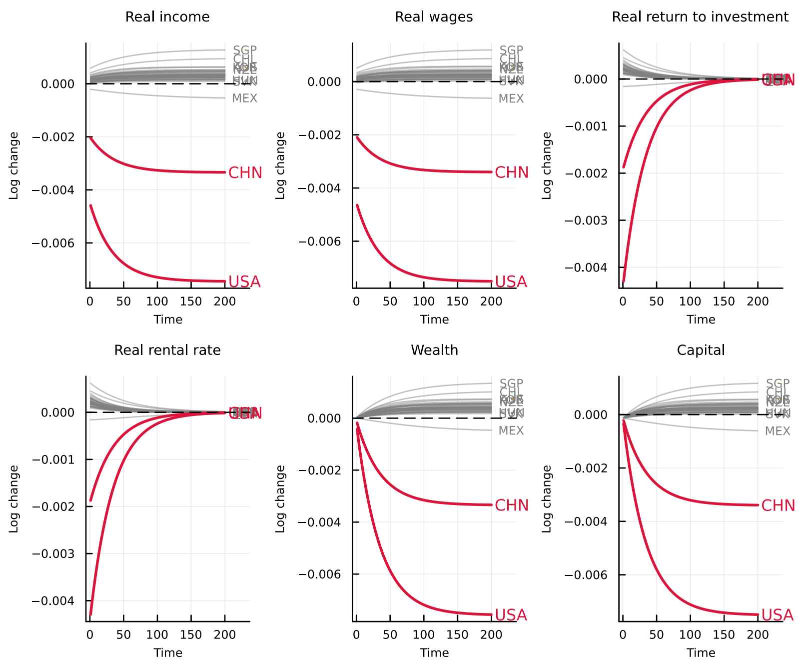
We now use our framework to evaluate three sets of counterfactuals for U.S.-China decoupling: (i) a 50 percent increase in bilateral trade frictions alone between China and the United States; (ii) a 50 percent increase in bilateral capital frictions alone between these two countries; (iii) a 50 percent increase in both bilateral trade and capital frictions between these two countries. We undertake these counterfactuals for our baseline model using our linearization and the observed trade and capital share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$, $\boldsymbol{B}$, $\boldsymbol{X}$) for 2019. We assume that agents at time $t = 0$ learn about a permanent increase in bilateral frictions from time $t = 1$ onwards. Using Propositions 3 and 4, we solve for the entire transition path of the wealth state variables and all other endogenous variables of the model from time $t = 1$ onwards.

We also compare the counterfactual predictions of our baseline open economy model to special cases with either capital autarky (and open trade) or trade autarky (and open capital markets), in order to highlight the interaction between goods and capital market integration. When we consider the special case with capital autarky, we replace the observed capital share matrices ($\boldsymbol{B}$, $\boldsymbol{X}$) with identity matrices, such that each country only invests domestically. Thus, we make sure to match the observed trade data in both cases, and only vary the degree of capital openness. Similarly, when we consider the special case with trade autarky, we replace the observed trade share matrices ($\boldsymbol{S}$, $\boldsymbol{T}$) with identity matrices, such that each country only consumes its own goods, while exactly matching observed data on capital flows.

Counterfactual 1: Trade Frictions (Open Goods Markets and Capital Autarky) In Figure 5, we undertake a counterfactual for a 50 percent increase in bilateral U.S.-China trade frictions for the special case of our model with openness in goods markets (trade frictions implied by the observed trade share matrices) and autarky in capital markets (prohibitive capital market frictions between all countries). We show counterfactual predictions for log deviations from the initial steady-state in real income ($\left(w_{it}\ell_i + v_{it}a_{it}\right)/p_{it}$,), real wages ($w_{it}/p_{it}$), the real return to investment (v_{it}/p_{it}), real rental rate (r_{it}/p_{it}), wealth (a_{it}), and capital (k_{it}). We show China and the United States by the solid labelled red lines and denote all the other countries in our sample by the solid gray lines.

In response to the higher trade frictions, we observe an immediate drop in real income and real wages at time $t = 1$ for both China and the United States (top-left and top-middle panels), which captures the foregone conventional static welfare gains from trade. This immediate welfare loss is greater for the United States, which reflects the fact that China is more important as a trade partner for the United States than the United States is as a trade partner for China.

Figure 5: Counterfactual for a 50 Percent Increase in Bilateral U.S.-China Trade Frictions (Open Goods Markets and Capital Autarky)



Note: Counterfactual for a permanent, 50 percent increase in bilateral trade frictions between China and the United States at time $t = 1$ using our closed-form solution for the economy's transition path, for the special case of the model with autarky in capital markets and open goods markets; each panel shows log deviations from the initial steady-state; top-left panel shows these log deviations for real income ($y_{it} = (w_{it}\ell_i + v_{it}a_{it})/p_{it}$); top-middle panel shows these log deviations for the real wage (w_{it}/p_{it}); top-right panel shows these log deviations for the real return to investment (v_{it}/p_{it}); bottom-left panel shows these log deviations for the real rental rate (r_{it}/p_{it}); bottom-middle panel shows these log deviations for wealth (a_{it}); and bottom-right panel shows these log deviations for capital (k_{it}).

In addition to these conventional static welfare losses, the increase in the consumption price index in China and the United States from higher trade frictions reduces the real return to investment (top-right panel), which leads to a gradual decumulation of wealth and capital in China and the United States (bottom-middle and bottom-right panels). This decumulation of capital further reduces real income (top-left panel) in these two countries, and gives rise to dynamic welfare losses, as wealth and capital in these two countries gradually converge to their new lower steady-state levels. The real rental rate in China and United States drops on impact (bottom-left panel), as the higher trade frictions between these two countries reduce the demand for their goods, before the decumulation of capital in China and the United States leads to a gradual recovery in their real rental rates.

In a number of other countries, we observe an immediate increase in real income, wages, rental rates and the return to investment at time $t = 1$, which reflects a static cross-substitution effect, as higher trade frictions between China and the United States make all other countries more competitive in these two markets. This increase in the real return to investment in other countries leads to a gradual accumulation of wealth and capital, and dynamic welfare gains, as the wealth state variables in these other countries converge to their new higher steady-state level. For Mexico, the positive cross-substitution effect is outweighed by a negative market size effect from lower income and expenditure in China and the United States. Therefore, Mexico experiences an immediate reduction in real income, wages, rental rates and the return to investment, and dynamic welfare losses from a gradual decumulation of wealth and capital.

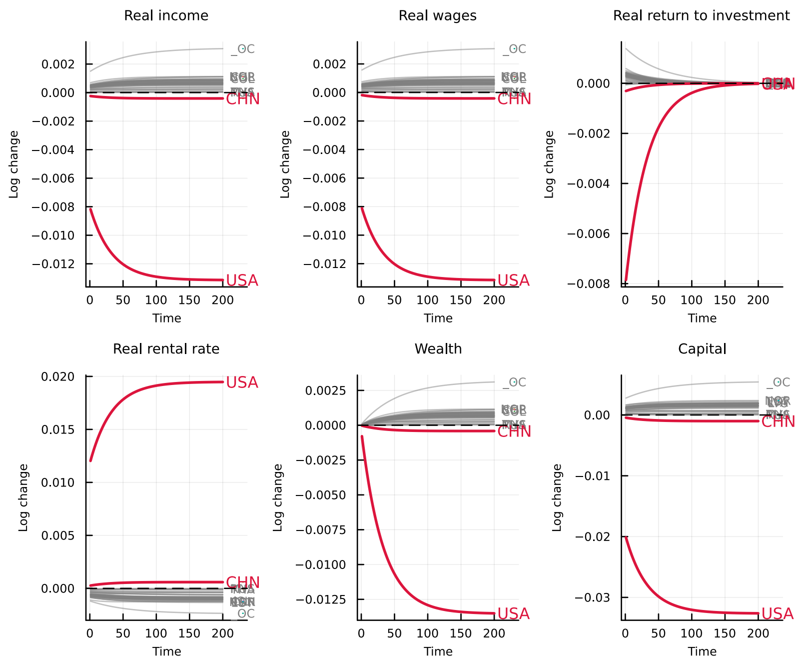
Counterfactual 2: Capital Frictions (Open Capital Markets and Trade Autarky) In Figure 6, we undertake a counterfactual for a 50 percent increase in bilateral U.S.-China capital frictions for the special case of our model with openness in capital markets (capital frictions implied by the observed capital share matrices) and autarky in goods markets (prohibitive trade frictions between all countries). We show counterfactual predictions for the same endogenous variables, and again denote China and the United States by the labelled solid red lines, and indicate the other countries in our sample by the solid gray lines.

In response to these higher bilateral capital frictions, we observe a reduction in the supply of capital at time $t = 1$ in both the United States and China (bottom-right panel). This reduction is larger for the United States than for China, because China is more important as a supplier of capital to the United States than the United States is as a supplier of capital to China. As a result of this greater reduction in the supply of capital, we observe a larger increase in rental rates at time $t = 1$ in the United States (bottom-left panel), and a larger reduction in both real income and real wages at time $t = 1$ in the United States (top-left and top-middle panels).

The increase in bilateral capital market frictions reduces the nominal return to investment at time $t = 1$, which is reflected in a fall in the real return to investment at time $t = 1$ in both China and the United States (top-right panel). However, this reduction in the real return to investment is much larger in the United States than in China, because of the greater decline in the supply of capital and output in the United States, which leads to a larger rise in its consumption price index. As a result of this greater reduction in the real return to investment in the United States, there is a larger decumulation of wealth in subsequent time periods $t > 1$ (bottom-middle panel), which implies a larger further reduction in capital in the United States in subsequent time periods $t > 1$ (bottom-right panel).

These dynamic effects of higher bilateral capital frictions for periods $t > 1$ are substantial relative to the initial static impact in period $t = 1$, and they occur over a prolonged interval of time, highlighting the importance of capital accumulation and transition dynamics.

Figure 6: Counterfactual for a 50 Percent Increase in Bilateral U.S.-China Capital Frictions (Open Capital Markets and Trade Autarky)



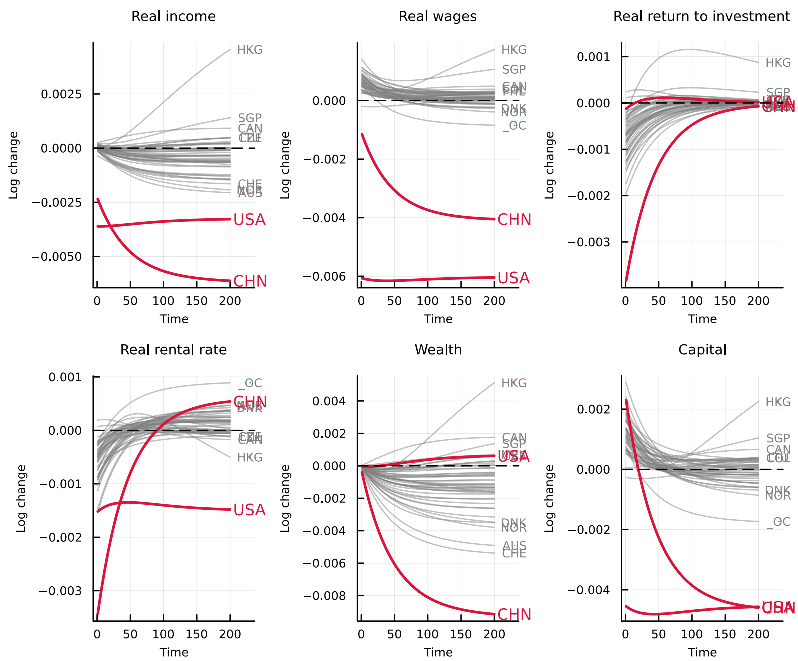
Note: Counterfactual for a permanent, 50 percent increase in bilateral capital frictions between China and the United States at time $t = 1$ using our closed-form solution for the economy's transition path, for the special case of the model with autarky in goods markets and open capital markets; each panel shows log deviations from the initial steady-state; top-left panel shows these log deviations for real income ($y_{it} = (w_{it}\ell_i + v_{it}a_{it})/p_{it}$); top-middle panel shows these log deviations for the real wage (w_{it}/p_{it}); top-right panel shows these log deviations for the real return to investment (v_{it}/p_{it}); bottom-left panel shows these log deviations for the real rental rate (r_{it}/p_{it}); bottom-middle panel shows these log deviations for wealth (a_{it}); and bottom-right panel shows these log deviations for capital (k_{it}).

Counterfactual 3: Trade Frictions (Open Goods and Capital Markets) In Figure 7, we again undertake a counterfactual for a 50 percent increase in bilateral U.S.-China trade frictions, but now undertake this counterfactual for our baseline model with openness in both goods and capital markets (the trade and capital frictions implied by the observed trade and capital share matrices). Comparing Figures 5 and 7, we find a sharply different pattern of counterfactual predictions for this increase in bilateral trade frictions once we allow for linkages between countries in both goods and capital markets, highlighting the important interaction between these two dimensions of international integration.

In response to this increase in trade frictions, we again observe an immediate drop in real income and real wages at time $t = 1$ for both China and the United States (top-left and top-middle panels), which is larger for the United States, and captures the foregone conventional static welfare gains from trade. But the drop in the real return to investment (top right panel) is much larger for China than the United States, which reflects the fact that the United States is much more important as a destination for investment from China than China is as a destination for investment for the United States. Therefore, the higher trade frictions between China and the

United States, and the resulting reduction in real income in the United States, make investment in the United States substantially less attractive for China, thereby reducing its real return to investment. Although the real return to investment also falls immediately in the United States, this drop is much more modest than for China. The resulting reallocation of capital back to the domestic market leads to a larger fall in the real rental rate in China than in the United States (bottom left panel).

Figure 7: Counterfactual for a 50 Percent Increase in Bilateral U.S.-China Trade Frictions (Open Goods and Capital Markets)



Note: Counterfactual for a permanent, 50 percent increase in bilateral trade frictions between China and the United States at time $t = 1$, using our closed-form solution for the economy's transition path for our baseline model with open goods and capital markets for the year 2019; each panel shows log deviations from the initial steady-state; top-left panel shows these log deviations for real income ($y_{it} = (w_{it}\ell_i + r_{it}k_{it}) / p_{it}$); top-middle panel shows these log deviations for the real wage (w_{it}/p_{it}); top-right panel shows these log deviations for the real return to investment (v_{it}/p_{it}); bottom-left panel shows these log deviations for the real rental rate (r_{it}/p_{it}); bottom-middle panel shows these log deviations for wealth (a_{it}); and bottom-right panel shows these log deviations for capital (k_{it}).

In response to the increase in trade frictions at time $t = 1$, there is now a static reallocation of capital across countries, as third countries, such as Canada, Hong Kong, Singapore and Mexico, become much more attractive investment destinations from which to serve the Chinese and United States markets (without the higher bilateral frictions between China and the United States). This reallocation of capital contributes to increases in real income and real wages in Hong Kong and Singapore, and leads to an increase on impact in the real return to investment in Canada, Mexico, Hong Kong and Singapore.

The reduction in the real return to investment in China leads to a gradual decumulation of wealth, and gives rise to dynamic welfare losses, as the economy gradually converges towards a new lower steady-state level of wealth. In contrast, the increase in the real return to investment in Canada, Mexico, Hong Kong and Singapore generates dynamic welfare gains, as these economies gradually converge to a new higher steady-state level of wealth. The resulting increases in real income in Canada and Mexico along the transition path gradually raise real income and the return to investment in the United States relative to their values at time $t = 1$. Ultimately, the United States experiences a small rise in the real return to investment relative to the initial steady-state, which is reflected in a small rise in wealth in the United States in the new steady-state.

Counterfactuals 4 and 5: Capital Frictions (Open Goods and Capital Markets) and Capital and Trade Frictions (Open Goods and Capital Markets) In Section K.2 of the online appendix, we report two further counterfactuals. In a fourth counterfactual, we consider a 50 percent increase in capital frictions in our baseline model with open goods and capital markets and show that find a substantial different pattern of counterfactual predictions from these higher capital frictions than under trade autarky (Counterfactual 2 above). In a fifth counterfactual, we evaluate a 50 percent increase in both trade and capital frictions in our baseline model with open goods and capital markets. Naturally, we tend to find negative welfare effects for the U.S. and China that are larger in absolute magnitude when we increase both frictions rather than only one friction alone. However, the interactions between countries in goods and capital markets lead to a quite different pattern of counterfactual predictions for disintegration in both goods and capital markets than for disintegration in only one of these markets alone.

Goods and Capital Market Linkages We find that this interaction between capital and goods market integration is both qualitatively and quantitatively important. In Counterfactual 1, the United States was more adversely affected by higher China-U.S. trade frictions in the special case of our model with openness in goods markets and autarky in capital markets. In contrast, in Counterfactual 3, China was more negatively affected by these higher bilateral trade frictions in our baseline model with openness in both goods and capital markets. This reversal of fortune between the models with and without open capital markets highlights the importance of studying trade and capital market integration in tandem. Although our modeling of goods and capital markets is necessarily stylized, it is heavily disciplined by the observed gravity equation relationships for trade and capital holdings, as captured in the observed trade and capital share matrices, suggesting that these interactions are likely to continue to be of relevance in other related models disciplined by these same key empirical relationships.

4 Conclusions

The textbook closed-economy neoclassical growth model (CNGM) remains central to our understanding of cross-country income dynamics. But the open-economy versions of this model make strong assumptions about substitutability in goods and capital markets. We generalize this canonical framework to incorporate costly trade and capital flows with imperfect substitutability between countries, such that our framework rationalizes the observed gravity equation relationships for trade and capital holdings in the data.

Our model simultaneously incorporates international goods trade and capital allocations within a given time period, as well as consumption-savings decisions over time. We show that it yields determinate predictions for both gross and net capital holdings, such that bilateral and multilateral imbalances emerge endogenously. It rationalizes empirical findings of home bias in international capital investments, because managing capital abroad is more costly than at home. It also provides a natural explanation for empirical findings of limited bilateral capital flows from rich to poor countries, because capital is imperfectly substitutable across countries, and even if poor countries offer higher rental rates, they can have less productive investment environments and higher costs of managing capital.

We quantify our model using readily-available data on national accounts, bilateral trade, and bilateral capital holdings. We show that our framework permits dynamic exact hat algebra counterfactuals, using only the observed values of endogenous variables in an initial equilibrium. We linearize the model to derive a closed-form solution for the economy's transition path, in terms of an impact matrix that captures the initial impact of shocks, and a transition matrix that governs the updating of the state variables over time. We undertake a spectral analysis of this transition matrix to provide an analytical characterization of the determinants of the speed of convergence to steady-state.

We find substantially slower convergence to steady-state than in the CNGM, helping to address the concern that the CNGM generates rates of convergence to steady-state for plausible intertemporal elasticities of substitution that are too fast relative to the slow transitions observed in the data. We show that goods and capital market integration interact with one another in non-trivial ways. Opening the closed economy to *either* free trade *or* free capital flows generates *faster* convergence than in the CNGM. In contrast, opening *both* free trade *and* free capital flows generates *slower* convergence than in the CNGM.

Since our framework incorporates gravity equations for trade and capital holdings, and allows for intertemporal consumption-savings decisions, it provides a suitable laboratory for evaluating counterfactual policies that affect bilateral frictions in both goods and capital markets (e.g., U.S.-China decoupling). We find that the impact of changes in goods market integration depends heavily on levels of capital market integration (and vice versa). Therefore, higher bilateral trade frictions give rise to conventional cross-substitution and market size effects in goods markets, as in conventional static trade models with capital market autarky. However, in our framework, these higher bilateral trade frictions also lead to a global reallocation of capital, as they alter the geography of market access between all pairs of countries. Furthermore, the resulting movements in the nominal return to investment and the consumption price index affect the real return to investment in each country, thereby giving rise to a rich pattern of dynamic welfare gains and losses along the economy's transition path to steady-state.

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