Cost Coordination*

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1 February 2022

Abstract

When firms engage in price discrimination under competition - due to selling in different geographic markets or to different customer types - they can face a trade-off when choosing to collude. In order to maintain price discrimination, upper-level executives may have to involve those lower-level employees with the demand information needed to tailor prices to markets and customers. However, that comes with an enhanced risk of the cartel's discovery. Alternatively, those executives could centralize pricing authority and coordinate on a more uniform price but that means foregoing some of the profits from price discrimination. Here we put forth another option which is for upper-level executives to coordinate on inflating the cost used in pricing by lower-level employees. Coordinating cost reports is shown to be more profitable than coordinating prices when market heterogeneity is sufficiently great or firms' products are sufficiently differentiated. Recent cartel episodes in which executives coordinated list prices or surcharges are explained to have the crucial features of this collusive scheme.

*I appreciate the excellent research assistance of Sherrie Cheng.

1 Introduction

Consider a market in which firms set many prices either because they sell multiple versions of a product, supply a collection of geographic markets, or offer customer-specific discounts. Collusion among firms in such a market is likely to be challenging. Coordinating on a large number of prices could require many meetings and negotiations and, should they succeed in coming to an agreement, monitoring for compliance will be its own challenge given the need to verify a multitude of prices. Furthermore, implementation of such a collusive scheme may require senior executives to include lower level employees in the collusive arrangement - such as sales managers and sales representatives - because they are needed for coordinating (due to those employees controlling some prices, such as customer-specific discounts) or monitoring (due to them having critical information about rival firms' prices). However, the expansion of the set of employees participating in an unlawful cartel comes with a higher risk of the cartel's discovery and prosecution.

One solution to this dilemma is for high-level executives to restrict the number of prices being charged so they are coordinating over a smaller set of prices. In particular, they could centralize pricing authority and make prices more uniform across markets and customers. However, such a scheme comes with a cost. Under competition, firms found it advantageous to have many prices, presumably because it allowed for more price discrimination. Some of those benefits would be lost by taking pricing authority away from those with the best demand information.

This paper proposes a different solution: firms' high-level executives coordinate on the "cost" that is used in firms' pricing decisions. If prices are set "as if" cost is higher than it actually is, this inflated cost will permeate all prices and the cartel will be able to achieve higher prices while maintaining some degree of price discrimination, and do so by avoiding the inclusion of lower-level employees in the collusive arrangement. A key element of the model which delivers this new collusive scheme is a more realistic representation of the internal pricing process of a firm. Rather than assume the canonical single-actor model of the firm, two levels are assumed: an upper level (such as senior managers) that has better information on cost and a lower level (such as sales managers) that has better information on demand. For this model, the competitive equilibrium has pricing authority delegated to the lower level. By coordinating to inflate the cost reports that the upper levels provide to their lower levels, those upper level executives can raise prices and continue to have price respond to the demand information possessed by lower-level agents. While the scheme sounds promising, there are two challenges. First, it is not clear how effectively coordinating on cost can emulate coordinating on an array of prices. Second, there is the prospect of a nondetectable deviation in the form of a high-level executive intervening in the pricing process and usurping the pricing authority of lower-level agents in order to lower prices.

While this theory has just been put forth as a new collusive scheme, I believe it captures in a stylized and parsimonious way some recently documented collusive practices. Canonically, collusion is with respect to final prices (i.e., the prices faced by customers), as illustrated by the many cartels described in Harrington (2006) and Marshall and Marx (2012). Departing from that scheme, there are instances in which firms coordinated on non-final prices such as list prices (e.g., cement, trucks, and urethane) and surcharges (e.g., air cargo, batteries, and rail freight).[1] The model of this paper captures three key elements of those episodes. First, collusion exclusively involved upper-level managers. Second, lower-level employees - such as sales managers and sales representatives - likely had some pricing authority. Combining these two elements, collusion did not involve some employees who were involved in determining the final prices charged to customers. Third, collusion effectively raised cost in the firm's pricing process. For example, a firm's list price is a signal through which cost is injected into the pricing process. It is well recognized that list prices are sensitive to costs so that when the list price is raised, lower-level employees (as well as buyers when the list price is public information) will infer that cost is higher and that will affect subsequent pricing outcomes. The introduction of a surcharge (e.g., for fuel) can similarly be interpreted as serving to raise perceived cost. Indeed, for both an increase in list prices and the introduction of a surcharge, it is common for the company to explain it as arising from higher cost. Though this third element is represented in a stylized way in the model of this paper, it still delivers a theory to explain the use of these well-documented collusive practices.

The paper makes two contributions. First, it develops a collusive theory in which firms agree to use an inflated cost in their pricing decisions. That coordinating costs can result in supracompetitive prices requires enriching the usual single-actor model to allow multiple agents within the firm to be involved in the pricing process. This model conforms closer to practice in its description of a firm's internal pricing process and delivers an explanation for why certain collusive practices are effective. Second, it provides insight into when firms would choose to coordinate their cost reports rather than collude in the more standard manner of coordinating prices. Coordinating cost reports has the advantage of allowing more tailored pricing because pricing remains delegated to those employees with the best demand information. While it will be assumed that executives can verify those cost reports to other firms' executives, there is the monitoring challenge that an executive may intervene in the pricing process so as to charge lower prices. In contrast, coordinating prices has the disadvantage of less price discrimination but the advantage of better monitoring since it is in terms of customers' prices and those are at least partially observable. It is found that coordinating on cost reports is preferable when firms' products are sufficiently differentiated or there is sufficient market heterogeneity.

Section 2 reviews some related papers. The static model is introduced in Section 3 where the competitive equilibrium is characterized, which has the upper level delegating pricing authority to the lower level. Section 4 offers sufficient conditions for the first-best outcome to be more profitable when firms coordinate their cost reports with decentralized pricing ("cost coordination") than when they coordinate final prices with centralized pricing ("price coordination"). The infinitely repeated game is described in Section 5 and the equilibrium conditions associated with two classes of collusive strategy profiles are provided in Section 6. Sections 7 and 8 characterize when upper-level executives would prefer to engage in cost coordination depending on the extent of product differentiation and market heterogeneity. Section 9 concludes.

[1] References on these cases can be found in Harrington (2021). Also see Boshoff and Paha (2021) for a general discussion of collusive practices involving list prices.

2 Related Research

This paper is related to two streams of research. The first stream is motivated by collusive practices involving coordination of list prices or surcharges. The second stream examines collusion when price discrimination is an option.

From the first stream, two papers are most directly relevant: Harrington and Ye (2019) and Chen (2021). Harrington and Ye (2019) considers firms coordinating their public announcements about cost - such as through list prices - which then affects how buyers bargain. Thus, it shares a feature with this paper's model in exploring how an inflated cost can result in supracompetitive final prices. In Harrington and Ye (2019), the inflated cost affects buyers' beliefs on sellers' costs which causes them to accept higher prices during the buyer-seller bargaining process. That paper's primary contribution is showing how coordinating on publicly observed list prices can be an effective form of collusion, even when firms do not coordinate on discounts off of those list prices. By comparison, this paper's model has an inflated cost report affect other agents in the firm who are involved in the pricing process, and compares coordinating costs to the more standard method of coordinating prices. It is applicable to when list prices are purely internal to a firm, as appears to be the case for the manufacturers involved in the EU trucks cartel.[2]

Though more tangential, collusion in list prices is also examined in Gill and Thanassoulis (2016) and Herold (2021). In those models, list prices are paid by some customers and thus act as more than a signal of a firm's cost. Gill and Thanassoulis (2016) assumes there is also coordination on discounts, which is not the case in Herold (2021). There are also a few papers encompassing a two-level model of the firm in which there is an agreement to *exchange* non-transaction prices (such as list prices) rather than *coordinate* those prices; see Harrington (2019), Andreu, Neven, and Piccolo (2020), and Janssen and Karamychev (2021).

Chen (2021) is motivated by cartel episodes in which firms coordinated surcharges. Similar to this paper, the internal pricing process of the firm is modelled though the upper level chooses a surcharge and the lower level chooses a base price. Demand depends only on the sum of the base price and surcharge. In the spirit of the strategic delegation literature,[3] the upper level designs the lower level's compensation scheme where the class of schemes depends on revenue but can differentiate between total revenue and base revenue (i.e., revenue from the base price which is what the lower level controls). Under competition, equilibrium has an upper level choosing a positive surcharge and making their manager's compensation more sensitive to base revenue. This is shown to cause a firm's lower-level agent to set a higher final price (= base price + surcharge) and, due to strategic complements, induces the rival firm's lower-level agent to set a higher final price. Assuming that the lower-level agents' compensation scheme remains unchanged from that under competition (which is an assumption also made in my model), the paper then explores when upper-level executives

[2] *Commission Decision of 19.7.2016 relating to a proceeding under Article 101 of the TFEA and Article 53 of the EEA Agreement, AT.39824 - Trucks* "From 1997 until the end of 2004, the [firms] participated in meetings involving senior managers of all Headquarters [where] the participants discussed and in some cases also agreed their respective gross price increases." [para. 51] There is no mention in the Decision that the gross list prices were shared with customers.

[3] See Vickers (1985), Ferhstman and Judd (1987), and Sklivas (1987).

coordinate surcharges while leaving lower-level agents to compete in their setting of base prices. Coordinating surcharges is shown to result in the same final prices - and, consequently, profits - as when firms coordinate final prices (i.e., both base prices and surcharges). Given that coordinating on surcharges is simpler and does not involve lower-level agents, the theory provides an explanation for why firms would adopt a collusive practice that has them coordinating surcharges.

Though relevant to some similar cartel episodes, the model and results of my paper are quite different from those in Chen (2021). First, there are multiple heterogeneous markets in this paper's model so firms are setting a vector of prices and, under competition, engaging in price discrimination. Second, the rationale for the lower-level employees having pricing power is not strategically based (i.e., how it affects rival firms' prices) but rather informationally based as they have some private demand information. Finally, this paper's findings are different in that we show coordinating costs can be strictly more profitable than coordinating prices, and industry conditions are identified under which cost coordination is more likely to be adopted.

The second stream investigates collusion when firms can engage in price discrimination. The central question is whether collusion is more or less difficult when firms segment markets and engage in price discrimination where difficulty is measured by the minimum discount factor for sustaining the joint profit maximum (either under price discrimination or a uniform price) assuming the grim punishment (i.e., infinite reversion to the static Nash equilibrium). While the joint profit maximum is higher with price discrimination, there are countervailing effects in that the deviation payoff can be higher and the grim punishment could be more or less severe (depending on the assumptions). Research in this stream includes Liu and Serfes (2007), Colombo (2010), and Helfrich and Herwig (2016). While I will also be comparing price discrimination and a uniform price, the model is quite different in assuming a two-level firm organization and considering a collusive scheme that has price discrimination implemented by coordinating cost reports (with competitive pricing) rather than coordinating prices. Due to the organizational structure, price discrimination as modelled in those papers is not an option.[4]

3 General Model

Consider a symmetric oligopoly setting with $n \geq 2$ firms offering differentiated products and a common cost $c \in [\underline{c}, \overline{c}]$ where $\underline{c} < \overline{c}$. c has a continuously differentiable cdf $H : [\underline{c}, \overline{c}] \rightarrow [0, 1]$ where $H'(c) > 0 \ \forall c \in [\underline{c}, \overline{c}]$. $D_i(p_1, ..., p_n, a) : \Re^n_+ \times [\underline{a}, \overline{a}] \rightarrow \Re_+$ is the (symmetric) firm demand function for market (or customer) type a given firms' prices $(p_1, ..., p_n)$. There is a collection of market types which, for analytical ease, is a continuum with a continuously differentiable cdf $G : [\underline{a}, \overline{a}] \rightarrow [0, 1]$, where $\underline{a} < \overline{a}$ and $G'(a) > 0 \ \forall a \in [\underline{a}, \overline{a}]$.

Each firm's organization has two levels where Ui (Li) denotes the upper (lower) level of firm i, $i = 1, ..., n$. One can think of the upper level as a senior executive and the lower level

[4]In addition, the analysis of this paper examines when the discount factor is close to one which is an uninteresting case for these other papers because price discrimination is always preferable to a uniform price when there are non-binding incentive compatibility constraints.

as a pricing manager or sales representative.[5] Ui has private information of cost and Li has private information of the market type. All other information is common knowledge. Ui chooses whether to have a centralized or decentralized pricing structure. Under centralization, Ui chooses price p_i which does not condition on a. Under decentralization, Ui conveys a (cheap talk) cost report x_i to Li. With knowledge of x_i and a, Li chooses price while conditioning on a.[6]

The extensive form is:

- Stage 1: Ui learns cost c, $i = 1, ..., n$.
- Stage 2: Ui chooses the pricing structure: centralization or decentralization.
- If Ui chose decentralization then:
  - Stage 3: Ui chooses cost report $x_i \in [\underline{c}, \overline{c}]$.
  - Stage 4: Li observes x_i and a and then chooses price $p_i(a)$, $\forall a \in [\underline{a}, \overline{a}]$.
- If Ui chose centralization then:[7]
  - Stage 3: Ui chooses price p_i.

Note that $U1, ..., Un$ make simultaneous pricing structure decisions and those decisions are private at the time when an organization chooses a cost report and price vector (under decentralization) or a price (under centralization). For the competitive solution, it does not matter that cost is learned prior to the pricing structure decision as the equilibrium is robust to switching the order. For the collusive solution, altering the sequence is likely to change the equilibrium but the main qualitative insight should be robust. The payoffs of Ui and Li are assumed to be proportional to the firm's profit. Thus, any agency problem has been solved and that solution is assumed fixed throughout the analysis.[8]

Let us begin by characterizing a symmetric separating perfect Bayes-Nash equilibrium for the one-shot game that will be the competitive solution. Consider the following symmetric strategy profile and beliefs:

- Ui chooses decentralization and $x_i = c \; \forall c \in [\underline{c}, \overline{c}]$.
- Li chooses $p_i = p^N(a, x_i) \; \forall a \in [\underline{a}, \overline{a}]$ where

$$p^N(a, x_i) \equiv \arg\max_{p_i} (p_i - x_i) D_i \left(p^N(a, x_i), ..., p_i, ..., p^N(a, x_i), a\right).$$

[5] There could be many lower-level agents and it is without loss of generality to assume there is one.

[6] Two comments are in order regarding the stark distinction between centralized and decentralized pricing. First, one can think of there being two types of demand variation; that which is observable to the upper (and possibly lower) level and that which is observable only to the lower level. For parsimony, the former is assumed away. Second, what is critical for the analysis is that price is more sensitive to the market type under decentralized pricing, which seems compelling.

[7] Under centralization, Ui also has the option of choosing a cost report though it will not be impactful. Having that option will be relevant when it comes to examining collusion.

[8] While one could allow for an agency problem, it is orthogonal to the issue at hand and I conjecture the qualitative findings are robust to a class of compensation schemes for lower-level agents.

- Li assigns probability one to $c = x_i \; \forall x_i \in [\underline{c}, \overline{c}]$.

Li's beliefs are correct given Ui's strategy. Note that if $x_i = x'$ then Li believes $x_j = x'$ $\forall j \neq i$ because there is a common cost. Given Lj is then expected to price at $p^N(a, x')$, it is optimal for Li to price at $p^N(a, x')$ given its payoff is proportional to[9]

$$(p_i - x') \, D_i \left(p^N(a, x'), ..., p_i, ..., p^N(a, x'), a \right).$$

Given c, Ui's payoff from decentralization and reporting x_i is proportional to

$$\int \left(p^N(a, x_i) - c \right) D_i \left(p^N(a, c), ..., p^N(a, x_i), ..., p^N(a, c), a \right) G'(a) da,$$

for which $x_i = c$ optimal. Finally, decentralization is optimal because

$$\begin{aligned} & \int \left(p^N(a, c) - c \right) D_i \left(p^N(a, c), ..., p^N(a, c), a \right) G'(a) da \\ > \; & \max_{p_i} \int (p_i - c) \, D_i \left(p^N(a, c), ..., p_i, ..., p^N(a, c), a \right) G'(a) da. \end{aligned}$$

Given there is no conflict of interest between the levels in a firm, the upper level will decentralize pricing and truthfully report cost to the lower level, and the lower level will then set price so as to maximize the firm's profit based on the cost report received. Consequently, equilibrium pricing is the same as when each firm has only one level that sets price knowing all cost and demand information.[10]

A1-A4 are presumed to hold $\forall (a, c) \in [\underline{a}, \overline{a}] \times [\underline{c}, \overline{c}]$. With one exception to be discussed below, A1-A4 are standard and, for example, hold for the case of linear demand examined in Section 8.

A1 $D_i(c, ..., c, a) > 0$.

A2 $p^N(a, x)$ exists and is twice differentiable and increasing in a and x. (Note: If $D_i(x, ..., x, a) > 0$ then $p^N(a, x) > x$.)

A3 $\pi(p, a, c) \equiv (p - c) \, D_i(p, ..., p, a)$ is differentiable and strictly quasi-concave in p. Hence, $p^M(a, c) \equiv \arg\max (p - c) \, D_i(p, ..., p, a)$ exists. $p^M(a, c)$ is differentiable and increasing in a and c.

A4 $p^M(a, c) > p^N(a, c) > c$.

[9] Given that cost reports are informative, the lower level's objective is the same whether it is assumed their compensation is based on "calculated" profit (using the cost report) or on actual realized profit. With the latter case, the agent acts to maximize expected profit given their belief on cost which puts unit mass on the cost report.

[10] This is the unique separating PBNE. As the centralization/decentralization decision is private information with respect to the other firms and the interests of the two levels are fully aligned, Ui always wants to delegate pricing to Li so that price can condition on a. Of course, there are pooling PBNE where cost reports are uninformative. However, it would always be in the mutual interest of a firm's two levels to coordinate on a separating strategy, regardless of what the other firms do. Given their interests are aligned, separating equilibria seem compelling.

The specification of market heterogeneity encompasses two substantive restrictions. First, firms' demands are symmetric. While this is a common assumption in the price discrimination literature (see, for example, Holmes 1989), it does imply that firms' competitive profits are higher with price discrimination compared to a uniform price. There are alternative demand specifications whereby firms' competitive profits are lower with price discrimination.[11] Second, the change in the monopoly and competitive prices with respect to the market type are of the same sign. As the monopoly price is assumed to be increasing in a, a higher value of a corresponds to a "stronger" market in the sense of a lower market-price elasticity of demand (i.e., less price elastic).[12] In assuming the competitive price is also increasing in a, the presumption is that a stronger market also has a lower firm-price elasticity of demand. Given that a firm's price elasticity of demand can be decomposed into the sum of the market-price elasticity of demand and the cross-price elasticity of demand, the assumption is that the market variation in the cross-price elasticity of demand is not sufficiently great so as to offset the variation in the market-price elasticity of demand. In particular, markets with more price-inelastic market demand do not have firms' products being sufficiently more substitutable. While this assumption does rule out some cases, it is still quite general.

4 First-Best Collusion

As in a number of cartels, suppose it is just firms' upper level executives who collude. Assume they do so in order to maximize firms' profits. The lower levels are not involved in collusion and, as they are unaware of it, continue to act according to the competitive solution. Two different collusive schemes are considered. The first scheme has the upper levels controlling price (centralization) and jointly choosing prices to maximize a typical firm's profit. The optimal collusive (uniform) price is

$$\widehat{p}(c) \equiv \arg\max \int (p-c)\, D_i\,(p,...,p,a)\, G'(a)da. \tag{1}$$

This scheme is referred to as "price coordination." The second scheme has the upper levels maintaining decentralized pricing while jointly choosing cost reports to maximize a typical firm's profit taking as given that the lower levels will price competitively based on the cost reports.[13] The optimal collusive cost report is

$$\widehat{x}(c) \equiv \arg\max_x \int \left(p^N(a,x)-c\right) D_i\left(p^N(a,x),...,p^N(a,x),a\right) G'(a)da \tag{2}$$

[11] This distinction is discussed in Corts (1998) where properties on the best response function determine whether price discrimination raises or lower profits compared to uniform pricing. In his terminology, our demand specification assumes best response symmetry.

[12] This discussion is based on Stole (2007), pp. 2234-2235.

[13] It is thus assumed that the colluding executives do not have control over the lower level's compensation scheme. One can suppose that is set by some other agents who reside between the lower and upper levels and are not part of the collusive scheme.

where it is straightforward to establish that $\widehat{x}(c) > c$.[14] Generally, both schemes are second-best. Price coordination is second-best because price does not condition on the demand state. Cost coordination is second-best because price is set competitively conditional on the cost report.

Theorem 1 offers a sufficient condition for first-best cost coordination to be more profitable than first-best price coordination; that is, maximal profit is higher by coordinating on an inflated cost used by pricing managers than that by taking control of price and coordinating on a supracompetitive uniform price. This sufficient condition is that the sensitivity to the market type of the monopoly price (evaluated at cost) exceeds that of the competitive price (evaluated at cost or an inflated cost report). Proofs are in the appendix.

Theorem 1 *If*

$$\frac{\partial p^M(a,c)}{\partial a} > \frac{\partial p^N(a,x)}{\partial a} > 0 \ \forall x \geq c, \ \forall a \in [\underline{a}, \overline{a}] \tag{3}$$

then first-best cost coordination is more profitable than first-best price coordination:

$$\int \left(p^N(a, \widehat{x}(c)) - c\right) D_i \left(p^N(a, \widehat{x}(c)), ..., p^N(a, \widehat{x}(c)), a\right) G'(a)da \tag{4}$$

$$> \int \left(\widehat{p}(c) - c\right) D_i \left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da.$$

Note that (3) in Theorem 1 holds if, evaluated at cost, the monopoly price is more sensitive to the market type than is the competitive price - $\frac{\partial p^M(a,c)}{\partial a} > \frac{\partial p^N(a,c)}{\partial a}$ - and the effect of the cost report on the competitive price's sensitivity to the market type, $\frac{\partial p^N(a,x)}{\partial a \partial x}$, is sufficiently bounded. Both of those conditions hold for the case of linear demand examined in Section 8. Also, it is not literally needed that (3) holds $\forall x \geq c$ but rather that it holds up to some upper bound (which could be the first-best cost report).

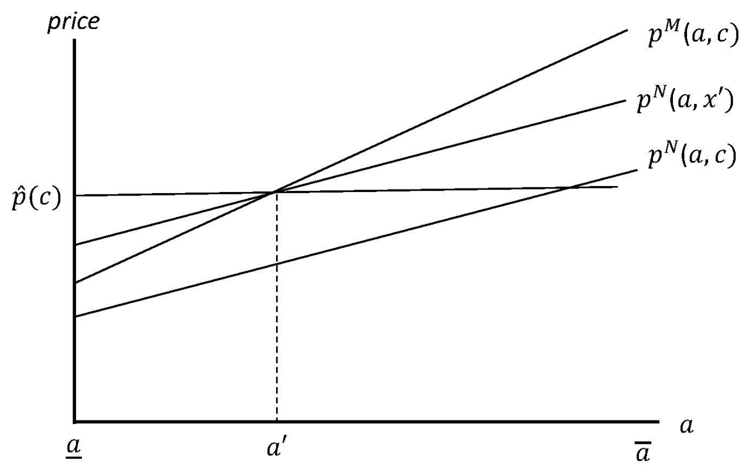
The proof strategy for Theorem 1 is as follows. First, find the demand state a' such that the monopoly price when conditioning on a equals the monopoly price when not conditioning on a: $p^M(a', c) = \widehat{p}(c)$; see Figure 1. Next find the cost report x' that equates the competitive price to the price under price coordination at demand state a': $p^N(a', x') = \widehat{p}$; again see Figure 1. Thus, at $a = a'$, cost coordination with cost report x' delivers the same profit as price coordination with uniform price $\widehat{p}(c)$. For stronger demand states $(a > a')$, $p^N(a, x') \in \left(\widehat{p}(c), p^M(a, c)\right)$ so cost coordination yields higher profit (than price coordination) by having price be higher (which follows from the strict quasi-concavity of the profit function with respect to a common price across firms). For weaker demand states $(a < a')$, $p^N(a, x') \in \left(p^M(a, c), \widehat{p}(c)\right)$ so cost coordination yields higher profit by having price be lower than $\widehat{p}(c)$.

It is interesting that these conditions for cost coordination to be more profitable do not require that market heterogeneity is sufficiently great even though the relative advantage of cost coordination is allowing price to condition on the market type. Rather, the conditions ensure that competitive pricing with an inflated cost is able to sufficiently approximate first-best price discrimination so that it is more profitable than the first-best uniform price. In

[14]This section focuses on first-best profits and, in particular, ignores the constraint that $x \leq \overline{c}$ so that lower levels assign a positive probability to a cost of x. This constraint will be taken into account in Section 6 when characterizing equilibrium.

a rough sense, the inflated cost is used to raise average price - achieving the same end as a high uniform price - and then there is the added benefit that price is responsive to demand (though how it is responsive is not under the control of the upper-level managers for that is determined by competitive pricing).

Figure 1



5 Infinitely Repeated Game

Now consider an infinitely repeated version of the game described in Section 4 where $\delta \in (0,1)$ is firms' common discount factor. There are two key informational assumptions. First, when firms engage in cost coordination, they are able to provide verifiable information about the cost reports that were internally distributed. This could be done by sharing internal memos or files that document the costs that were conveyed from the upper to the lower level. Second, at the end of each period, a firm observes a finite random sample of rival firms' prices. This is to capture the ability to engage in partial monitoring of prices. In order to avoid the complications of private monitoring, the random sample of a firm's prices is common knowledge to all firms.[15]

Our focus is on comparing the two classes of collusive schemes described in the preceding section: price coordination (so the upper levels centralize and then coordinate on a common price) and cost coordination (so the upper levels decentralize, coordinate on a common cost report, and lower levels competitively set prices based on those cost reports). While this will be made more formal below, it is useful to describe the types of deviations that can occur. Under price coordination, the upper levels agree to centralize and then coordinate on a common high price p'. An upper level could deviate by continuing to centralize but setting a price below p'. Alternatively, it could deviate by decentralizing and strategically selecting a cost report. Both forms of deviations are off-path (in that they result in prices different

[15] The stage game is such that, in each period, there is a fresh cost draw after which firms choose prices. It is probably more natural to suppose that cost changes less frequently than price. Assuming the same frequency for changes in costs and prices is a simplifying assumption in that all ensuing results hold if instead firms select prices (and imperfectly observe those prices) multiple times between cost changes.

from equilibrium prices) and, with a random sampling of prices, are observed (at least with positive probability). As shown below (and for the usual reasons), these off-path incentive compatibility constraints (ICCs) are satisfied when $\delta \simeq 1$.

Under cost coordination, the upper levels decentralize, coordinate on a common cost report x^o, and lower levels set prices based on that cost report so prices are $\left\{p^N(a, x^o)\right\}_{a\in[\underline{a},\overline{a}]}$. Deviation can take one of two forms. An upper level could deviate by continuing to decentralize and setting a lower cost report than x^o. Given cost reports are verifiable, that deviation would be detected and, consequently, the associated ICC is satisfied when $\delta \simeq 1$.[16] The other form of deviation has an upper level choosing the collusive cost report x^o but centralizing (so the cost report becomes irrelevant for pricing) and sets some uniform price p''. If $p'' \notin \left[p^N(\underline{a}, x^o), p^N(\overline{a}, x^o)\right]$ then again it is an off-path deviation that is detected and, therefore, the ICC is satisfied when $\delta \simeq 1$. The challenging deviation is centralization and $p'' \in \left[p^N(\underline{a}, x^o), p^N(\overline{a}, x^o)\right]$ as then it is an on-path deviation. That is, a random sampling of a firm's prices that turns up p'' is consistent with compliance and having sampled the firm's price in market type a'' where $p^N(a'', x^o) = p''$.[17,18]

One type of deviation that is not permitted is changing the incentive scheme for the lower level. It is assumed to be fixed throughout the analysis; that is, a lower level's compensation is always proportional to profit. One can imagine that it is set by some unmodelled level such as a mid-level manager that resides between the upper and lower levels.[19] The focus of this paper is on other forms of deviation which I conjecture are empirically more relevant. It is also assumed lower levels do not suspect collusion which is reasonable given the upper levels' intent not to involve them.[20]

The analysis will focus on when $\delta \simeq 1$ so all off-path ICCs are satisfied. The strategies will assume the grim punishment as that will be sufficient for our purpose. As price coordination involves only off-path ICCs then $\delta \simeq 1$ implies the first-best outcome is achieved with price coordination. However, that is not necessarily the case with cost coordination because of the possibility of an on-path deviation.

[16] That deviation would also be observable through price monitoring. If an upper level deviates with cost report $x'(< x^o)$ then non-equilibrium prices $\left[p^N(\underline{a}, x'), p^N(\underline{a}, x^o)\right)$ will be observed.

[17] Note that it is the upper level that is engaged in price monitoring and does not observe the market's type, which is private information to the lower level. Monitoring is then more difficult compared to when the lower level is participating in the collusive scheme.

[18] Of course, if a firm is complying then it is a probability zero event to observe the same price p'' more than once. However, if that were to be seen as an off-path event and punished for sure, a deviating firm could easily avoid it by adding a small amount of noise to its deviation price which is randomly assigned to differnet market types.

[19] Assume there are three levels. The upper level observes cost and provides a cost report to the lower level. The middle level's task is to choose a compensation scheme for the lower level. Given the compensation scheme, the cost report, and demand information, the lower level chooses prices. Assume that the compensation of the upper and middle levels is proportional to firm profit and only the upper level is engaged in collusion. Collusion will then not affect the compensation scheme and the same equilibrium outcome will occur as is described here.

[20] There is a similar informational structure in Harrington and Ye (2019) in that sellers are colluding but buyers do not suspect such collusion. It is shown that results are robust to allowing buyers to assign a small probability to sellers colluding. That same robustness may hold here.

6 Equilibrium Conditions

6.1 Price coordination

Consider the following symmetric strategy for the upper level of firm i where the collusive price is the first-best price as defined in (1) and c^t is the common cost in period t.

- In period 1, centralize and price at $\widehat{p}(c^1)$.
- In period $t = 2, 3, ...$
  - centralize and price at $\widehat{p}(c^t)$ if all period τ sampled prices equalled $\widehat{p}(c^\tau)$, $\forall \tau = 1, ..., t-1$.
  - decentralize and submit cost report $x_i^t = c^t$ otherwise.

Consider some period and let c' be the current period's cost. By symmetry, we can state the equilibrium conditions in terms of firm 1. The ICC associated with deviating by maintaining centralization and charging a price different from $\widehat{p}(c')$ is:[21]

$$\begin{aligned} & \int \left(\widehat{p}(c') - c'\right) D_1\left(\widehat{p}(c'), ..., \widehat{p}(c'), a\right) G'(a)da \\ & + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da\right] \\ \geq \ & \max_{p_1} \int \left(p_1 - c'\right) D_1\left(p_1, \widehat{p}(c'), ..., \widehat{p}(c'), a\right) G'(a)da \\ & + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(p^N(a,c) - c\right) D_1\left(p^N(a,c), ..., p^N(a,c), a\right) G'(a)da\right] \\ \forall c' \ \in \ & \left[\underline{c}, \overline{c}\right]. \end{aligned} \tag{5}$$

[21] It is assumed that a deviation occurs with respect to all markets which implies detection of it occurs for sure. Alternatively, deviation could occur with respect to a subset of markets. If the deviation entails continued centralization and a lower price, a firm could apply that lower price in a (random) fraction $1-\theta$ of markets. If w prices of a firm are randomly sampled then the probability of detection is $1-\theta^w$. Results are robust to this modification.

Assuming the lower level best responds to the prices of the other firms, the ICC associated with decentralizing and choosing cost report x_1 is[22]

$$\begin{aligned}
& \int \left(\widehat{p}(c') - c'\right) D_1\left(\widehat{p}(c'), ..., \widehat{p}(c'), a\right) G'(a)da \\
& + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da\right] \\
\geq \; & \max_{x_1} \int \left(\psi_1\left(\widehat{p}(c'), ..., \widehat{p}(c'), x_1, a\right) - c'\right) D_1\left(\psi_1\left(\widehat{p}(c'), ..., \widehat{p}(c'), x_1, a\right), \widehat{p}(c'), ..., \widehat{p}(c'), a\right) G'(a)da \\
& + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(p^N(a,c) - c\right) D_1\left(p^N(a,c), ..., p^N(a,c), a\right) G'(a)da\right] \\
\forall c' \; \in \; & [\underline{c}, \overline{c}]
\end{aligned} \tag{6}$$

where

$$\psi_1\left(p_2, ..., p_n, x_1, a\right) \equiv \arg\max_{p_1}\left(p_1 - x_1\right) D_1\left(p_1, p_2, ..., p_n, a\right).$$

If $\delta \simeq 1$ then it is straightforward to show that (5)-(6) are satisfied if:

$$\begin{aligned}
& E_c\left[\int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da\right] \\
> \; & E_c\left[\int \left(p^N(a,c) - c\right) D_1\left(p^N(a,c), ..., p^N(a,c), a\right) G'(a)da\right]
\end{aligned} \tag{7}$$

which holds if

$$\begin{aligned}
& \int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da \\
> \; & \int \left(p^N(a,c) - c\right) D_1\left(p^N(a,c), ..., p^N(a,c), a\right) G'(a)da, \forall c \in [\underline{c}, \overline{c}];
\end{aligned} \tag{8}$$

in other words, first-best (centralized) price coordination is more profitable than (decentralized) competition. (8) need not always be true as it is possible that firms do better by competing and engaging in price discrimination than colluding with a uniform price. However, there are conditions satisfying (8) in which case price coordination is preferred to competition. For the case of linear demand, such conditions are derived in Section 8.

6.2 Cost Coordination

Consider the following symmetric strategy for the upper level of firm i where the collusive cost report is $\widetilde{x}(c) : [\underline{c}, \overline{c}] \rightarrow [\underline{c}, \overline{c}]$. It is not presumed that the cost report is the first-best cost report (2).

[22] It is not immediate how the lower level should price in this scenario. However, note that the deviation payoff in (6) is the maximal deviation payoff from the perspective of the upper level. Hence, if (6) holds then the upper level will not want to deviate for any other possible assumption it makes as to how the lower level will price.

- In period 1, decentralize and submit cost report $x_i^1 = \widetilde{x}(c^1)$.
- In period t,
  - decentralize and submit cost report $x_i^t = \widetilde{x}(c^t)$ if $x_j^\tau = \widetilde{x}(c^\tau)$ $\forall j = 1, ..., n$ and all period τ sampled prices are in $\left[p^N(\underline{a}, \widetilde{x}(c^\tau)), p^N(\overline{a}, \widetilde{x}(c^\tau))\right]$, $\forall \tau = 1, ..., t-1$.
  - decentralize and submit cost report $x_i^t = c^t$ otherwise.

Note that $\widetilde{x}(c)$ could equal c for some values of c which would mean that firms do not charge supracompetitive prices for those cost realizations. Also note that an off-path deviation is punished with the grim punishment but an on-path deviation is not punished. Alternatively, imperfect public monitoring could be used to discipline on-path deviations. For example, a statistical test could be constructed based on past sampled prices where a punishment occurs when those prices are sufficiently unlikely should all firms have complied. We chose not to deploy punishments based on imperfect monitoring in order to maintain a realistic simplicity to firms' strategies.[23] Alternatively, one can view our results as providing a lower bound on the maximal equilibrium payoff under cost coordination. In that case, the conditions such that cost coordination is preferred to price coordination are sufficient (though the conditions whereby price coordination is preferred may not be robust to allowing for a richer set of punishments with cost coordination).

Given cost c', consider an upper level deviating by choosing a cost report different from $\widetilde{x}(c')$. As cost reports are assumed to be verifiable, this deviation would be detected. The ICC is

$$\begin{aligned} & \int \left(p^N(a, \widetilde{x}(c')) - c'\right) D_1\left(p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a) da \\ & + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(p^N(a, \widetilde{x}(c)) - c\right) D_1\left(p^N(a, \widetilde{x}(c)), ..., p^N(a, \widetilde{x}(c)), a\right) G'(a) da\right] \\ \geq \; & \max_{x_1} \int \left(p^N(a, x_1) - c'\right) D_1\left(p^N(a, x_1), p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a) da \\ & + \left(\frac{\delta}{1-\delta}\right) E_c\left[\int \left(p^N(a, c) - c\right) D_1\left(p^N(a, c), ..., p^N(a, c), a\right) G'(a) da\right] \\ \forall c' \; \in \; & [\underline{c}, \overline{c}]. \end{aligned} \tag{9}$$

It is straightforward to establish that if $\delta \simeq 1$ then (9) is satisfied as long as

$$\begin{aligned} & E_c\left[\int \left(p^N(a, \widetilde{x}(c)) - c\right) D_1\left(p^N(a, \widetilde{x}(c)), ..., p^N(a, \widetilde{x}(c)), a\right) G'(a) da\right] \\ > \; & E_c\left[\int \left(p^N(a, c) - c\right) D_1\left(p^N(a, c), ..., p^N(a, c), a\right) G'(a) da\right], \end{aligned} \tag{10}$$

[23] My knowledge of cartel conduct makes me more comfortable engaging in equilibrium selection based on (an admittedly subjective notion of) simplicity than on complex optimal equilibria that maximize equilibrium payoffs. While documentary evidence from cartels show firms give considerable attention to specifying the collusive outcome and designing the monitoring protocol, discussions about punishments are highly incomplete or absent, as evidenced by the many cartel episodes covered in Harrington (2006). For this reason, there is little empirical evidence justifying the selection of equilibria with complex punishments. I conjecture that firms see a heightened chance of cartel collapse and a return to competition as the implicit punishment.

which is true by construction of $\widetilde{x}(\cdot)$ (assuming $\widetilde{x}(c) > c$).

Next consider a deviation in which an upper level centralizes and charges a uniform price. A uniform price outside of $\left[p^N(\underline{a}, \widetilde{x}(c)), p^N(\overline{a}, \widetilde{x}(c))\right]$ is an off-path deviation. It is straightforward to show the associated ICC is satisfied when $\delta \simeq 1$ and (10) holds. The on-path ICC is when the deviation price lies in $\left[p^N(\underline{a}, \widetilde{x}(c')), p^N(\overline{a}, \widetilde{x}(c'))\right]$:

$$\begin{aligned} & \int \left(p^N(a, \widetilde{x}(c')) - c'\right) D_1\left(p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a)da \\ \geq & \max_{p_1 \in [p^N(\underline{a}, \widetilde{x}(c')), p^N(\overline{a}, \widetilde{x}(c'))]} \int \left(p_1 - c'\right) D_1\left(p_1, p^N(a, \widetilde{x}(c')), ..., p^N(a, \widetilde{x}(c')), a\right) G'(a)da, \ \forall c' \in [\underline{c}, \overline{c}]. \end{aligned} \tag{11}$$

Assuming $\delta \simeq 1$ so the other ICCs are satisfied, the cost coordination problem is then, for each $c \in [\underline{c}, \overline{c}]$, to choose the cost report that maximizes profit subject to satisfying (11). Note that there is little room to inflate cost when $c \simeq \overline{c}$ so there won't be much if any collusion for high cost realizations. I will return to this point later.

6.3 Profit Comparison

In identifying conditions under which firms would prefer to engage in cost coordination than price coordination, our focus will be on when $\delta \simeq 1$. In that case, cost coordination is more profitable than price coordination iff:

$$\begin{aligned} & \int \left(\int \left(p^N(a, \widetilde{x}(c)) - c\right) D_1\left(p^N(a, \widetilde{x}(c)), ..., p^N(a, \widetilde{x}(c)), a\right) G'(a)da \right) H'(c)dc \\ > & \int \left(\int \left(\widehat{p}(c) - c\right) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da \right) H'(c)dc \end{aligned} \tag{12}$$

where

$$\widehat{p}(c) \equiv \arg\max \int (p - c)\, D_i\, (p, ..., p, a)\, G'(a)da,$$

and

$$\widetilde{x}(c) \equiv \arg\max_{x \in [\underline{c}, \overline{c}]} \int \left(p^N(a, x) - c\right) D_1\left(p^N(a, x), ..., p^N(a, x), a\right) G'(a)da \tag{13}$$

subject to

$$\begin{aligned} & \int \left(p^N(a, x) - c\right) D_1\left(p^N(a, x), ..., p^N(a, x), a\right) G'(a)da \\ \geq & \max_{p_1 \in [p^N(\underline{a}, x), p^N(\overline{a}, x)]} \int \left(p_1 - c\right) D_1\left(p_1, p^N(a, x), ..., p^N(a, x), a\right) G'(a)da, \ \forall c \in [\underline{c}, \overline{c}]. \end{aligned} \tag{14}$$

As H is continuous, it is sufficient to show the integrands in (12) satisfy the inequality:

$$\begin{aligned} & \int_{\underline{a}}^{\overline{a}} (p^N(a, \widetilde{x}(c)) - c) D_1\left(p^N(a, \widetilde{x}(c)), ..., p^N(a, \widetilde{x}(c)), a\right) G'(a)da \\ > & \int_{\underline{a}}^{\overline{a}} (\widehat{p}(c) - c) D_1\left(\widehat{p}(c), ..., \widehat{p}(c), a\right) G'(a)da, \end{aligned} \tag{15}$$

for almost all c. It is also the case that if (15) does not hold for almost all c that (12) is not true and, therefore, price coordination is preferred to cost coordination (presuming that price coordination is more profitable than competition). This will be the proof strategy used in the next section.

7 Relative Performance of Cost Coordination and Price Coordination

In determining when colluding firms prefer to coordinate cost reports than prices, two market traits are considered: product differentiation and market (or customer) heterogeneity. A measure of product differentiation is defined in the next sub-section. More market heterogeneity is captured by greater dispersion in G which is the distribution on market types.

7.1 Effect of Product Differentiation

The variable $\gamma \in [0,1]$ is introduced to represent the degree of product similarity where $\gamma = 0(1)$ is independent (homogeneous) products. Recall that $\widehat{p}(c)$ and $p^M(a,c)$ is the joint profit-maximizing or monopoly price when pricing is uniform (does not condition on a) and when it involves third-degree price discrimination (price does condition on a), respectively. $\widehat{p}(c)$ and $p^M(a,c)$ will be assumed to be independent of γ. The symmetric Nash equilibrium price is

$$p^N(a,x,\gamma) \equiv \arg\max_{p_1} (p_1 - x)\, D_1\left(p_1, p^N(a,x,\gamma), ..., p^N(a,x,\gamma), a, \gamma\right),$$

and assume: if $p > (<) p^N(a,x,\gamma)$ then $\psi_1(p,...,p,a,x,\gamma) < (>) p$. That higher values of γ correspond to less differentiated products is reflected in the following three assumptions. $p^N(a,x,\gamma)$ is continuously decreasing in γ, the Nash equilibrium price approaches cost when products become homogeneous (minimally differentiated),

$$\lim_{\gamma \to 1} p^N(a,x,\gamma) = x,$$

and approaches the monopoly price when products become independent (maximally differentiated),

$$\lim_{\gamma \to 0} p^N(a,x,\gamma) = p^M(a,x).$$

Theorem 2 shows if products are sufficiently differentiated then colluding firms prefer to coordinate on injecting an inflated cost into their decentralizing pricing process than coordinate on a uniform price. Recall that cost coordination is said to be preferred to price coordination when (12) holds.

Theorem 2 $\exists \gamma' > 0$ *such that if* $\gamma \in (0, \gamma')$ *then cost coordination is preferred to price coordination.*

Theorem 2 is proven by the following argument. As products become maximally differentiated, the profits from competition converge to those from first-best collusion. At the same time, the profits from third-degree price discrimination are bounded above the profits from a uniform price. Thus, when products are sufficiently differentiated, competition with third-degree price discrimination is more profitable than collusion with a uniform price. Hence, price coordination is inferior to competition. At the same time, there exists an inflated cost report for which cost coordination is incentive compatible and more profitable than competition. Though this proof strategy is based on deriving sufficient conditions for price coordination to be less profitable than competition, the result is more general in that an intermediate level of product differentiation can make cost coordination more profitable than price coordination when price coordination is more profitable than competition. Though this is difficult to prove analytically, it is established using numerical methods in Section 8.

The next result shows that price coordination is preferred when products are sufficiently similar. Thus, if firms are offering commodities then they will prefer the more standard method of coordinating the prices faced by consumers.

Theorem 3 $\exists \gamma' < 1$ *such that if* $\gamma \in (\gamma', 1)$ *then price coordination is preferred to cost coordination.*

As products are less differentiated, the symmetric Nash equilibrium price becomes more sensitive to cost and less sensitive to market type. When products are near homogeneous, prices are approximately uniform and equal to cost. Thus, the advantage of price discrimination from cost coordination is small when product differentiation is low. Furthermore, for cost coordination to be at least as profitable as price coordination, the inflated cost report must result in a price in the neighborhood of the optimal uniform price $\widehat{p}$. As the competitive price is close to cost when products are near homogeneous, this means the cost report $\widetilde{x}(\gamma)$ must be close to $\widehat{p}$. However, as shown in the proof, this leaves room for a profitable on-path deviation. If firms are coordinating on a common cost report of $\widetilde{x}(\gamma)$, a firm can profitably deviate by centralizing pricing authority and setting a uniform price of $p^N(\underline{a}, \widetilde{x}(\gamma), \gamma)$; that is, pricing as if $a = \underline{a}$. This on-path deviation is shown to yield a higher profit for all values of a when products are sufficiently similar. Even for markets with strong demand (i.e., high values of a), a firm's profit is higher by undercutting with price $p^N(\underline{a}, \widetilde{x}(\gamma), \gamma)$ than charging the market-specific collusive price $p^N(a, \widetilde{x}(\gamma), \gamma)$ under cost coordination.

This preference for price coordination can also be described as follows. When products are sufficiently similar, prices are almost uniform under cost coordination so the first-best outcome under cost coordination is only slightly better than under price coordination. At the same time, monitoring with cost coordination limits how much the cost report can be inflated and thereby limits the collusive markup. In short, monitoring is more effective under price coordination and, by being able to sustain a higher average price, compensates for setting a uniform price.

7.2 Effect of Market Heterogeneity

Here it is established that price coordination is preferred when market heterogeneity is sufficiently low. The next section shows, under the assumption of linear demand, that firms prefer cost coordination when market heterogeneity is sufficiently large.

In order to consider when market heterogeneity is small, define an extreme distribution that puts all mass on one market type:

$$\widehat{G}(a) = \begin{cases} 0 & \text{if } a \in [\underline{a}, \widehat{a}) \\ 1 & \text{if } a \in [\widehat{a}, \overline{a}] \end{cases} .$$

Low market heterogeneity will be represented by distributions that are close to $\widehat{G}$. For this purpose, let $\widehat{a} \sim \widehat{G}$ and $a_k \sim G_k : [\underline{a}, \overline{a}] \to [0,1]$ where G_k is continuously differentiable and $G_k'(a) > 0 \ \forall a \in [\underline{a}, \overline{a}]$.

Theorem 4 *If $\{a_k\}_{k=1}^{\infty}$ converges in distribution to $\widehat{a}$ then $\exists k'$ such that if $k > k'$ then price coordination is preferred to cost coordination.*

When G puts sufficient mass in a sufficiently small neighborhood around $\widehat{a}$ then, in order to be as profitable as price coordination, cost coordination must price close to $p^M(\widehat{a}, c)$. However, a firm can then engage in an on-path deviation by pricing just below $p^M(\widehat{a}, c)$. As in the case with minimal product differentiation, price coordination does not forego much potential profit with its uniform price but is superior in terms of monitoring which results in the incentive compatible (first-best) uniform price under price coordination outperforming the price discrimination that is supported by the incentive compatible cost report under cost coordination.

8 Linear Demand

Towards understanding when a cartel would choose to coordinate cost reports rather than prices, this section goes beyond the extreme conditions in the previous section by assuming linear demand.

Assuming two products (and firms) for simplicity, a representative agent's utility function is specified to be

$$\theta\left(q_1 + q_2\right) - \left(\frac{1}{2}\right)\left(\beta\left(q_1^2 + q_2^2\right) + 2\gamma q_1 q_2\right),$$

which results in the following firm demand function:

$$D_1\left(p_1, p_2, \theta\right) \tag{16}$$
$$\equiv \begin{cases} \frac{\theta}{\beta} - \left(\frac{1}{\beta}\right) p_1 & \text{if } p_1 \leq \left(\frac{1}{\gamma}\right)\left(\beta p_2 - (\beta - \gamma)\theta\right) \\ \frac{\theta}{\beta+\gamma} - \left(\frac{\beta}{\beta^2-\gamma^2}\right) p_1 + \left(\frac{\gamma}{\beta^2-\gamma^2}\right) p_2 & \text{if } \left(\frac{1}{\gamma}\right)\left(\beta p_2 - (\beta-\gamma)\theta\right) \leq p_1 \leq \left(\frac{1}{\beta}\right)\left(\gamma p_2 + (\beta - \gamma)\theta\right) \\ 0 & \text{if } \left(\frac{1}{\beta}\right)\left(\gamma p_2 + (\beta-\gamma)\theta\right) \leq p_1. \end{cases}$$

The approach is to derive conditions for cost coordination to be more profitable than price coordination given a particular value for cost, call it c'. By continuity, cost coordination is more profitable $\forall c \in [c', c' + \varepsilon]$ for $\varepsilon > 0$ and small. Thus, if H puts sufficient mass on $[c', c' + \varepsilon]$ then (12) will hold. While this approach minimizes the significance of variation in cost over time, cost variation is not central to the insight of the paper; it simply serves to provide a reason for the upper level to provide a cost report to the lower level.

8.1 Analytical Results

In deriving the closed-form solutions of this sub-section, it is presumed that firms' demands are interior for all relevant prices. I will return to this qualification at the end of Section 8.1. Referring to (16), the implication of that presumption is that firm demand is

$$\frac{\theta}{\beta+\gamma}-\left(\frac{\beta}{\beta^2-\gamma^2}\right)p_1+\left(\frac{\gamma}{\beta^2-\gamma^2}\right)p_2.$$

To economize on notation, define

$$a\equiv\frac{\theta}{\beta+\gamma},b\equiv\left(\frac{\beta}{\beta^2-\gamma^2}\right),d\equiv\left(\frac{\gamma}{\beta^2-\gamma^2}\right) \tag{17}$$

so firm demand is

$$D_1\left(p_1,p_2,a\right)=a-bp_1+dp_2, \tag{18}$$

where $b>d\geq 0$. $a\sim G:[\underline{a},\overline{a}]\rightarrow[0,1]$ and let μ_a and σ_a^2 denote the mean and variance of a, respectively. σ_a^2 measures the degree of market heterogeneity.

If all firms decentralize pricing and have a common cost report x then the symmetric Nash equilibrium price is

$$p^N(a,x)=\frac{a+bx}{2b-d}. \tag{19}$$

Under competition, $x=c$ and profit is

$$\frac{b\left(\mu_a-(b-d)c\right)^2+b\sigma_a^2}{\left(2b-d\right)^2}. \tag{20}$$

The first-best price under price coordination is

$$\widehat{p}(c)=\frac{\mu_a+(b-d)c}{2(b-d)} \tag{21}$$

and profit is

$$\frac{\left(\mu_a-(b-d)c\right)^2}{4(b-d)}. \tag{22}$$

It is more profitable for firms to centralize and coordinate on a common price (price coordination) than to decentralize and compete if and only if (22) exceeds (20):

$$\frac{\left(\mu_a-(b-d)c\right)^2}{4(b-d)}>\frac{b\left(\mu_a-(b-d)c\right)^2+b\sigma_a^2}{\left(2b-d\right)^2}\Leftrightarrow\sigma_a^2<\frac{d^2\left(\mu_a-(b-d)c\right)^2}{4b\left(b-d\right)}\equiv\omega_2. \tag{23}$$

Thus, if market heterogeneity is not too great then firms prefer collusion with a uniform price to competition with price discrimination.

Turning to cost coordination, the profit from cost report x is

$$\begin{aligned}V\left(x\right) &\equiv \int\left(\frac{a+bx}{2b-d}-c\right)\left(a-(b-d)\left(\frac{a+bx}{2b-d}\right)\right)G'(a)da \\ &= \frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{\left(2b-d\right)^2}+\frac{b\sigma_a^2}{\left(2b-d\right)^2}.\end{aligned}$$

The equilibrium collusive cost report is the solution to the following constrained optimization problem:

$$\widetilde{x}(c) \equiv \arg\max_{x} \frac{b\left(\mu_a - (b-d)x\right)\left(\mu_a - (b-d)c + b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2} \tag{24}$$

subject to

$$\frac{b\left(\mu_a - (b-d)x\right)\left(\mu_a - (b-d)c + b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2} \tag{25}$$
$$\geq \max_{p_1 \in \left[\frac{\underline{a}+bx}{2b-d}, \frac{\overline{a}+bx}{2b-d}\right]} (p_1 - c)\left(\mu_a - bp_1 + d\left(\frac{\mu_a + bx}{2b-d}\right)\right).$$

As a benchmark, the unconstrained optimum of (24) is

$$\widehat{x}(c) \equiv c + \frac{d(\mu_a - (b-d)c)}{2b(b-d)} \tag{26}$$

with a price of

$$\frac{a + b\left(c + \frac{d(\mu_a - (b-d)c)}{2b(b-d)}\right)}{2b-d} = \frac{\mu_a + (b-d)c}{2(b-d)} + \frac{a - \mu_a}{2b-d}.$$

Note that the average price equals (21) and thus is the same as under price coordination.

It is shown in the appendix that if $\sigma_a^2 \leq (\mu_a - \underline{a})^2$ and

$$\sigma_a^2 \leq \frac{d^2(2b-d)^2(\mu_a - (b-d)c)^2}{16b^2(b-d)^2} \equiv \omega_1 \tag{27}$$

then the solution to (24)-(25) is

$$\widetilde{x}(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a. \tag{28}$$

$\sigma_a^2 \leq (\mu_a - \underline{a})^2$ is a relatively weak condition; for example, it strictly holds for any G with a symmetric density. When market heterogeneity is sufficiently low - as specified in (27) - the ICC is binding at the optimal cost report and, consequently, (28) is the highest cost report satisfying (25).

Given cost report (28), the cost coordination price for market type a is

$$\frac{a + bc}{2b-d} + \frac{2b\sigma_a}{(2b-d)^2}. \tag{29}$$

It is straightforward to show, under (27), that average price under cost coordination is lower than the uniform price under price coordination. Though a lower average price reduces profit, cost coordination can counterbalance that effect with price discrimination which raises profit.

The associated profit from cost coordination is

$$V\left(c+\left(\frac{2}{2b-d}\right)\sigma_a\right)=\frac{b\left(2b\mu_a+d\sigma_a-d\mu_a-2b^2c-cd^2+3bcd\right)^2}{(2b-d)^4}. \tag{30}$$

Using (30) and (22), cost coordination is more profitable than price coordination iff

$$\frac{b\left(2b\mu_a+d\sigma_a-d\mu-2b^2c-cd^2+3bcd\right)^2}{(2b-d)^4}>\frac{\left(\mu_a-bc+cd\right)^2}{4(b-d)}\Leftrightarrow$$

$$\sigma_a^2>\frac{(2b-d)^2(\mu_a-(b-d)c)^2\left((2b-d)-\sqrt{4b(b-d)}\right)^2}{4b(b-d)d^2}\equiv\omega_3. \tag{31}$$

To summarize, the solution to (24)-(25) is (28) when $\sigma_a^2\le(\mu_a-\underline{a})^2$ and $\sigma_a^2\le\omega_1$. Price coordination is more profitable than competition iff $\sigma_a^2<\omega_2$, and price coordination is more profitable than cost coordination iff $\sigma_a^2<\omega_3$. It is shown in the appendix that $\omega_1>\omega_2>\omega_3$ (assuming $d>0$). We then have the following findings:

- If $\sigma_a^2\in(0,\omega_3)$ then price coordination is more profitable than cost coordination and competition.
- If $\sigma_a^2\in(\omega_3,\omega_2)$ then cost coordination is more profitable than price coordination (which is more profitable than competition).
- If $\sigma_a^2\in(\omega_2,\omega_1)$ then cost coordination is more profitable than price coordination (which is less profitable than competition).

The key finding is that if market heterogeneity is sufficiently great then firms prefer to coordinate their cost reports, rather than their prices. Though intuitive, the result is not as immediate as one might suppose. Note that the case of linear demand satisfies the conditions in Theorem 1 which means the first-best outcome under cost coordination is always more profitable than the first-best outcome under price coordination, regardless of the value of σ_a^2 (as long as it is positive). However, if σ_a^2 is low then *incentive compatible* price coordination is more profitable than *incentive compatible* cost coordination. In order to make cost coordination immune to an executive centralizing pricing authority and lowering the average price, the collusive cost report must be set below the first-best cost report. That results in average price being lower than under price coordination (where the uniform monopoly price is incentive compatible). The higher profit from the higher average price under price coordination is balanced against the higher profit from price discrimination under cost coordination. As the latter effect is small when σ_a^2 is low, firms prefers to coordinate on a higher uniform price; they are willing to forego price discrimination in order to be able to sustain a higher average price. When instead σ_a^2 is high, the additional profit from price discrimination more than offsets the lower average price under cost coordination so firms prefer to coordinate cost reports.

In concluding this section, it must be emphasized that these closed-form solutions are correct if and only if firms' demands are interior for all prices relevant to the derivations of those solutions. Where that may be problematic is when price is uniform - as with price coordination or considering a uniform deviation price under cost coordination - because both firms' demands may not be positive when there is sufficient variation in a and the realized value of a is low. Thus, requiring σ_a^2 to be sufficiently great, as specified in (31), could imply that firms' demands are not positive for some prices and demand realizations which would then invalidate the analysis. To allay that concern, numerical analysis is conducted in the next sub-section which does not make the supposition that firms' demands are always interior. That analysis supports the preceding findings.

8.2 Numerical Results

The numerical analysis uses the demand specification in (16). Though it is not presumed that firms' demands are interior for all prices, I will focus on parameterizations for which equilibrium prices under cost coordination and price coordination result in positive demand for all market types. Details are in the appendix.

The degree of product differentiation is decreasing in γ where products are independent at $\gamma = 0$ and identical at $\gamma = \beta$. The market type is represented by θ which is uniformly distributed on $[\mu_\theta - (r/2), \mu_\theta + (r/2)]$. r is the range of θ and captures market heterogeneity. An increase in r raises the variance $\sigma_\theta^2 = r^2/12$ but leaves the mean unchanged. The model's parameters are $(\mu_\theta, r, \beta, \gamma, c)$ and are chosen so that $\gamma \in [0, \beta]$ and $\mu_\theta - (r/2) > c \geq 0$.

Results are reported in Figures 2 and 3 where the horizontal axis measures the degree of market heterogeneity r and the vertical axis measures the degree of product similarity γ.[24] Figure 2 assumes $(\mu_\theta, \beta) = (10, 1)$ where $(r, \gamma) \in \{.1, .2, ..., 4.9, 5.0\} \times \{.00, .01, ..., .98, .99\}$. For $c = 1$, the solid line partitions the space between values of (r, γ) where cost coordination is more profitable (below the line) and price coordination is more profitable (above the line). Also shown are the thresholds when $c = 3$ (dashed line) and $c = 5$ (dotted line). Consistent with the results in Sections 7 and 8.1, cartels facing greater market heterogeneity and more product differentiation are more likely to coordinate cost reports than prices. This finding is confirmed in Figure 3 for $(\mu_\theta, \beta) = (15, 1)$ and $c \in \{1, 3, 5, 7\}$ where $(r, \gamma) \in \{.1, .2, ..., 9.9, 10.0\} \times \{.00, .01, ..., .98, .99\}$.

[24] It is verified that, for some parameterizations, condition (27) is not satisifed which means the cases considered by the numerical analysis go beyond those allowed for in deriving the analytical results. Specifically, σ_θ^2 is allowed to be so high that the optimal cost report is such that the optimal deviation price is constrained; see the analysis in section 10.2.1.

Figure 2

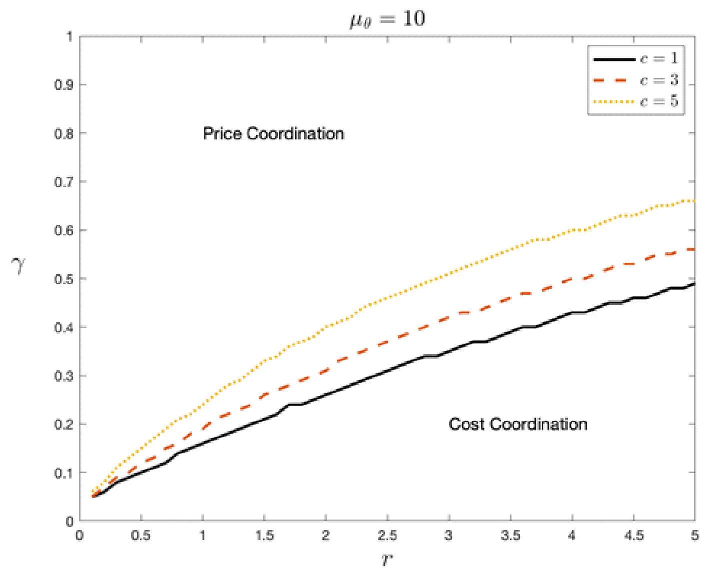
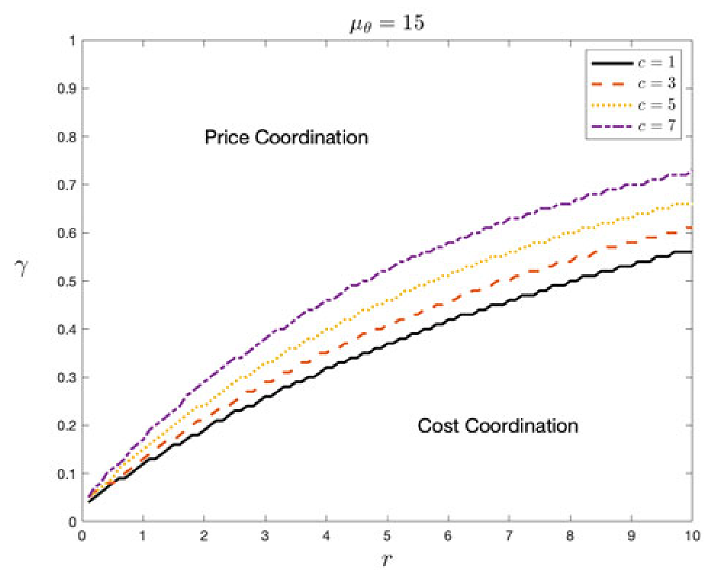


Figure 3



9 Concluding Remarks

Collusion is generally viewed as firms agreeing to charge higher prices. What that means is relatively straightforward when each firm sets one or a small number of prices. It is far less so when firms tailor their prices to market-specific traits or offer customer-specific discounts. It could be difficult for a cartel to coordinate on such a large number of prices and, should colluding executives decide to do so, it could require involving lower level employees; e.g., sales representatives who are able to identify those customers willing to pay more. However, expanding the set of employees with knowledge of the cartel brings forth an added risk of discovery and conviction. That risk could cause colluding executives to choose to charge a uniform price and forego the additional profits from price discrimination.

This paper proposes an alternative collusive scheme which allows for price discrimination while keeping knowledge of the cartel exclusive to high-level executives. With pricing authority delegated under competition, the executives maintain that decentralized structure and coordinate on the cost reports provided to the levels with pricing authority. Competitive price discrimination occurs but with a "supracompetitive" cost. In comparing coordinating on a uniform price and coordinating on an inflated cost report, the trade-off is that monitoring for compliance is less effective with cost coordination (because a colluding executive could secretly intervene in the internal pricing process and set a low uniform price) but the potential profit that can be earned is higher (because of price discrimination). In short, average price is higher with price coordination but cost coordination allows price discrimination. When market heterogeneity is high, the profit gain from price discrimination is sufficient to offset the lower average price so the cartel coordinates cost reports. With low market heterogeneity, the cartel pursues the more standard method of coordinating prices. When products are highly substitutable, the extent of price discrimination under cost coordination is low because, given prices are competitive prices, they will tend to be close to firms' (inflated) cost. Thus, price is close to being uniform even when firms coordinate cost reports. Furthermore, the average price with cost coordination is below the uniform price with price coordination because less effective monitoring constrains how high a cost report the cartel can sustain. Hence, the cartel coordinates prices when products are commodities but coordinates cost reports when products are reasonably differentiated.

The key starting point to this paper's analysis is recognizing the relevance of a firm's internal pricing process. That recognition was reached while puzzling over how certain collusive practices could be effective. How could a cartel that coordinates list prices be effective when it does not coordinate discounts off of list prices? How could a cartel that coordinates on introducing a surcharge be effective when it does not coordinate on fixing other components of the final price? Harrington and Ye (2019) explains that coordinating list prices can be effective when it affects buyers' beliefs on firms' costs and consequently the final negotiated prices. However, that theory requires list prices to be public and thus cannot explain how coordinating list prices in the EU trucks cartel was effective given those list prices were not publicly revealed. As in Chen (2021), the approach taken in this paper is to encompass a more descriptively realistic model of a firm's internal pricing process. High-level executives who coordinate list prices or surcharges - but not final prices - have an impact on final prices because they do not control (or choose not to control) final prices. In Chen (2021), colluding executives coordinate surcharges but delegate base prices to lower-level employees who are not part of the cartel. In the current paper, colluding executives coordinate cost reports (which could be implemented by coordinating list prices or surcharges) but delegate final prices to lower-level employees who are not part of the cartel. The insight delivered in these papers runs contrary to current understanding which is that a cartel is less effective when it does not fully control prices (e.g., due to imperfect monitoring as in Green and Porter, 1984). Here we see that certain practices are *only* effective because the cartel does *not* fully control prices. The general takeaway is that collusive conduct can be better understood by getting inside the firms comprising the cartel.

10 Appendices

10.1 Appendix: Proofs

Proof of Theorem 1. I begin with some preliminary results (where the dependence on c is dropped to avoid extraneous notation). Let us show $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$ where $\widehat{p}$ is defined by:

$$\int_{\underline{a}}^{\overline{a}} \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da = 0. \tag{32}$$

Suppose not. If $\widehat{p} \leq p^M(\underline{a})$ then, by strict quasi-concavity and $p^M(a)$ is increasing in a,

$$\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p} > 0 \; \forall a \in (\underline{a}, \overline{a}]$$

and, therefore,

$$\int_{\underline{a}}^{\overline{a}} \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da > 0$$

which contradicts (32). If $\widehat{p} \geq p^M(\overline{a})$ then, by strict quasi-concavity and $p^M(a)$ is increasing in a,

$$\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p} < 0 \; \forall a \in [\underline{a}, \overline{a})$$

and, therefore,

$$\int_{\underline{a}}^{\overline{a}} \left(\frac{\partial \pi\left(\widehat{p}, a\right)}{\partial p}\right) G'(a)da < 0$$

which contradicts (32). Hence, $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$.

Define a' as the market type such that the monopoly price when conditioning on a equals the monopoly price when not conditioning on a: $p^M(a') = \widehat{p}$. Note that $\widehat{p} \in \left(p^M(\underline{a}), p^M(\overline{a})\right)$ implies $a' \in (\underline{a}, \overline{a})$. Next define the cost report x' that equates the competitive price to the price under price coordination at market type a': $p^N(a', x') = \widehat{p}$. To show that x' exists, first note that $D_i\left(c, ..., c, a'\right) > 0$ implies $D_i\left(p^M(a'), ..., p^M(a'), a'\right) > 0$. Therefore, if $x = p^M(a')$ then $p^N(a', p^M(a')) > p^M(a')$ by A2. We then have:

$$p^N(a', p^M(a')) > p^M(a') > p^N(a', c).$$

By continuity of $p^N(a, x)$ in x, $\exists x' \in \left(c, p^M(a')\right)$ such that $p^N(a', x') = p^M(a')$ and thus $p^N(a', x') = \widehat{p}$.

Based on the definitions of a' and x' and (3), we have Figure 1. Note that $p^N(a, x') \in \left(p^M(a), \widehat{p}\right) \; \forall a \in [\underline{a}, a')$ and $p^N(a, x') \in \left(\widehat{p}, p^M(a)\right) \; \forall a \in (a', \overline{a}]$. By strict quasi-concavity of $(p - c)\, D_i\,(p, ..., p, a)$ in p and that

$$p^M(a) \equiv \arg\max_p\, (p - c)\, D_i\,(p, ...p, a)\,,$$

then

$$\left(p^N(a, x') - c\right) D_i\left(p^N(a, x'), ..., p^N(a, x'), a\right) > \left(\widehat{p} - c\right) D_i\left(\widehat{p}, ..., \widehat{p}, a\right) \forall a \in [\underline{a}, a')$$

and

$$\left(p^N(a,x') - c\right) D_i \left(p^N(a,x'), ..., p^N(a,x'), a\right) > (\widehat{p} - c)\, D_i\, (\widehat{p}, ..., \widehat{p}, a)\, \forall a \in (a', \overline{a}]\, .$$

Therefore,

$$\begin{aligned} & \int \left(p^N(a,x') - c\right) D_i \left(p^N(a,x'), ..., p^N(a,x'), a\right) G'(a)da \\ > & \int (\widehat{p} - c)\, D_i\, (\widehat{p}, ..., \widehat{p}, a)\, G'(a)da. \end{aligned} \tag{33}$$

Given

$$\widehat{x} = \arg\max_x \int \left(p^N(a,x) - c\right) D_i \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da,$$

it follows from (33) that (4) is true. ■

Proof of Theorem 2. I will show that if products are sufficiently differentiated then $\exists x > c$ such that (15) holds for almost all c which then implies (12). To begin, consider (15) and (14) for any x :

$$\begin{aligned} & \int_{\underline{a}}^{\overline{a}} (p^N(a,x) - c) D_1 \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da \\ > & \int_{\underline{a}}^{\overline{a}} (\widehat{p}(c) - c) D_1\, (\widehat{p}(c), ..., \widehat{p}(c), a)\, G'(a)da, \end{aligned} \tag{34}$$

$$\begin{aligned} & \int \left(p^N(a,x) - c\right) D_1 \left(p^N(a,x), ..., p^N(a,x), a\right) G'(a)da \\ \geq & \max_{p_1 \in [p^N(\underline{a},x), p^N(\overline{a},x)]} \int (p_1 - c)\, D_1 \left(p_1, p^N(a,x), ..., p^N(a,x), a\right) G'(a)da,\ \forall c \in [\underline{c}, \overline{c}]\, . \end{aligned} \tag{35}$$

Let us start with $x = c$. $x = c$ implies (35) holds strictly because $p^N(a,x)$ is the best response to other firms pricing at $p^N(a,x)$ given a. Hence, the integrand on the LHS exceeds the integrand on the RHS for almost all a. Given $\lim_{\gamma \to 0} p^N(a,c,\gamma) = p^M(a,c)$ then, for γ close to 0, the LHS of (34) is close to the monopoly profit from third-degree price discrimination and the RHS of (34) is close to the monopoly profit from a uniform price. Hence, (34) hold strictly as $\gamma \to 0$.

Thus far, it has been shown $\exists \gamma' > 0$ such that if $\gamma \in (0, \gamma')$ then (34)-(35) hold strictly for $x = c$. By continuity, if $\gamma \in (0, \gamma')$ then $\exists \varepsilon > 0$ such that (34)-(35) hold for $x = c + \varepsilon$. Also note that cost coordination with $c + \varepsilon$ (as long as ε is sufficiently small) is more profitable than competition (i.e., $x = c$):

$$\begin{aligned} & \int_{\underline{a}}^{\overline{a}} (p^N(a, c+\varepsilon) - c) D_1 \left(p^N(a, c+\varepsilon), ..., p^N(a, c+\varepsilon), a, \gamma\right) G'(a)da \\ > & \int_{\underline{a}}^{\overline{a}} (p^N(a,c) - c) D_1 \left(p^N(a,c), ..., p^N(a,c), a, \gamma\right) G'(a)da. \end{aligned} \tag{36}$$

The preceding condition follows from the strict quasi-concavity of $(p-c)D_1(p,...,p,a,\gamma)$ and that $p^N(a,x)$ is increasing in x which then implies the integrand of the LHS of (36) exceeds the integrand of the RHS for $\varepsilon > 0$ and close to zero.

In sum, when products are sufficiently differentiated, cost coordination with $x = c + \varepsilon$, where $\varepsilon > 0$ and small, is preferable to price coordination. Recognizing that x is required not to exceed $\overline{c}$, then this condition holds $\forall c \in [\underline{c}, \overline{c} - \varepsilon]$. By setting ε close to zero, (12) can be assured of holding. ■

Proof of Theorem 3. Let us show $\exists \gamma' < 1$ such that if $\gamma \in (\gamma', 1)$ then $\nexists x > c$ such that (34)-(35) hold and this is true $\forall c \in [\underline{c}, \overline{c}]$. (It will also be true that (34)-(35) does not hold for $x = c$.) To prove this claim, let us suppose the contrary and derive a contradiction. Thus, suppose $\exists x^o(\gamma) > c$ satisfying (34)-(35). Recall that $\widehat{p}$ is the uniform price charged under price coordination (which, by assumption, is independent of γ).

Given $\lim_{\gamma\to 1} p^N(a,x,\gamma) = x \ \forall a$ then, for γ close to one, $p^N(a,x,\gamma)$ is close to a uniform price in that it is in a small neighborhood around x. Given that $\widehat{p}$ is the optimal uniform price then, for (34) to hold when γ is close to one, x must be close to $\widehat{p}$ so that $p^N(a,x,\gamma)$ is close to $\widehat{p}$. We then have: $\lim_{\gamma\to 1} x^o(\gamma) = \widehat{p}$.

Next consider (35). Given $\lim_{\gamma\to 1} p^N(a,c,\gamma) = c$ and it was just shown that (34) implies $\lim_{\gamma\to 1} p^N(a,x^o(\gamma),\gamma) = \widehat{p}(> c)$ then $p^N(a,x^o(\gamma),\gamma)$ is bounded above $p^N(a,c,\gamma)$ as $\gamma \to 1$. By assumption, if $p > p^N(a,c,\gamma)$ then $\psi_1(p,...,p,a,c,\gamma) < p$. It then follows:

$$\lim_{\gamma\to 1} p^N(a,x^o(\gamma),\gamma) > \lim_{\gamma\to 1} \psi_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a,c,\gamma\right).$$

Given $\lim_{\gamma\to 1} p^N(a,x,\gamma) = x$ then, as $\gamma \to 1$, $p^N(\underline{a},x,\gamma)$ converges to $p^N(a,x,\gamma)$ $\forall a \in (\underline{a},\overline{a}]$ and does so from below (because $p^N(a,x,\gamma)$ is increasing in a). Combining the previous two results: $\exists \gamma' < 1$ such that if $\gamma \in (\gamma', 1)$ then

$$p^N(\underline{a},x^o(\gamma),\gamma) \in \left(\psi_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a,c,\gamma\right), p^N(a,x^o(\gamma),\gamma)\right) \ \forall a \in (\underline{a},\overline{a}].$$

By strict quasi-concavity of $(p_1 - c)D_1(p_1,...,p_n,a)$ in p_1, the previous condition implies

$$\begin{aligned} &(p^N(\underline{a},x^o(\gamma),\gamma) - c)D_1\left(p^N(\underline{a},x^o(\gamma),\gamma),p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right) \\ > \ &(p^N(a,x^o(\gamma),\gamma) - c)D_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right) \ \forall a \in (\underline{a},\overline{a}]. \end{aligned}$$

Taking the integral of each side of the preceding equation, we have:

$$\begin{aligned} &\int_{\underline{a}}^{\overline{a}} (p^N(\underline{a},x^o(\gamma),\gamma) - c)D_1\left(p^N(\underline{a},x^o(\gamma),\gamma),p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right) G'(a)da \\ > \ &\int_{\underline{a}}^{\overline{a}} (p^N(a,x^o(\gamma),\gamma) - c)D_1\left(p^N(a,x^o(\gamma),\gamma),...,p^N(a,x^o(\gamma),\gamma),a\right) G'(a)da, \end{aligned}$$

which contradicts (35). ■

Proof of Theorem 4. There are two properties associated with convergence in distribution that will be used. First, given $(p^N(a,x) - c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)$ is bounded and continuous in a then

$$\begin{aligned}&\lim_{k\to\infty}\int_{\underline{a}}^{\overline{a}}(p^N(a,x)-c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)G_k'(a)da\\=\;&(p^N(\widehat{a},x)-c)D_1\left(p^N(\widehat{a},x),...,p^N(\widehat{a},x),\widehat{a}\right).\end{aligned}$$

Second, given $(p-c)\,D\,(p,...,p,a)$ is bounded and continuous in a then

$$\lim_{k\to\infty}\int_{\underline{a}}^{\overline{a}}(p-c)\,D\,(p,...,p,a)\,G_k'(a)da=(p-c)D_1\,(p,...,p,\widehat{a})\,.$$

Defining

$$\widehat{p}(G_k)\equiv\arg\max\int_{\underline{a}}^{\overline{a}}(p-c)\,D\,(p,...,p,a)\,G_k'(a)da,$$

it follows:

$$\lim_{k\to\infty}\widehat{p}(G_k)=p^M(\widehat{a}).$$

It will be shown: if $\{a_k\}_{k=1}^{\infty}$ converges in distribution to $\widehat{a}$ then $\exists k'$ such that $\nexists x$ satisfying (34)-(35) $\forall k>k'$. To prove it, suppose the contrary - $\exists\,\{\widehat{x}_k\}_{k=1}^{\infty}$ and k' such that (34)-(35) is satisfied $\forall k>k'$ - and let us derive a contradiction.

(34) is reproduced here:

$$\begin{aligned}&\int_{\underline{a}}^{\overline{a}}(p^N(a,x)-c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)G_k'(a)da\\>\;&\int_{\underline{a}}^{\overline{a}}(\widehat{p}(G_k)-c)D_1\,(\widehat{p}(G_k),...,\widehat{p}(G_k),a)\,G_k'(a)da,\end{aligned}\tag{37}$$

Considering the RHS of (37), $\{a_k\}_{k=1}^{\infty}\to\widehat{a}$ implies: $\forall\varepsilon>0$, $\exists k'$ such that if $k>k'$ then

$$\begin{aligned}&\int_{\underline{a}}^{\overline{a}}(\widehat{p}(G_k)-c)D_1\,(\widehat{p}(G_k),...,\widehat{p}(G_k),a)\,G_k'(a)da\\\in\;&\left((p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon,(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon\right).\end{aligned}\tag{38}$$

Considering the LHS of (37), $\{a_k\}_{k=1}^{\infty}\to\widehat{a}$ implies: $\forall\varepsilon>0$, $\exists k'$ such that if $k>k'$ then

$$\begin{aligned}&\int_{\underline{a}}^{\overline{a}}(p^N(a,\widehat{x}_k)-c)D_1\left(p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a\right)G'(a)da\\\in\;&\left((p^N(\widehat{a},\widehat{x}_k)-c)D_1\left(p^N(\widehat{a},\widehat{x}_k),...,p^N(\widehat{a},\widehat{x}_k),\widehat{a}\right)-\varepsilon,\right.\\&\left.(p^N(\widehat{a},\widehat{x}_k)-c)D_1\left(p^N(\widehat{a},\widehat{x}_k),...,p^N(\widehat{a},\widehat{x}_k),\widehat{a}\right)+\varepsilon\right).\end{aligned}\tag{39}$$

Given (37) is assumed to be satisfied and (38) provides a lower bound on the RHS of (37) and (39) provides an upper bound on the LHS of (37), we then have: $\forall\varepsilon>0$, $\exists k'$ such that if $k>k'$ then

$$(p^N(\widehat{a},\widehat{x}_k)-c)D_1\left(p^N(\widehat{a},\widehat{x}_k),...,p^N(\widehat{a},\widehat{x}_k),\widehat{a}\right)+\varepsilon>(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon$$

or

$$(p^N(\widehat{a},\widehat{x}_k)-c)D_1\left(p^N(\widehat{a},\widehat{x}_k),...,p^N(\widehat{a},\widehat{x}_k),\widehat{a}\right)+2\varepsilon>(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right). \tag{40}$$

As the RHS of (40) is the unique maximum of $(p-c)D_1(p,...,p,a)$ then, for (40) to hold $\forall\varepsilon>0$, it must be true:

$$\lim_{k\to\infty}p^N(\widehat{a},\widehat{x}_k)=p^M(\widehat{a}).$$

Next consider (35) which is reproduced here:

$$\int_{\underline{a}}^{\overline{a}}(p^N(a,x)-c)D_1\left(p^N(a,x),...,p^N(a,x),a\right)G_k'(a)da \tag{41}$$
$$\geq \max_{p_1\in[p^N(\underline{a},x),p^N(\overline{a},x)]}\int_{\underline{a}}^{\overline{a}}(p_1-c)D_1\left(p_1,p^N(a,x),...,p^N(a,x),a\right)G_k'(a)da.$$

Given it has been shown $\lim_{k\to\infty}p^N(\widehat{a},\widehat{x}_k)=p^M(\widehat{a})$ then $\{a_k\}_{k=1}^{\infty}\to\widehat{a}$ implies: $\forall\varepsilon>0$, $\exists k'$ such that if $k>k'$ then, referring to the LHS of (41),

$$\int_{\underline{a}}^{\overline{a}}(p^N(a,\widehat{x}_k)-c)D_1\left(p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a,\gamma\right)G_k'(a)da \tag{42}$$
$$\in\left((p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon,(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon\right);$$

and, referring to the RHS of (41),

$$\int_{\underline{a}}^{\overline{a}}(p_1-c)D_1\left(p_1,p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a,\gamma\right)G_k'(a)da$$
$$\in\left((p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon,(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon\right)\ \forall p,$$

which implies:

$$\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}\int_{\underline{a}}^{\overline{a}}(p_1-c)D_1\left(p_1,p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a\right)G'(a)da \tag{43}$$
$$\in\Bigg(\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon,$$
$$\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon\Bigg).$$

It follows from (42)-(43): $\forall\varepsilon>0$, $\exists k'$ such that if $k>k'$ then

$$(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon>\int_{\underline{a}}^{\overline{a}}(p^N(a,\widehat{x}_k)-c)D_1\left(p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a\right)G_k'(a)da \tag{44}$$

and

$$\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}\int_{\underline{a}}^{\overline{a}}(p_1-c)D_1\left(p_1,p^N(a,\widehat{x}_k),...,p^N(a,\widehat{x}_k),a\right)G_k'(a)da \tag{45}$$
$$>\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon.$$

As the LHS of (44) is greater then the LHS of (41) and the RHS of (45) is less than the RHS of (41), (41) holding $\forall k$ implies: $\forall \varepsilon > 0$, $\exists k'$ such that if $k > k'$ then

$$(p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+\varepsilon > \max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)-\varepsilon. \tag{46}$$

Let us show (46) does not hold; that is, $\exists \varepsilon > 0$ such that $\forall k$,

$$\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p_1-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right) > (p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right)+2\varepsilon, \tag{47}$$

which will be our contradiction. As $p^N(a,x)$ is increasing in a and $\lim_{k\to\infty} p^N(\widehat{a},\widehat{x}_k) = p^M(\widehat{a})$ then $\widehat{a} \in (\underline{a},\overline{a})$ implies $p^M(\widehat{a}) \in \left(p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)\right)$ as $k \to \infty$. Also note that $p^N(\underline{a},\widehat{x}_k)$ is bounded below $p^M(\widehat{a})$. Hence, for $\eta > 0$ and small, $p^M(\widehat{a}) - \eta \in \left[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)\right]$ as $k \to \infty$ which implies

$$\max_{p_1\in[p^N(\underline{a},\widehat{x}_k),p^N(\overline{a},\widehat{x}_k)]}(p-c)D_1\left(p_1,p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right) > (p^M(\widehat{a})-c)D_1\left(p^M(\widehat{a}),...,p^M(\widehat{a}),\widehat{a}\right).$$

Therefore, (47) holds for ε small. ■

10.2 Appendix: Linear Demand

10.2.1 Analytical

Reproducing (24)-(25) here, we want to solve:

$$\widetilde{x}(c) \equiv \arg\max \frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2}$$

subject to

$$\frac{b\left(\mu_a-(b-d)x\right)\left(\mu_a-(b-d)c+b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2}$$
$$\geq \max_{p_1\in\left[\frac{\underline{a}+bx}{2b-d},\frac{\overline{a}+bx}{2b-d}\right]}(p_1-c)\left(\mu_a-bp_1+d\left(\frac{\mu_a+bx}{2b-d}\right)\right).$$

The proof strategy is as follows. First, it is shown that if the constraint on the deviation price in the ICC is binding at the optimal solution - that is, the unconstrained optimal deviation price does not lie in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d},\frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$ - then $\sigma_a^2 > (\mu_a-\underline{a})^2$. Hence, if $\sigma_a^2 \leq (\mu_a-\underline{a})^2$ then a solution must have the unconstrained optimal deviation price lying in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d},\frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$. Second, under that assumption about that deviation price, the ICC is solved for a constraint on x. Third, necessary and sufficient conditions are derived for the ICC to be binding at the optimal solution. Given the strict quasi-concavity of the objective function, $\widetilde{x}(c)$ is then the highest value of x satisfying the ICC.

To implement the first step, let us suppose the optimal deviation price is constrained at the solution to (24)-(25). Maximizing the LHS of (25), the unconstrained optimal deviation price is

$$p_1 = \frac{2(\mu_a + bc) + d(x - c)}{2(2b - d)}. \tag{48}$$

Suppose it was to exceed the upper bound to the choice set for p_1:

$$\frac{2(\mu_a + bc) + d(x - c)}{2(2b - d)} > \frac{\overline{a} + bx}{2b - d}.$$

From this condition is derived:

$$\begin{aligned} \frac{2(\mu_a + bc) + d(x - c)}{2(2b - d)} &> \frac{\overline{a} + bx}{2b - d} \Leftrightarrow 2(\mu_a + bc) + d(x - c) > 2\overline{a} + 2bx \\ 2(\mu_a - \overline{a}) + (2b - d)c &> (2b - d)x \Leftrightarrow c - \frac{2(\overline{a} - \mu_a)}{2b - d} > x. \end{aligned}$$

$x < c$ is inconsistent with a solution to (24)-(25) because it would deliver profit lower than that from $x = c$ which is assured of satisfying the ICC. The more relevant constraint is the lower bound to the choice set for p_1, which is violated by the unconstrained optimal deviation price iff

$$\frac{2(\mu_a + bc) + d(x - c)}{2(2b - d)} < \frac{\underline{a} + bx}{2b - d},$$

which gives us this constraint on x:

$$\begin{aligned} \frac{2(\mu_a + bc) + d(x - c)}{2(2b - d)} &< \frac{\underline{a} + bx}{2b - d} \Leftrightarrow 2(\mu_a + bc) + d(x - c) < 2\underline{a} + 2bx \\ 2(\mu_a - \underline{a}) + (2b - d)c &< (2b - d)x \end{aligned}$$

$$\frac{2(\mu_a - \underline{a})}{2b - d} + c < x. \tag{49}$$

If x satisfies (49) then the (constrained) optimal deviation price is $\frac{\underline{a}+bx}{2b-d}$ and the associated deviation profit is

$$\left(\frac{\underline{a} + bx}{2b - d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b - d}\right) + d\left(\frac{\mu_a + bx}{2b - d}\right)\right).$$

Consequently, the ICC (25) is

$$\begin{aligned} &\frac{b\,(\mu_a - (b - d)x)\,(\mu_a - (b - d)c + b(x - c))}{(2b - d)^2} + \frac{b\sigma_a^2}{(2b - d)^2} \\ \geq &\left(\frac{\underline{a} + bx}{2b - d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b - d}\right) + d\left(\frac{\mu_a + bx}{2b - d}\right)\right) \end{aligned}$$

$$\begin{aligned} \frac{b\sigma_a^2}{(2b - d)^2} \geq &\left(\frac{\underline{a} + bx}{2b - d} - c\right)\left(\mu_a - b\left(\frac{\underline{a} + bx}{2b - d}\right) + d\left(\frac{\mu_a + bx}{2b - d}\right)\right) \\ &- \frac{b\,(\mu_a - (b - d)x)\,(\mu_a - (b - d)c + b(x - c))}{(2b - d)^2} \end{aligned}$$

$$x \leq c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)}. \tag{50}$$

In sum, a value for x results in the optimal deviation price being constrained and the ICC being satisfied iff (49) and (50) hold:

$$\frac{2(\mu_a - \underline{a})}{2b-d} + c < x \leq c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)}. \tag{51}$$

A necessary condition for (51) to hold is:

$$\frac{2(\mu_a - \underline{a})}{2b-d} + c < c + \frac{(\mu_a - \underline{a})^2 + \sigma_a^2}{(\mu_a - \underline{a})(2b-d)} \Leftrightarrow 2(\mu_a - \underline{a})^2 < (\mu_a - \underline{a})^2 + \sigma_a^2 \Leftrightarrow (\mu_a - \underline{a})^2 < \sigma_a^2.$$

Thus, if $(\mu_a - \underline{a})^2 \geq \sigma_a^2$ then the optimal deviation price must not be constrained at a solution to (24)-(25); that is, the unconstrained optimal deviation price must lie in $\left[\frac{\underline{a}+b\widetilde{x}(c)}{2b-d}, \frac{\overline{a}+b\widetilde{x}(c)}{2b-d}\right]$. From hereon, this assumption is made.

Let us consider the ICC with the unconstrained optimal deviation price. Evaluating the RHS of (25) at the price in (48), the deviation profit is

$$\frac{b\left(2(\mu_a - (b-d)c) + d(x-c)\right)^2}{4(2b-d)^2}$$

which results in (25) taking the form:

$$\begin{aligned} & \frac{b\left(\mu_a - (b-d)x\right)\left(\mu_a - (b-d)c + b(x-c)\right)}{(2b-d)^2} + \frac{b\sigma_a^2}{(2b-d)^2} \\ \geq \quad & \frac{b\left(2(\mu_a - (b-d)c) + d(x-c)\right)^2}{4(2b-d)^2} \\ \Leftrightarrow \quad & x \leq c + \left(\frac{2}{2b-d}\right)\sigma_a. \end{aligned}$$

Note that the unconstrained optimal value of x in (26) exceeds the RHS in the preceding condition when:

$$c + \frac{d(\mu_a - (b-d)c)}{2b(b-d)} \geq c + \left(\frac{2}{2b-d}\right)\sigma_a \Leftrightarrow \sigma_a \leq \frac{d(2b-d)(\mu_a - (b-d)c)}{4b(b-d)}.$$

Under that condition on σ_a and given the strict concavity of $V(x)$, the solution to (24)-(25) is $\widetilde{x}(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a$.

To verify the conjecture that the optimal deviation price lies in the choice set, we need to show:

$$\frac{2(\mu_a + bc) + d(x-c)}{2(2b-d)} \in \left[\frac{\underline{a} + bx}{2b-d}, \frac{\overline{a} + bx}{2b-d}\right]$$

when $x = \widetilde{x}(c)$. That condition is equivalent to

$$x \in \left[c + \frac{2(\mu_a - \overline{a})}{2b-d}, c + \frac{2(\mu_a - \underline{a})}{2b-d}\right].$$

Given $x = c + \left(\frac{2}{2b-d}\right)\sigma_a$, we then need

$$\frac{2(\mu_a - \overline{a})}{2b-d} \leq \left(\frac{2}{2b-d}\right)\sigma_a \leq \frac{2(\mu_a - \underline{a})}{2b-d} \Leftrightarrow \mu_a - \overline{a} \leq \sigma_a \leq \mu_a - \underline{a}.$$

Clearly, the LHS inequality holds since $\mu_a - \overline{a} < 0$, and the RHS inequality is equivalent to $\sigma_a^2 \leq (\mu_a - \underline{a})^2$.

In sum, if $\sigma_a^2 \leq (\mu_a - \underline{a})^2$ and

$$\sigma_a \leq \frac{d(2b-d)(\mu_a - (b-d)c)}{4b(b-d)}$$

then the cost coordination solution is

$$\widetilde{x}(c) = c + \left(\frac{2}{2b-d}\right)\sigma_a.$$

Let us pull together the various conditions on σ_a^2. The derived solution to (24)-(25) is valid iff $\sigma_a^2 \leq \omega_1$. Price coordination is more profitable than competition iff $\sigma_a^2 < \omega_2$. Price coordination is more profitable than cost coordination iff $\sigma_a^2 < \omega_3$. Let us prove that if $d > 0$ then $\omega_1 > \omega_2 > \omega_3$.

$$\omega_1 \equiv \frac{d^2(2b-d)^2(\mu_a - (b-d)c)^2}{16b^2(b-d)^2} > \frac{d^2(\mu_a - (b-d)c)^2}{4b(b-d)} \equiv \omega_2$$

$$(2b-d)^2 > 4b(b-d) \Leftrightarrow d^2 > 0.$$

$$\omega_2 \equiv \frac{d^2(\mu_a - (b-d)c)^2}{4b(b-d)} > \frac{(2b-d)^2(\mu_a - (b-d)c)^2\left((2b-d) - \sqrt{4b(b-d)}\right)^2}{4b(b-d)d^2} \equiv \omega_3$$

$$\begin{aligned} d^4 &> (2b-d)^2\left((2b-d) - \sqrt{4b(b-d)}\right)^2 \\ d^2 &> 4b^2 - 4bd + d^2 - (2b-d)\sqrt{4b(b-d)} \\ (2b-d)\sqrt{4b(b-d)} &> 4b(b-d) \Leftrightarrow (2b-d)^2 > 4b(b-d) \\ 4b^2 - 4bd + d^2 &> 4b^2 - 4bd \Leftrightarrow d^2 > 0 \end{aligned}$$

10.2.2 Numerical

Using the expressions in Section 8.1 and substituting with (17), price under price coordination is:

$$\widehat{p} = \frac{\mu_a + (b-d)c}{2(b-d)} = \frac{\frac{\mu_\theta}{\beta+\gamma} + \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)c}{2\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)} = \frac{\frac{\mu_\theta}{\beta+\gamma} + \left(\frac{1}{\beta+\gamma}\right)c}{2\left(\frac{1}{\beta+\gamma}\right)} = \frac{\mu_\theta + c}{2}.$$

Parameterizations are considered such that firm demand is positive for all market types:

$$\frac{\underline{\theta}}{\beta+\gamma} - \left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)\left(\frac{\mu_\theta + c}{2}\right) > 0 \Leftrightarrow \underline{\theta} - \frac{\mu_\theta + c}{2} > 0.$$

In that case, profit is

$$\frac{(\mu_a-(b-d)c)^2}{4(b-d)}=\frac{\left(\frac{\mu_\theta}{\beta+\gamma}-\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)c\right)^2}{4\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)}=\frac{\left(\frac{\mu_\theta}{\beta+\gamma}-\left(\frac{1}{\beta+\gamma}\right)c\right)^2}{4\left(\frac{1}{\beta+\gamma}\right)}=\frac{\left(\frac{1}{\beta+\gamma}\right)^2(\mu_\theta-c)^2}{4\left(\frac{1}{\beta+\gamma}\right)}$$

$$\frac{(\mu_a-(b-d)c)^2}{4(b-d)}=\frac{\left(\frac{\mu_\theta}{\beta+\gamma}-\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)c\right)^2}{4\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)}=\frac{(\mu_\theta-c)^2}{4(\beta+\gamma)}\equiv V^{pc}. \tag{52}$$

The symmetric Nash equilibrium price is

$$p^N(a,x)=\frac{a+bx}{2b-d}=\frac{\frac{\mu_\theta}{\beta+\gamma}+\frac{\beta}{\beta^2-\gamma^2}x}{\frac{2\beta-\gamma}{\beta^2-\gamma^2}}=\frac{(\beta-\gamma)\theta+\beta x}{2\beta-\gamma}$$

and demand is positive iff

$$\frac{\theta}{\beta+\gamma}-\left(\frac{\beta-\gamma}{\beta^2-\gamma^2}\right)\left(\frac{(\beta-\gamma)\theta+\beta x}{2\beta-\gamma}\right)>0\ \forall\theta\Leftrightarrow\left(\frac{\beta(\theta-x)}{2\beta-\gamma}\right)>0\ \forall\theta\Leftrightarrow\underline{\theta}>x.$$

If $\underline{\theta}>c$ then profit under competition ($x=c$) is

$$V^{comp}\equiv\left(\frac{\beta(\beta-\gamma)}{(\beta+\gamma)(2\beta-\gamma)^2}\right)\left((\mu_\theta-c)^2+(\beta+\gamma)^2\sigma_\theta^2\right). \tag{53}$$

Price coordination is more profitable than competition iff

$$V^{pc}>V^{comp}\Leftrightarrow\sigma_\theta^2<\frac{(\mu_\theta-c)^2\gamma^2}{4\beta(\beta-\gamma)(\beta+\gamma)^2}.$$

Turning to cost coordination, if $\underline{\theta}>x$ then the profit from cost coordination is

$$\frac{\beta(\mu_\theta-x)((\beta-\gamma)(\mu_\theta-c)+\beta(x-c))+\beta(\beta-\gamma)\sigma_\theta^2}{(2\beta-\gamma)^2(\beta+\gamma)}.$$

In that case, (14)-(15) take the form:

$$\widetilde{x}=\arg\max_x\frac{\beta(\mu_\theta-x)((\beta-\gamma)(\mu_\theta-c)+\beta(x-c))+\beta(\beta-\gamma)\sigma_\theta^2}{(2\beta-\gamma)^2(\beta+\gamma)} \tag{54}$$

subject to

$$\frac{\beta(\mu_\theta-x)((\beta-\gamma)(\mu_\theta-c)+\beta(x-c))+\beta(\beta-\gamma)\sigma_\theta^2}{(2\beta-\gamma)^2(\beta+\gamma)}$$

$$\geq\max_{p_1\in\left[\frac{(\beta-\gamma)\underline{\theta}+\beta x}{2\beta-\gamma},\frac{(\beta-\gamma)\overline{\theta}+\beta x}{2\beta-\gamma}\right]}\int_{\underline{\theta}}^{\overline{\theta}}(p_1-c)\,D_1\left(p_1,\frac{(\beta-\gamma)\theta+\beta x}{2\beta-\gamma},\theta\right)\left(\frac{1}{\overline{\theta}-\underline{\theta}}\right)d\theta.$$

In solving this constrained optimization problem, the RHS of the constraint allows for p_1 such that firms' demands are not interior. Substituting $\widetilde{x}$ and $\sigma_\theta^2=r^2/12$ in (54), the profit from cost coordination is:

$$V^{cc}\equiv\frac{\beta(\mu_\theta-\widetilde{x})((\beta-\gamma)(\mu_\theta-c)+\beta(\widetilde{x}-c))+\beta(\beta-\gamma)(r^2/12)}{(2\beta-\gamma)^2(\beta+\gamma)}. \tag{55}$$

Cost coordination is more (less) profitable than price coordination when $V^{cc}>(<)V^{pc}$.

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