The International Medium of Exchange*

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Abstract

We propose a model of persistent coordination on a dominant international medium of exchange. An asset becomes dominant because it is widely held, and remains widely held because it is dominant. The central country is a net debtor, but earns an "exorbitant privilege" on its position. Calibrating the model, we find that steady states with one dominant asset are stable and equilibrium paths are unique. The dominant country enjoys a significant welfare gain, most of which is accrued during its rise to dominance. The model can rationalize the history of the international monetary system over the past century, and highlights the importance of economic policy and path dependence.

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1 Introduction

The US dollar plays a central role in international trade of both goods and financial assets.[1] At the same time, the US has a unique external position: it is the world's largest debtor country, but earns positive net income from its positions vis-a-vis the rest of the world, largely thanks to the "exorbitant privilege" of being able to pay a low return on the assets it issues.[2] These two phenomena, transaction dollarization and exorbitant privilege, are puzzling from the viewpoint of standard international macroeconomic models, and could potentially have large welfare implications.

In this paper, we propose a new theory of these facts, based on endogenous coordination on the international medium of exchange. The theory's central insight is that local asset availability matters: when more than one asset might serve as a medium of exchange, traders tend to coordinate on using the asset that is most readily available to all counterparties. In the context of international exchange, this means that when safe dollar assets are widely held across the world, more cross-border transactions will be conducted using dollars. On the other hand, if dollars are heavily used in international transactions, then investors outside the US will choose to hold more dollar assets in order to satisfy the higher demand for dollar liquidity. In this way, the portfolio holdings of an asset and its use in transactions mutually reinforce coordination on that asset as the medium of exchange.

We begin with an analytical model that illustrates the key insights of our asset availability/asset usage feedback mechanism. The model is composed of three country/regions – the US, the Eurozone (EZ), and a continuum of rest-of-world (RW) small open economies. In each country, there is a continuum of traders seeking to engage in profitable transactions with traders from other countries. Imperfect contract enforcement across borders requires traders to collateralize transactions with safe assets that serve as performance guarantees. To obtain collateral, traders seek an intra-period loan of either US or EZ bonds in local bond-specific search and matching markets.[3] On the other side of these credit markets are households, who form optimal portfolios and offer intra-period loans of their assets for a fee.

[1] Gopinath (2015) shows that dollar-invoiced trade is five times larger than trade directly involving the US, and the dollar by far the largest third-party currency used for trade invoicing. Goldberg (2011) and Maggiori et al. (2017) show that 85% of foreign exchange rate transactions involve the USD, 60% international debt securities are issued in dollars, and that foreign investors have a particular affinity for dollar safe assets.

[2] Estimates of the asset/liabilities return differential range from 1% to 3% (Gourinchas and Rey, 2007).

[3] Such financing frictions are prevalent in real world trade of both goods and financial instruments. Trade finance — the banking activities that fund imports and exports — is one of the oldest domains of banking, and remains a large and thriving business today (BIS, 2014). Similarly, much of foreign financial institutions' dollar funding is obtained via international repurchasing agreements using US Treasuries as collateral.

Because of search frictions in the collateral credit markets, traders prefer ceteris paribus to search for an asset that forms a substantial portion of local household portfolios, and is thus relatively plentiful in local funding markets. Conversely, households are aware that an asset that is actively used by traders is more likely to be loaned in the collateral markets and earn the associated fee. Thus, the incentives of households and traders are mutually reinforcing: wide holdings of an asset drive its adoption as the medium of exchange, while higher adoption encourages households everywhere to maintain large positions in that asset.

We characterize the stationary equilibrium in the analytical model in a series of propositions. In particular, we show that the feedback between households and traders described above leads the model to have three steady-state equilibria: a dollar-dominant one in which international transactions use dollars and US assets are widely held across the world, a mirror-image euro-dominant one, and a symmetric multipolar equilibrium in which each currency intermediates one half of transactions and portfolios are perfectly diversified.

Dominant-currency equilibria exist even though traders have no incentive to coordinate currency choices, but those equilibria are not necessarily stable. To ensure stability, we introduce a currency mismatch cost that is incurred by trading pairs who use different types of collateral.[4] This cost creates a strategic complementarity between traders' currency choices, similar to the complementarities that serve as the basis for most existing currency competition models. While this type of complementarity alone can generate dominant-currency equilibria, it generally only does so when there is equilibrium indeterminacy.

By contrast, dominant-currency equilibria in our model do not need to coincide with indeterminacy, because the core household-trader feedback helps to anchor the equilibrium. In particular, we prove that for intermediate levels of cross-trader complementarity, the dominant-currency equilibria we obtain are both locally stable and not subject to self-fulfilling sunspot shocks to traders' beliefs. This happens because asymmetric asset positions serve as a coordination device for international traders – if one asset is broadly held by households worldwide, its availability to potential counterparties encourages coordination on that asset as the medium of exchange. Thus, a shift in equilibrium currency use requires a corresponding shift in international asset holdings – this allows the model to generate fully-coordinated currency use equilibria, without necessarily implying indeterminacy. As a result, the dominant-currency regimes are persistent, consistent with the historical record.

Moreover, dominant-currency equilibria also fit several other important features of the data. First, the dominant international safe asset earns a liquidity premium, derived from the fees paid by traders who use the asset as collateral. This premium generates a failure of

[4] Amiti et al. (2018) find evidence of such coordination incentives in firm-level trade data from Belgium.

interest parity that leads to an "exorbitant privilege" for the central country.[5] Furthermore, the dominant asset is widely held in portfolios around the world, and this large external demand leads to a permanent negative net foreign asset (NFA) position for the issuing country, consistent with the US experience. Importantly, this imbalance is not a symptom of an economic problem – it both helps generate and is supported by the resulting exorbitant privilege. Relatedly, while the aggregate portfolio holdings in the model exhibit significant home bias, the dominant country's portfolio is substantially less biased, as in the data.

To quantify our mechanism and study dynamics away from steady state, we embed our model of collateralized exchange within a broader, generalized international macroeconomic framework. For this, we focus in particular on the real goods trade channel, and ignore transaction frictions in financial assets trade, because of the availability of data on financing frictions in real goods trade which allow us to discipline the model tightly. Overall, this strategy means our results provide a lower bound on the strength of our mechanism.

In our quantitative model, each country receives an endowment of a differentiated tradable good, and the governments of the two big countries issue a safe asset denominated in their respective good. Households trade financial assets in integrated world markets, and consume both domestic and foreign goods, with all imports and exports subject to the international exchange frictions detailed above.[6] We calibrate the model by selecting parameters to match target moments on the size of government debt, trade, invoicing currency, and import markups. The model exactly matches the target moments and, we show, a number of additional, non-targeted moments are also very consistent with the data.

At the benchmark calibration, we find that the two dominant-currency steady states are indeed dynamically stable, and lie within large regions of the state space that uniquely converge to their respective dominant-currency steady state.[7] Within those regions, the equilibrium paths of the economy are determinate (i.e. not subject to sunspot shocks) and the currency regime is uniquely determined by initial economic states. This makes the model particularly well-suited for quantitative counter-factual analysis.

Using the calibrated model, we compute the welfare implications of issuing the dominant asset. At the steady-state, the welfare gain of the dominant country, relative to the large non-dominant country, is small: only 0.03% of permanent consumption. This happens despite

[5]Interest parity violations, and the exorbitant privilege specifically, are well documented in the data (e.g. Gourinchas et al. (2017), Jiang et al. (2018), Engel and Wu (2018), Valchev (Forthcoming)).

[6]This is akin to Antras and Costinot (2011) with trade financing frictions, in addition to their trade frictions. Since we are focused on third-party use of a dominant asset, however, our model would work in the same way if we assumed no such frictions in the US/EZ.

[7]To study dynamics, we introduce small portfolio adjustment costs that prevent large jumps in asset holdings, but otherwise have negligible local effects.

the central country's "exorbitant privilege," because the concomitant foreign demand for the dominant asset means its issuer has a *negative* NFA position. Thus, in steady state, the benefits of the lower interest rate are mostly offset by the dominant country's large external debt. However, factoring the transition path from a multipolar to a dominant-currency steady state changes the result by an order of magnitude. We find that the dominant country gains 0.75% of permanent consumption over the transition, as the increasing external demand for its assets allows it to increase its external debt, leading to temporarily increased consumption. Thus, our model shows that the benefits of currency dominance are much more nuanced than commonly perceived, and are concentrated in the transition period.

We conclude the paper with two counter-factual analyses that highlight the role of history and economic policy in determining the dominant currency. First, we consider why the world today finds itself near the dollar-dominant steady state in particular. Our model emphasizes the importance of path dependence – the currency equilibrium depends on the distribution of asset holdings around the world, and those have well-defined stable dynamics depending on initial conditions. We provide a detailed characterization of the history of the international monetary system since the nineteenth century through the lens of our model, but in short, the world is currently dollarized because at the end of the Bretton Woods era, foreign asset holdings heavily favored US assets, starting the world on the path to the dollar steady state.

The model predicts the currency regime would not change unless a sufficiently large shock pushes the economy out of the attraction region of the dollar steady state. We provide a quantitative exercise where we study one such development from recent history – the formation of the Eurozone. Consistent with the data, we find that the mere introduction of the Euro itself is not enough to precipitate a switch away from the dollar-dominant steady state. For this to have happened, the Eurozone's capacity of issuing safe assets would have needed to exceed that of the US by 30%; a scenario that could well have occurred if the UK had joined the Eurozone. Looking forward, the model predicts that another potential challenger, the Chinese renminbi, cannot play a significant role until the Chinese capital account is sufficiently liberalized. According to our theory, broad international access to a given safe asset is essential for it to gain dominant status.

Lastly, we consider the consequences of trade policy. We find that an indefinite trade war in which the US raises tariffs on all imports, and its trading partners respond in kind, could be enough to threaten the dominance of the dollar. Interestingly, we find that a severe-but-temporary trade war could also permanently destroy the special role of dollar assets. The effect on the international currency equilibrium represents an additional, and potentially important, welfare cost of a temporary trade war.

Relation to existing literature

In this paper, we provide a model where the dominant position of US assets in the international monetary system is due to their endogenous role as the main *medium of exchange*. Several previous authors have explored other explanations of the special role of the dollar, focusing on the implications of the unit of account and store of value roles of money. Gourinchas et al. (2019) provide an excellent overview of this literature.

Among the papers that focus on the unit of account role of money, concurrent work by Gopinath and Stein (2018) is most closely related to our own. To our knowledge, it is the only other paper that explicitly connects the dollarization of trade invoicing with exorbitant privilege. The unit of account function is also highlighted by Casas et al. (2016), who consider the effect of dollar-denomination on shock pass-through and expenditure switching.

A larger literature focuses on the capacity of countries to generate store-of-value assets. Caballero et al. (2008) argue that the United States' superior ability to produce safe assets can explain the US experience of persistent trade deficits, falling interest rates, and rising portfolio share of US assets in developing countries. Mendoza et al. (2009) and Maggiori (2017) focus on differences in financial development as a driver of global imbalances and the emergence of a dominant currency. Brunnermeier and Huang (2018) explore the impact of this mechanism in emerging market crises. Gourinchas et al. (2017) propose a framework, where US households have lower risk-aversion than foreigners, thus end up holding most of the world's risky assets. Bocola and Lorenzoni (2017) provide a framework where dollarization of world financial markets occurs because of the dollar's unique risk profile. Meanwhile, Farhi and Maggiori (2016) consider the positive and normative implications of a single dominant reserve asset versus a multipolar system. He et al. (2016) use a global-game in a world with two ex-ante identical assets to model how a single safe asset emerges in equilibrium.

Several features differentiate our work from the literature cited above. First, compared to models with ex-ante asymmetries, currency dominance emerges endogenously here in an otherwise symmetric world, allowing us to study and understand transitions from one currency regime to another. Second, currency choice is uniquely determined by endogenous economic states, as asset availability plays the role of a coordination device. Third, we model a fully dynamic economy and consider transitions between currency regimes. We view the last two features as especially desirable, because (i) historically currency regimes have changed only infrequently and (ii) accounting for transitions changes welfare implications substantially. Previous work has included some of these features, but not all together.

This paper is also related to the long literature on search-based theories of money, surveyed by Lagos et al. (2017). Many such papers examine equilibrium multiplicity and the co-existence of multiple currencies in an international setting (e.g. Matsuyama et al. (1993), Zhou (1997), Wright and Trejos (2001), Rey (2001), Ravikumar and Wallace (2002), Kannan (2009), Devereux and Shi (2013), Zhang (2014), Doepke and Schneider (2017)). A separate strand of literature, including Vayanos and Weill (2008) and Weill (2008), uses search frictions to explain variation in liquidity premia for over-the-counter asset markets. A central difference in our work, is that our endogenous asset availability channel can uniquely determine which asset emerges as "money" for given initial conditions, thus we do not have to introduce exogenous shocks to either beliefs or tastes to select the equilibrium.

2 Analytical Results

We begin by providing several key steady-state results in the context of a simple, analytically tractable model. Later, in Sections 3 and 4, we embed the mechanism in a rich dynamic model and evaluate it quantitatively, both in and out of steady state.

The environment

The world consists of two symmetric big countries, the United States (US) and the Eurozone (EZ), of equal size $\mu_{us} = \mu_{ez}$, and a continuum of small open economies with total mass μ_{rw} making up the *rest of the world* (RW). Countries are indexed by $j \in \{us, ez, [0, \mu_{rw}]\}$.

Within each country j, there is a continuum of risk-neutral international traders that can engage in a profitable transaction with a foreign counterparty. Contract enforcement across borders is imperfect, however, so each trader must post collateral to guarantee their side of the transaction. Two ex-ante identical and universally-recognized safe assets can serve this collateral role: US and EZ government bonds (i.e. dollar and euro safe assets, respectively).

To obtain collateral, traders seek an intra-period loan of one of the two safe assets in local bond-specific search and matching markets. On the other side of these credit markets are the domestic households, who make their holdings of safe assets available for such intra-period loans. We assume that traders look for a fixed amount of funding, which we normalize to one, and that they make the binary choice of either seeking dollar or euro collateral. Since our aim is to explain third-party use of a dominant currency, we focus on characterizing the optimal choice of collateral type for traders residing in a rest-of-world SOE, while assuming that the traders in the US and the EZ seek to use their respective domestic safe assets.

The probability that a country-j trader seeking to borrow a US asset is successful is given by $p_j^{\$}$, while the probability of borrowing a EZ asset is $p_j^{€}$. If a trader finds collateral, it pays a fee, $r_j^{\$}$ or $r_j^{€}$ respectively, to the household for the use of the asset, and proceeds to the international transactions market. If the trader does not find collateral, he exits the market.

Conditional on obtaining collateral, the trader seeks a trading counterparty from another country $j' \neq j$. Upon matching with a partner, the matched pair transacts using their collateral to clear any payments needed and splits the gross transaction surplus, totaling 2π, equally. For now, we assume that π is exogenous and that traders meet at random, but in Sections 3 and 4, we relax both of these assumptions.

In the event that the two counterparties' collateral is mismatched — i.e. that one side of the match uses dollar assets and the other side euro assets as collateral — the transaction's surplus is reduced by a currency mismatch cost of 2κ. Throughout we assume that $\kappa < \pi - \max\{r_j^{\$}, r_j^{€}\}$, so that the transaction is profitable even in the event of a mismatch.

This mismatch cost captures, in reduced-form, any potential reasons that traders may wish to coordinate the currency in which they transact, including (i) multiple transactions within a period, in which case traders want to stabilize the relative prices of their different trades, (ii) the risk that collateral values might not be sufficient ex-post if relative valuation of safe asset guarantees shifts, (iii) the added complication of negotiating a contract that includes a currency conversion or is conditioned on exchange rates.

A number of existing papers leverage such cross-trader complementarity that emerges when $\kappa > 0$. Our key mechanism, however, is different, and lies with the interaction between households and traders, which we highlight by showing our main results hold even if $\kappa = 0$.

RW traders' collateral choice

In deciding which asset to seek as collateral, traders in the RW compare the expected relative payoff of using US and EZ safe assets. The choice of collateral, affects a trader's probability of obtaining funding (and thus operating) and their probability of facing a currency mismatch. Letting X_j be the fraction of country j traders that choose to use dollar collateral, the average dollar use among international traders (across all countries) is:

$$\bar{X} \equiv \mu_{us} X_{us} + \mu_{ez} X_{ez} + \int_{j \in \mu_{rw}} X_j dj = \mu_{us} + \int_{j \in \mu_{rw}} X_j dj,$$

where to obtain the second expression we use the fact that the big country traders are assumed to always use their domestic currency and thus $X_{us} = 1$ and $X_{ez} = 0$.

The average dollar use, $\bar{X}$, is the probability that any given trader will be matched with a counterparty who uses dollars, and thus determines the probability of incurring the currency mismatch cost in a transaction. Hence, for a trader in country $j \in [0, \mu_{rw}]$ (i.e. one of the small open economies), the relative payoff of choosing to seek dollar over euro funding is:

$$V_j^{\$} = p_j^{\$}\left[\pi - r_j^{\$} - \kappa(1-\bar{X})\right] - p_j^{€}\left[\pi - r_j^{€} - \kappa\bar{X}\right]. \tag{1}$$

The first term in equation (1) is expected profits when operating in dollars, and the second term is expected profits when operating in euros, net of mismatch and financing costs. If $V_j^{\$} > 0$, a trader in country j strictly prefers to seek dollar funding, while if $V_j^{\$} < 0$ he strictly prefers to seek euro funding. Otherwise, he is indifferent between the two currencies.

In each country j, there are two collateral search markets, one for dollars and one for euros, and the funding probabilities $p_j^{\$}$ and $p_j^{€}$ are endogenously determined. On one side of these markets are the domestic traders, who seek to borrow an asset for use in an international transaction, while on the other side are domestic households, who offer intra-period loans of their safe asset holdings. For simplicity, we assume that the funding fees are symmetric: $r_j^{\$} = r_j^{€} = r$ for all j. Note that this makes no difference to our results – e.g. assuming that the fees are determined by Nash bargaining between households and traders would simply mean that equation (1) is scaled down by the Nash-bargaining parameter of the traders.[8]

The number of matches that emerge in a given country-currency credit market is governed by the den Haan et al. (2000) matching function, according to which the number of matches formed in a market with B units of the asset on offer and X traders searching is

$$M^f(B, X) = \frac{BX}{(B^{\frac{1}{\varepsilon_f}} + X^{\frac{1}{\varepsilon_f}})^{\varepsilon_f}}.$$

We call the parameter ε_f the matching function elasticity, and for tractability we set this parameter to unity for the remainder of this section (later we calibrate it to the data). Thus, a country-j trader searching for dollar funding succeeds with probability

$$p_j^{\$} = \frac{M^f(B_j^{\$}, X_j)}{X_j} = \frac{B_j^{\$}}{B_j^{\$} + X_j},$$

where $B_j^{\$}$ and $B_j^{€}$ are the holdings of US and EZ safe assets of the country j households. A similar expression describes the probability of obtaining euro funding. Substituting the

[8] Suppose households Nash bargain with share α. Then, financing fees are $r_j^{\$} = \alpha(\pi - \kappa(1-\bar{X}))$ and $r_j^{€} = \alpha(\pi - \kappa\bar{X})$ and a trader's net payoff of seeking dollars is $V_j^{\$} = p_j^{\$}(1-\alpha)\left[\pi - \kappa(1-\bar{X})\right] - p_j^{€}(1-\alpha)\left[\pi - \kappa\bar{X}\right]$.

expressions for the funding probabilities into equation (1) yields

$$V_j^{\$} = \frac{B_j^{\$}}{B_j^{\$} + X_j}\left[\pi - r - \kappa(1-\bar{X})\right] - \frac{B_j^{€}}{B_j^{€} + 1 - X_j}\left[\pi - r - \kappa\bar{X}\right]. \tag{2}$$

Equation (2) captures the key interaction between the collateral choice of traders and the asset holdings of households. And in particular, it shows that these choices are complements: e.g. the probability of obtaining dollar funding — and therefore a trader's payoff of choosing to seek dollars — increases with the household's holdings of US safe assets ($B_j^{\$}$).

Equation (2) also summarizes two additional strategic incentives with respect to other traders. First, with respect to other domestic traders, collateral choices are strategic substitutes: when more domestic traders look for dollar collateral (higher X_j), the local dollar funding market becomes crowded, lowering the probability of any given trader obtaining dollar funding. Second, with respect to the action of traders in other countries ($\bar{X}$), collateral choices are strategic complements when κ is strictly positive.

Lemma 1 characterizes the optimal collateral choices of country-j traders, taking as given household portfolio positions, $B_j^{\$}$ and $B_j^{€}$, and the average dollar use by foreign traders, $\bar{X}$.

Lemma 1. *Let*

$$X_j^* \equiv \frac{B_j^{\$}\left[\pi - r - \kappa\left(B_j^{€} + 1 - \bar{X}(2B_j^{€}+1)\right)\right]}{\left(B_j^{\$} + B_j^{€}\right)(\pi - r) + \kappa\left(\bar{X}(B_j^{\$} - B_j^{€}) - B_j^{\$}\right)}.$$

Given household bond holdings, $B_j^{\$}$ and $B_j^{€}$, and rest-of-world currency choice, $\bar{X}$, the optimal currency choice by traders in country $j \in [0, \mu_{rw}]$ implies $X_j = \min\{\max\{X_j^*, 0\}, 1\}$.

Proof. Proved in Appendix A. ■

Equation (1) nets out the effects of the within-country substitutability in collateral choice, leaving only the effects of within- and across-country complementarities mentioned above. To emphasize our contribution — the interaction between asset usage in transactions and asset holdings — Corollary 1 eliminates cross-country complementarities by setting $\kappa = 0$.

Corollary 1. *When $\kappa = 0$, traders' optimal currency choice depends only on $B_j^{\$}$ and $B_j^{€}$:*

$$X_j = \frac{B_j^{\$}}{B_j^{\$} + B_j^{€}}$$

In this case, the optimal currency choice is directly determined by the bond holdings of the domestic household, with the average use of dollar collateral simply equal to the share of US safe asset in the household's portfolio.

Household's optimal asset holdings

Households solve a standard consumption-savings problem, with the choice of saving in both US and EZ safe assets. For tractability, we assume there is a single consumption good and both risk-free bonds promise one unit of that good. As a result, the bonds only differ ex-ante because of their issuer.[9] Nevertheless, we show they may serve different intermediation roles in equilibrium and, therefore, earn different equilibrium interest rates.

Households in each country $j \in \{us, ez, [0, \mu_{rw}]\}$ solve

$$\max_{C_{jt}, B_{jt}^{\$}, B_{jt}^{€}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$$

subject to

$$C_{jt} + (Q_t^{\$} - \Delta_{jt}^{\$})B_{jt}^{\$} + (Q_t^{€} - \Delta_{jt}^{€})B_{jt}^{€} = B_{jt-1}^{\$} + B_{jt-1}^{€} + Y_{jt},$$

and a non-negativity constraint on bond-holdings. In the above, Q_t^c is the price of a c-denominated bond for $c \in \{\$, €\}$, Δ_j^c is the liquidity premium earned by safe asset c in country j, and Y_{jt} is an endowment of consumption goods.

The liquidity premia Δ_j^c are endogenous and derive from the fees that borrowing traders pay to lending households. The liquidity premium an asset earns is equal to the probability that the household successfully lends it in its respective credit market, multiplied by the funding fee r that it receives when it does so. Thus, the premia can be expressed as

$$\Delta_j^{\$} = r \times \frac{M^f(B_j^{\$}, X_j)}{B_j^{\$}} = \frac{X_j}{B_j^{\$} + X_j} r, \tag{3}$$

$$\Delta_j^{€} = r \times \frac{M^f(B_j^{€}, 1 - X_j)}{B_j^{€}} = \frac{(1 - X_j)}{B_j^{€} + (1 - X_j)} r. \tag{4}$$

Equation (3) shows that higher dollar use by the traders of a given country (i.e. higher X_j) implies a higher liquidity premium on US safe assets in that country. On the other hand, if more dollar bonds are held by the country j households ($B_j^{\$}$), the lower is the resulting liquidity premium on US safe assets.

For the analytical results in this section, we focus on steady-state equilibria, i.e. situations in which both households and traders would find it optimal to maintain their current strategy

[9] In the quantitative model analyzed in Sections 3 and 4, each country's bond promises repayment in terms of that country's differentiated good and exchange rates are endogenously determined. This additional richness has little effect on our steady-state analysis.

indefinitely. For example, at an interior steady state, optimal bond holdings are governed by a set of country-specific Euler equations equating the returns of US and EZ assets:

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta_j^{\$}} = \frac{1}{Q^{€} - \Delta_j^{€}}. \tag{5}$$

Since the Euler equations hold for all j that hold the asset, the liquidity premia earned by asset c are equalized across those countries.

We close the model by assuming that the total supply of US and EZ assets are both exogenous and equal to $\bar{B}$. Using expressions (3) and (4) for the liquidity premia, steady-state Euler equations, and bond market clearing ($\bar{B} = \int_{\mu_{rw}} B_j^c dj + \mu_{us} B_{us}^c + \mu_{ez} B_{ez}^c$), we can solve for the households' optimal portfolio holdings as a function of traders' actions X_j.

Lemma 2. *Equilibrium household portfolios, as a function of traders' currency choices, are:*

$$B_j^{\$} = \bar{B} \frac{X_j}{\int_{\mu_{rw}} X_j dj + \mu_{us}} \tag{6}$$

$$B_j^{€} = \bar{B} \frac{1 - X_j}{\int_{\mu_{rw}} (1 - X_j) dj + \mu_{ez}}. \tag{7}$$

for all $j \in \{us, ez, [0, \mu_{rw}]\}$.

Proof. Proved in Appendix A. ■

Equations (6) and (7) summarize the link between currency use and household portfolios. Crucially, the equations imply that there is a positive feedback between household asset allocations and the traders' currency choice within a country: e.g. higher X_j implies higher $B_j^{\$}$. Intuitively, an asset that is heavily used for funding the international transactions of a given country's traders will deliver a higher liquidity premium, thereby giving that country's households an incentive to increase holdings of this kind of bond.

Together, Lemma 2 with Corollary 1 show that household asset allocations and trader currency choices are strategic complements *even when* $\kappa = 0$. Thus, this complementarity, which is the key force in our mechanism, is distinct from the cross-country complementarity between traders (captured by κ) often emphasized by previous work.

Steady-state equilibria

The complementarity between households and traders in the rest-of-world countries results in multiple steady-state equilibria, including dominant-currency ones, even when $\kappa = 0$.

We consider symmetric steady states in which the strategies of the ex-ante identical RW traders and households are the same (i.e. $X_j = X$, $B_j^{\$} = B^{\$}, B_j^{€} = B^{€}$ for all $j \in [0, \mu_{rw}]$).

Definition 1 (Steady-state Equilibrium). *A steady-state equilibrium is a rest-of-world currency usage X and set of asset holdings $\{B^{\$}, B^{€}, B_{us}^{\$}, B_{us}^{€}, B_{ez}^{\$}, B_{ez}^{€}\}$ such that*

1. *The optimality conditions of the rest-of-world traders' are satisfied (Lemma 1)*

2. *The optimality conditions of household bond holdings are satisfied (Lemma 2).*

3. *Bond markets clear:*

$$\bar{B} = \int_{\mu_{rw}} B_j^c dj + \mu_{us} B_{us}^c + \mu_{ez} B_{ez}^c, \text{ for } c \in \{\$, €\}.$$

In a steady-state, bond prices $\{Q^{\$}, Q^{€}\}$ and liquidity premia $\{\Delta^{\$}, \Delta^{€}\}$ are then pinned down by equations (3)-(5), together with the fact that the big-country traders use their domestic currency: $X_{us} = 1 - X_{eu} = 1$. In addition, clearing in real goods markets will determine equilibrium consumptions. We delay our discussion of consumption and associated welfare, however, until we examine our quantitative model in Section 3 and 4.

Proposition 1 establishes that the economy *always* has dominant currency equilibria:

Proposition 1. *For any $\kappa \geq 0$, the economy has three steady-state equilibria:*

(i) a dollar-dominant steady state with $X = 1$ and $B^{€} = 0$;

(ii) a euro-dominant steady state with $X = 0$ and $B^{\$} = 0$;

(iii) a multipolar steady state with $X = 1/2$ and $B^{\$} = B^{€}$.

Moreover, these are the only steady states except when $\kappa = \bar{\kappa}$,

$$\bar{\kappa} \equiv \frac{(\pi - r)}{\bar{B} + 1},$$

in which case there is a continuum of steady-state equilibria with $X \in [0, 1]$.

Proof. Proved in Appendix A. ■

The fact that steady-state multiplicity emerges even when $\kappa = 0$ is one of the key results of our framework. It highlights the feedback between household and trader choices generated by financial frictions *within* countries. The dollar-dominant steady state exists not because

RW traders seek to match behavior with their trading counter-parties, but because the RW households happen to hold a lot of US assets, making dollar financing the most convenient for RW traders. RW households, in turn, are happy to concentrate their holdings in US assets because the traders' demand for dollar funding gives rise to a liquidity premium on dollar assets. Naturally, this argument applies to all three potential steady states — traders are willing to play an equal-weight mixed strategy if the household holds a diversified portfolio, while the households will indeed hold a diversified portfolio if there is no asymmetry in traders' demand for US and EZ collateral.

The logic with $\kappa = 0$ also shows why the steady states described in the Proposition are robust to generalizing many details of our model of international exchange. For example, any strategy that traders may take to avoid paying currency mismatch costs — such as directing their search to counterparties holding a particular type of collateral or renegotiating the settlement currency ex-post — will not eliminate any of these steady states. Similarly, the result is robust to allowing households to lend their assets as collateral in foreign markets, since the relative optimal shares of assets listed in every market will still satisfy (6) and (7).

Cross-country complementarity: $\kappa > 0$

In addition to being sufficient for generating coordinated equilibria, the complementarity between households and traders interacts in an important way with the cross-trader complementarity that emerges when $\kappa > 0$. To analyze this interaction, it is useful to define the notion of quasi-equilibrium in currency use, in analogy to Mas-Colell et al. (1995, p. 551).

Definition 2 (Quasi-equilibrium). *Given asset holdings of the RW households $\{B^{\$}, B^{€}\}$, a symmetric quasi-equilibrium is a rest-of-world traders' currency usage choice X such that*

1. *The optimality conditions of the rest-of-world traders' are satisfied (Lemma 1).*

We denote the set of quasi-equilibria by the correspondence $X(B^{\$}, B^{€})$.

The definition of quasi-equilibrium captures the notion of equilibrium currency choice among traders, taking as given HH asset holdings. The correspondence $X(B^{\$}, B^{€})$ thus allows us to explore how optimal currency usage shifts with changes in asset allocations.

Proposition 2 uses the concept of quasi-equilibrium to formalize the manner in which household bond holdings have an *anchoring effect* on the optimal currency choice. It also shows that moderate levels of cross-trader complementarity (κ) serve to reinforce traders' coordination on using the asset that is relatively more widely-held across the RW countries.

Proposition 2. *Suppose* $\kappa < \frac{\pi - r}{\mu_{rw}\left(\min\{B^{\$}, B^{€}\} + \frac{1}{2}\right) + \frac{1}{2}}$. *Then*

(*i*) *the currency quasi-equilibrium is unique, with* $X(B^{\$}, B^{€}) > \frac{1}{2}$ *if and only if* $B^{\$} > B^{€}$.

(*ii*) *for interior asset holdings, higher* κ *reinforces coordination on the relatively more widely-held asset. That is*

$$\frac{\partial X(B^{\$}, B^{€})}{\partial \kappa} \geq 0$$

whenever $B^{\$} > B^{€}$, *and vice-versa. The inequality is strict when* $X(B^{\$}, B^{€}) \in (0, 1)$.

Proof. Proved in Appendix A. ■

Part (i) of Proposition 2 shows that asymmetric bond holdings favor a quasi-equilibrium with higher use of the relatively more widely-held asset. Intuitively, an asymmetry in household bond holdings acts as a *coordination device* for international traders, helping them to synchronize their currency choices. Part (ii) of the proposition shows that increasing the cross-trader coordination motive interacts with the availability mechanism to amplify this anchoring effect of bond holdings.

For example, if RW households hold more dollar assets, on the one hand an individual RW trader will shift her usage toward dollar collateral because it is easier to obtain. On the other hand, that same trader realizes that her counterparties are similarly more likely to fund themselves with dollar collateral due to its greater availability across RW household portfolios. This further increases her own incentives to seek dollar funding. The Proposition shows that increasing κ increases the strength of this feedback channel, leading to more concentrated currency use for any given RW bond holdings, $B^{\$}$ and $B^{€}$.

A suggested by Proposition 2, it is possible that the cross-trader coordination incentives are so strong that they swamp the anchoring effects of assets holdings, so that there are multiple currency quasi-equilibria for a given set of bond holdings. In particular, the uniqueness threshold for κ becomes tighter (i.e. falls) as the size of $\min\{B^{\$}, B^{€}\}$ increases. This happens because larger and more symmetric household asset holdings make any type of funding less difficult for traders to obtain, reducing the anchoring effects of the friction in credit markets.

Due to market clearing in bond markets, the asset holdings of RW households are bounded above by $\frac{\bar{B}}{\mu_{rw}}$. Using this observation, we can derive the following condition that ensures an unique currency quasi-equilibrium, for any feasible asset allocation:

$$\kappa < \kappa^{sunspot} \equiv \frac{\pi - r}{\mu_{rw}\left(\frac{\bar{B}}{\mu_{rw}} + \frac{1}{2}\right) + \frac{1}{2}}.$$

If this condition is not satisfied, it is possible for pure "sunspot shocks" to traders' beliefs about what other traders would do, to shift the currency quasi-equilibrium.

By contrast, the anchoring effect of household bond positions can help explain why historically currency regimes have tended to be persistent. In particular, so long as the cross-country coordination motive is not too strong ($\kappa < \kappa^{sunspot}$), the currency quasi-equilibrium is unique, and thus a change in the equilibrium currency use also requires a corresponding shift in household bond positions. We exploit this feature in our quantitative model (Section 4), where asset holdings are slow-moving state variables. This allows us to generate determinate equilibrium paths, and thus characterize tightly dynamics out of steady state. Existing currency competition models based only on high κ-complementarity, instead, typically generate dominant currency equilibria only in the presence of equilibrium indeterminacy, and thus cannot as readily analyze transitions or speak to the persistence of currency regimes.

Stability of steady states

In this section, we explore the local stability of our steady-state equilibria. Our notion of local stability is both analytically tractable and provides intuition about the later results on dynamic stability. We consider a steady state locally stable if the best response functions of the traders and the households jointly define a contraction mapping in the neighborhood of that steady state. Intuitively, this ensures that a one-time unilateral deviation in the strategy of one type of player does not lead to a change in the steady-state equilibrium.

We find that when the currency coordination incentives across traders, κ, is sufficiently high, the two dominant currency steady states are stable and the symmetric steady state is not. The converse holds when κ is low. Importantly, however, the dominant-currency steady states become stable at a relatively low level of κ, which is below the threshold at which sunspots appear. This result is stated formally in Proposition 3.

Proposition 3. *For $\kappa < \bar{\kappa}$ only the symmetric steady state is locally stable, while for $\kappa > \bar{\kappa}$ only the two coordinated, dollar and euro-dominant steady states, are locally stable, where*

$$\bar{\kappa} \equiv \frac{\pi - r}{\bar{B} + 1} < \kappa^{sunspot}.$$

Proof. Proved in Appendix A. ■

The result can be understood by considering the two different kinds of incentives traders face in their currency choice – the asset availability incentive due to the feedback between household and trader choices, and the cross-country coordination incentive across traders.

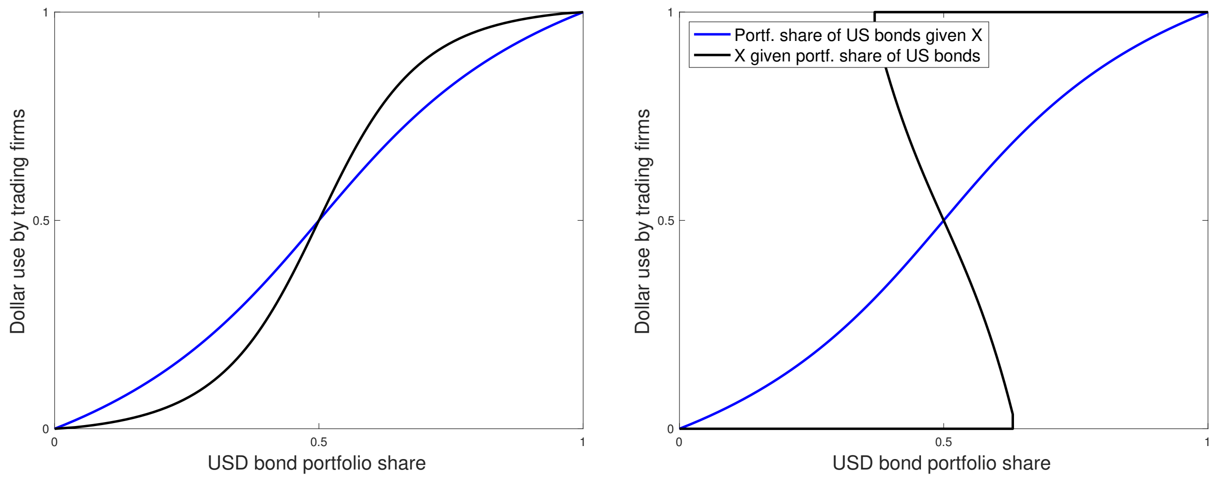
The basic feedback between households and traders operates through congestion externalities in the credit markets, which tends to favor the symmetric steady state. Intuitively, both households and traders have incentives to try and maximize their chances of matching in credit markets, and thus their optimal responses to this force tend to orient them towards the symmetric steady state where neither the US nor the EZ collateral markets are too congested and both sources of liquidity are used equally. On the other hand, the symmetric steady state is not desirable when κ is high, because it maximizes the traders' expected losses due to currency mismatch. As a result, higher κ will tend to push traders away from the symmetric steady states. These two forces are exactly equal to one another when $\kappa = \bar{\kappa}$, and thus when κ is above that threshold, the symmetric steady state is not locally stable, but the two coordinated steady states (where $X = 1$ or $X = 0$) are locally stable.

Crucially, thanks to the interaction between the cross-trader and trader-household complementarities, the coordinated steady states can be locally stable, even if the cross-trader coordination incentives are too weak to create dominant currency equilibria by themselves (and thus be subject to sunspots). Intuitively, when bond holdings are asymmetric and thus favor a given currency, the effect of κ amplifies the reaction of traders who shift towards that currency. As a result, even a relatively small κ can make coordinated steady states stable.

For $\kappa \in [\bar{\kappa}, \kappa^{sunspot})$ the model delivers locally stable dominant-currency steady states and currency choice remains uniquely determined *given* bond holdings. For $\kappa > \kappa^{sunspot}$, the dominant-currency steady states are still locally stable, but the quasi-equilibrium in currency markets is no longer uniquely determined given household's actions.

This result is illustrated graphically in Figure 1, which plots the optimal actions of households and traders as a function of the other's action. The solid black line plots the currency quasi-equilibrium, X, as a function of the share of US bonds held in the portfolios of RW households, and the blue line is the optimal choice of portfolio holdings, given X. In the left panel, we show the case where $\bar{\kappa} < \kappa < \kappa^{sunspot}$. In that case, (i) the asset availability channel is strong enough to ensure a unique currency choice equilibrium for any portfolio composition and (ii) κ is still high enough to ensure that the coordinated steady states are locally stable. The local stability can be inferred from the fact that the back line crosses the blue line from below (above) at the left (right) corners of the graph.

In contrast, in the right panel we illustrate the case $\kappa > \kappa^{sunspot}$. Whereas in the left panel the black line was increasing everywhere, and thus there was a unique currency choice quasi-equilibrium in the game between traders, in the right panel the black line is a correspondence. This signifies that for some allocations of assets, the model is subject to sunspot multiplicity in currency markets – i.e. there might be different quasi-equilibrium values of X, for the



(a) $\bar{\kappa} < \kappa < \kappa^{sunspot}$ (b) $\kappa^{sunspot} < \kappa$

Figure 1: Equilibrium in the simple game.

exact same household asset positions $B^{\$}$ and $B^{€}$.

The interaction between the two mechanisms is crucial in order for our model to deliver persistent currency regimes. For neither the asset availability mechanism nor the cross-country complementarity can do so on their own: If $\kappa = 0$, the dominant currency steady state equilibria are unstable, hence unlikely to persist. While if $\kappa > 0$ and bond supplies are infinite (so credit frictions are non-existent), currency choices are subject to sunspot shocks.

In Section 4, we calibrate the generalized, fully dynamic model to the data, and quantify the net effect of these forces, and find that it indeed results in both dynamically stable dominant-currency steady states and determinate transition paths. These features are crucial for the empirical realism of the model, since historically international currency regimes have exhibited a single dominant currency and strong persistence.

Implications

Although our model is stylized, the dollar-dominant steady state (i.e. $X = 1$) already captures several important features of the real world. First, dollar assets are more widely used in international transactions, and thus serve as the dominant international medium of exchange. At the dollar steady state, the overall international uses of dollars and euros are, respectively, $\bar{X} = \mu_{rw} + \mu_{us}$ and $1 - \bar{X} = \mu_{ez}$. Hence, since $\mu_{us} = \mu_{ez}$, it follows that

$$\bar{X} > \frac{1}{2}$$

Importantly, the dollar dominates international exchange because of its outsize role in facilitating *third-party* transactions (i.e. dollar use by RW traders), as is true in the data.

An implication of the higher demand for US assets is that the dollar liquidity premium is higher than that of the euro. Using $X = 1$, equations (3) and (4), and Lemma 2:

$$\Delta^{\$} = \frac{r\bar{X}}{\bar{B} + \bar{X}} > \frac{r(1 - \bar{X})}{\bar{B} + 1 - \bar{X}} = \Delta^{€}.$$

As more traders use dollars for international trade, a larger proportion of funding fees are paid to dollar assets, increasing their equilibrium liquidity premium. Thus, the US bond also has a higher equilibrium price; using the above result and equation (5):

$$Q^{\$} - Q^{€} = \Delta^{\$} - \Delta^{€} > 0.$$

Naturally, this difference in bond prices leads to a failure of interest parity at steady state, in favor of the (high-interest rate) euro-bond:

$$\frac{1}{Q^{€}} - \frac{1}{Q^{\$}} > 0.$$

Intuitively, since the dollar liquidity premium is higher, the euro asset must be compensated with a higher interest rate, and thus the high interest rate currency earns positive excess returns relative to the low interest rate currency. At the deterministic steady state we analyze here, there is no distinction between covered and uncovered interest parity, but we note that both have been shown to fail in the data.

A second important implication is that dollar assets are also more broadly *held* around the world than euro assets: substituting $X = 1$ into (6) and (7) it follows that $B^{\$} > B^{€}$. This is due to the feedback between household and trader choices — the currency choice of traders gives RW households an incentive to concentrate their portfolios in US assets.

The large external demand for US assets leads to a negative net foreign asset (NFA) position in the US. The aggregate foreign assets of US agents are simply the US household's holdings of EZ bonds ($B^{€}_{us}$, the only non-US asset), while the aggregate foreign liabilities of the US are the sum total of US bonds held outside of the US – the sum of US bonds held by EZ households ($B^{\$}_{ez}$) and RW households ($B^{\$}$). Using the closed form solutions for per-capita bond holdings (6) and (7) at the dollar steady state (i.e. plug-in $X = 1$), and weighting by the relative measures of the different types of households, we have:

$$NFA_{us} = \underbrace{\mu_{us} B_{us}^{€}}_{\text{US foreign assets}} - \underbrace{\left(\mu_{ez} B_{ez}^{\$} + \int_0^{\mu_{rw}} B_j^{\$} dj \right)}_{\text{US foreign liab.}} = -\bar{B} \frac{\mu_{rw}}{\mu_{rw} + \mu_{us}} < 0. \tag{8}$$

Equation (8) underscores the tight link between international asset positions and currency dominance: coordination on the dominant currency is sustained by large holdings of that asset outside of its issuing country. This result is consistent with the historical experience of the US: As it has risen as the dominant provider of international safe assets, the US has also developed a persistent negative net foreign asset position. Through the lens of our model, the negative NFA is not a temporary "imbalance" that would correct in the long-run, but is an inherent part of being the supplier of the dominant international currency.

Lastly, we observe that due to the failure of interest parity described above, the net return that the dominant country earns on its negative NFA position could in fact be positive. In the case of a dollar-dominant steady state, the high external demand for US assets both increases US foreign liabilities (generating a negative NFA position) and *decreases* the rate of return paid on these liabilities (i.e. a lower $\frac{1}{Q^{\$}}$). These effects go in opposite directions, and the total effect of these forces is a quantitative question to which we now turn.

3 Dynamic General Equilibrium Model

Having illustrated several key implications of our mechanism, we now relax the simplifying assumptions of the previous section and embed the mechanism in a rich dynamic general equilibrium model. This serves both to confirm that the basic insights of the analytical model carryover to a more general context, and to quantify the effects of the mechanism. After calibrating the model, we use it to (i) quantify the welfare implications of currency dominance, (ii) analyze transition dynamics away from steady state, and (iii) explore reasons a dollar-dominant regime may have emerged and how policy choices could change it.

Quantifying our mechanism requires data on international transactions and the associated frictions and returns. Though our model could be important for both trade in goods and financial assets, empirical measurement has historically focused on the financial frictions in goods trade.[10] Due to the resulting lack of appropriate data to discipline the frictions in

[10]While the literature has long recognized that international transactions face imperfect contracting problems and thus often require financing, there has been little effort at systematically surveying and quantifying the extend to these frictions until recently. The global financial crisis lead to a precipitous fall in inter-

international financial transactions, we model the mechanism as applying only to real goods trade. Thus, our results provide a lower bound on the total effect of our mechanism, since they ignore the likely substantial effects of frictions in trade in financial assets.

3.1 Motivating Evidence

Before describing the general model, we provide a brief overview of the empirical evidence that motivates our modeling choices. The data show that international trade in goods trade is large, and that (i) the vast majority of trade transactions require financing (on both import and export sides); (ii) that a substantial share of trade financing happens through banks; and (iii) that international trade financing is highly dollarized, perhaps even more so than trade invoicing. Moreover, these phenomena are especially pronounced in emerging markets, consistent with our focus on the currency choice of RW traders.

Lastly, we conclude this section by showing that, at the country level, higher dollar invoicing of trade is indeed strongly associated with higher holdings of dollar-denominated assets. Thus, trade does not only face frictions like the ones we postulate, but the strong link between asset holdings and currency choice is borne out by the data.

Trade Finance in the Data

A recent study by the World Trade Organization indicates that 80-90% of world trade is supported by some form of "trade financing", defined as external financing that is directly tied to facilitating an international trade transaction (Auboin, 2016). In 2017, this amounted to somewhere between \$20 and \$25 trillion worth of international trade intermediation. Meanwhile, a detailed study of the bank-intermediated portion of trade finance in particular (BIS, 2014) finds that direct bank involvement finances about 30-40% of world trade (\$8-10 trillion dollars of annual financing). Much of this activity is concentrated in emerging markets, in which up to 60% of total trade flows are directly financed by banks. Moreover, their data shows that in the case of emerging markets trade finance is almost exclusively locally sourced, with domestic banks supporting more than 80% of all trade financing activity.

Similar to the evidence on the currency of denomination of trade invoices, the BIS report also finds that trade finance products are highly dollarized, concluding that international trade is not only priced, but also predominantly *settled* in dollars. This is again especially

national transactions (in both goods and financial assets), and this has galvanized a newfound interest in documenting the financing frictions involved in greasing the wheels of international flows. These efforts have so far centered on understanding the friction in goods trade (e.g. Amiti and Weinstein (2011), Ahn (2014), BIS (2014), Antras and Foley (2015)), due to the extensive availability on export and import flows data.

true for emerging markets, which tend to be more reliant on the Letter of Credit (LC) form of trade financing (more than 80% of which are denominated in dollars), and where trade finance loans (an alternative to LCs) are also predominantly denominated in dollars. For example, the report finds that in China such loans are twice as likely to be denominated in dollars than in renminbi (with no other foreign currency playing a role), while in India 90% of import and export loans are in dollars. The US and the Eurozone, on the other hand, primarily use their respective local currency – for example 80-90% of the export/import loans in Italy are in euros. Both of these findings align with the basic results of our mechanism.

The report concludes that, given the dollarized nature of trade finance, "a key condition for the ability of many banks to provide trade finance is their access to US dollar funding." Combined with the fact that the bulk of trade finance in emerging markets is locally sourced, this implies that for firms to have access to trade finance, the local financial industry itself must have access to dollar funding. This is consistent with the model, where local access to different types of safe assets plays a major role in the ability of obtaining trade financing.

Lastly, a number of studies (e.g. Amiti and Weinstein (2011), Ahn (2014), Antras and Foley (2015), Niepmann and Schmidt-Eisenlohr (2017)) have shown that trade finance is not only large, but that it plays a quantitatively important role in supporting real goods flows. For example, Amiti and Weinstein (2011) find that when the access to dollar funding of banks engaged in trade finance is restricted, this leads to a significant negative impact on the international trade activities of the clients of those banks. On the other hand, the Asian Development Bank (ADB) estimates that there is a roughly $2 trillion trade finance "gap" of unmet trade finance needs, leading to many potential international transactions not being consummated (see also Auboin (2016)). Such shortages have important economic effects, and are in line with our assumption of search frictions in credit markets.

The emerging evidence on the importance of trade finance is perhaps not surprising, given that the international trade literature has long recognized that trade faces unique challenges caused by imperfect cross-border contracting, transportation lags, and etc. (Antràs, 2013). Trade finance helps overcome such frictions, though it is not the only way firms cope: e.g. transactions between subsidiaries within multinational firms form a substantial share of US trade flows (Antràs and Yeaple, 2014). Nevertheless, the evidence cited above suggests trade finance remains a crucial part of trade globally, especially in emerging markets. This is consistent with our modeling approach, which seeks to characterize the currency choice of firms in countries like China and India, rather than the currency choices of US multinationals.

Table 1: Dollar invoice share and portfolio share of dollar bond holdings

	(1)	(2)	(3)	(4)	(5)
$\frac{B_i^{us}}{\sum_{c\in Countries} B_i^c}$	0.70***		0.62***		
	(0.16)		(0.2)		
$\frac{EX_i^{us}+IM_i^{us}}{\sum_c (EX_i^c+IM_i^c)}$		1.00***	0.26		0.44
		(0.34)	(0.39)		(0.32)
$\frac{B_i^{\$}}{\sum_{c\in currencies} B_i^c}$				0.72***	0.65***
				(0.10)	(0.13)
R^2	0.34		0.35	0.67	0.70
N	42	42	42	28	28

Holdings of US bonds and invoicing

Here we show that the share of US bonds in a country's aggregate portfolio is indeed highly correlated with the share of dollar invoicing of its trade, thus directly confirming one important implication of our mechanism.[11] In particular, we estimate the regression

$$X_i = \alpha + \beta_{B_{usd}} \frac{\text{Holdings of US bonds}_i}{\text{Total Foreign Bond Holdings}_i} + \beta_{UStrade} \frac{\text{Trade with US}_i}{\text{Total Trade}_i} + \varepsilon_i$$

where X_i is the share of dollar invoicing in country i's trade (exports plus imports) as per Gopinath (2015). The first regressor is the holdings of US bonds as a share of all foreign bond holdings of country i, and the second regressor is country i's direct trade with the US as a share of its total international trade.[12] The estimates are presented in Table 1.

As predicted by the model, we find that the portfolio share of US bond holdings is highly positively correlated with the share of trade invoiced in dollars (column (1)). Moreover, US bond holdings do not simply proxy for the share of direct trade with the US – controlling for the share of direct trade with the US does not change the magnitude or strength of the relationship with bond holdings (column (3)). In fact, direct US trade is not significantly associated with invoicing once we control for bond holdings.

[11] As documented by BIS (2014), the currency of invoicing is closely related to the currency in which trade is settled and financed, but we do not have direct data on the settlement currency. We use Gopinath (2015)'s data on invoicing instead to proxy for it.

[12] For portfolio composition, we use data from the IMF's "Coordinated Investment Portfolio Survey" and for trade levels, we use data from the US census and World Bank.

Lastly, for a subset of the countries, we also have data on the currency composition of their foreign bonds holdings. Our theory predicts that what truly matters for invoicing is the currency denomination of the safe asset holdings, not that they are underwritten by the US in particular.[13] And indeed, we find that the relationship between bond holdings and invoicing is even stronger, once we use the share of dollar denominated bonds (see columns (4) and (5)). Thus, any way we slice the data we find that higher concentration holdings of US/USD safe assets is closely associated with a higher use of the dollar in international trading transactions, as predicted by the model.

3.2 Model Overview

The model consists of households, governments, and international trade firms who perform export and import transactions. International trade faces financing frictions, where import/export firms need to collateralize their transactions with either US or EZ safe assets. We describe the key model elements here, and leave detailed derivations to Appendix B.

Households

The household sector in any country $j \in \{us, ez, [0, \mu_{rw}]\}$ consists of a representative consumer who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}.$$

Each country supplies its own differentiated good, and consumption baskets C_{jt} are Cobb-Douglas aggregators of home and foreign goods. The consumption share of the domestic good is α_h, and the consumption shares for foreign goods are proportional to the respective size of the origin country. So, for example, the US consumption basked is:

$$C_{us,t} = (C_{us,t}^{us})^{a_h} \left((C_{us,t}^{ez})^{\frac{\mu_{ez}}{\mu_{ez}+\mu_{rw}}} (C_{us,t}^{rw})^{(\frac{\mu_{rw}}{\mu_{ez}+\mu_{rw}})} \right)^{1-a_h}$$

where $C_{jt}^{rw} = (\int (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$ is an aggregate of RW goods. Throughout we use the notation C_{jt}^{i} to denote consumption of good i in country j.

[13] Maggiori et al. (2018) show that investors have an affinity for dollar denominated foreign assets, even when the issuer has a different local currency.

Aggregate price indexes, P_{jt}, follow the standard formulas, e.g. for the US we have:

$$P_{us,t} = \frac{1}{K}(P^{us}_{us,t})^{a_h}\left((P^{ez}_{us,t})^{\frac{\mu_{ez}}{\mu_{ez}+\mu_{rw}}}(P^{rw}_{us,t})^{\frac{\mu_{rw}}{\mu_{ez}+\mu_{rw}}}\right)^{1-a_h}.$$

where K is the standard proportionality constant (derived in the Appendix), and $P^{j}_{us,t}$ is the price of country j's differentiated good in the US. Notice that due to the frictions in international trade, the law of one price would not hold in our economy, and goods will have different prices in different localities, depending on the equilibrium patterns of trade.

In addition to consumption, households also choose how much to save and how to allocate savings among US and EZ bonds, each of which has a payoff equal to the domestic price of their respective good — i.e. a US bond purchased at time $t-1$ yields a payment of $P^{us}_{us,t}$ at time t. Thus, the household in country j faces the budget constraint:

$$\begin{aligned} P_{jt}C_{jt} + (1-\Delta^{\$}_{jt})P^{us}_{us,t}Q^{\$}_t B^{\$}_{jt} + (1-\Delta^{€}_{jt})P^{ez}_{ez,t}Q^{€}_t B^{€}_{jt} + P^{us}_{us,t}Q^{\$}_t\tau(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1}) + \\ P^{ez}_{ez,t}Q^{€}_t\tau(B^{€}_{jt}, \underline{B}^{€}_{j,t-1}) = P^{us}_{us,t}B^{\$}_{jt-1} + P^{ez}_{ez,t}B^{€}_{jt-1} + P^{j}_{jt}Y_{jt} + \Pi^{T}_{jt} + T_{jt}, \end{aligned} \tag{9}$$

where $Q^{\$}_t$ and $Q^{€}_t$ are the prices of the US and the EZ bonds respectively, $\Delta^{\$}_{jt}$ and $\Delta^{€}_{jt}$ represent the endogenous liquidity premia earned by the bonds lent within the period to trading firms, Y_{jt} is the household's endowment of its domestic good, Π^{T}_{jt} is the total profit of country j's trading sector (gross of fixed and mismatch costs), and T_{jt} are lump-sum taxes.

Lastly, we assume that households face external portfolio adjustment costs. These costs are parameterized by the function $\tau(B, \underline{B}) \equiv \frac{\tau}{2}\left(\frac{B-\underline{B}}{\underline{B}}\right)^2 \underline{B}$, which is quadratic in terms of percent deviations from country-wide bond holdings entering the period, $\underline{B}^{\$}_{j,t-1}$ and $\underline{B}^{€}_{j,t-1}$. These adjustment costs are zero at (any) steady state, and simply serve to prevent excess volatility of capital flows without affecting the average level of bond holdings.

Intertemporal optimality implies the following household Euler equations:

$$1 = \beta E_t\left[\left(\frac{C_{jt+1}}{C_{jt}}\right)^{-\sigma}\frac{P_{jt}}{P_{jt+1}}\frac{P^{us}_{us,t+1}}{P^{us}_{us,t}}\frac{1}{Q^{\$}_t(1-\Delta^{\$}_{jt}+\tau'(B^{\$}_{jt}, \underline{B}^{\$}_{j,t-1}))}\right] \tag{10}$$

$$1 = \beta E_t\left[\left(\frac{C_{jt+1}}{C_{jt}}\right)^{-\sigma}\frac{P_{jt}}{P_{jt+1}}\frac{P^{ez}_{ez,t+1}}{P^{ez}_{ez,t}}\frac{1}{Q^{€}_t(1-\Delta^{€}_{jt}+\tau'(B^{€}_{jt}, \underline{B}^{€}_{j,t-1}))}\right]. \tag{11}$$

The Import-Export Sector

Each period, new import-export firms are formed, operate, return profits to local households, and then disband. Exporting firms look to match with importers from another country in order to ship goods across borders. This market structure aligns well with micro data, which shows significant churn in bilateral trade relationships (Eaton et al., 2016).[14] Overall, the model is akin to combining the intermediated trade mechanism of Antras and Costinot (2011), with trade financing and collateralization frictions.

There are three stages to the life of each firm. In stage one, prospective firms choose whether or not to pay a fixed cost ϕP_{jt} and become operational for period t. Upon entry, firms do not know whether they are going to be importing or exporting, or the country with which they will eventually trade, but optimally choose the probability of each one of these activities ex-ante.[15] Intuitively, importing or exporting opportunities with any partner country arrive stochastically, and firms can choose how hard to look for each type of opportunity.

In stage two, firms look for funding and have the choice of seeking either dollars or euros (i.e. US or EZ safe assets). The firms face a search friction in obtaining funding, and are assumed to look for a fixed amount of funding, normalized to one unit of the numeraire.

If a firm obtains funding, it proceeds to stage three, where it discovers whether it is an importer or an exporter and where from/to, and then searches for an appropriate foreign trading partner. If a trading match is formed, the importer buys goods from the exporter, sells them in its domestic market, and the resulting surplus is split between the two. The transaction is settled using the collateral guarantees, but if the two counter-parties have mismatched collateral, they incur an additional cost of a κ fraction of the transaction value.

We solve the firm's problem backwards and thus detail the three stages in reverse.

Trading Round and Profits (Stage 3): The importer and exporter in a trading match split the surplus of their transaction using Nash bargaining. For example, the resulting profit for a RW importer who uses dollars transacting with a US exporter is:

$$\pi^{\$,im}_{(rw,us),t} = \frac{(1-\alpha)}{P^{whol}_{(us,rw),t}} \left[P^{us}_{rw,t} - P^{us}_{us,t} - \kappa P^{whol}_{(us,rw),t}(1 - \tilde{X}_{us,t}) \right]. \tag{12}$$

In the above, α is the Nash bargaining share of the exporter, $P^{us}_{rw,t}$ is the price of the US good when sold in RW, and $P^{us}_{us,t}$ is the price at which the US good can be procured at in the US – the difference between the two is the gross markup on exports. $P^{whol}_{(us,rw),t}$ is the price at which the US exporter and the RW importer transact with each other (determined by the Nash bargain over the surplus), $\tilde{X}_{us,t}$ is the fraction of US trading firms funded with dollars,

[14] We have also experimented with allowing for persistent trading matches. This makes currency use a state variable, reinforcing the persistence of dominant-currency steady states, and amplifying our results.

[15] This assumption is convenient because it us allows to analyze a game with a single player type, but it is not necessary – the basic result holds even if the firm knew what trading opportunity it will have.

and thus $\kappa P^{whol}_{(us,rw),t}(1-\tilde{X}_{us,t})$ is the expected currency mismatch cost. General formulas for the surplus for all possible bilateral match combinations are provided in the appendix.

Funding Stage (Stage 2): Before engaging in trade, the import/export firms seek safe asset collateral in local dollar or euro credit markets.

On the other side of these trade finance markets are domestic households who make their holdings of US and EZ bonds available for lending. Matches in the funding market are generated by the den Haan et al. (2000) matching function, with an additional parameter ν that exogenously governs the velocity of collateral. For example, the probability that a country j trading firm seeking US bonds finds a funding match is

$$p^{\$}_{jt} = \frac{M^f\left(m_{jt}X_{jt}, \nu P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_t\right)}{m_{jt}X_{jt}},$$

where m_{jt} is the equilibrium mass of trading firms operating in country j. The matching probabilities for other markets can be computed similarly, and are listed in the appendix.

In making their funding currency choice, firms compare the expected profits of using dollars ($\Pi^{\$}_{jt}$) or euros ($\Pi^{€}_{jt}$). The net benefit to firm i in country j of choosing dollars is

$$V^{\$}_{jt} = \Pi^{\$}_{jt} - \Pi^{€}_{jt} + \theta_{it},$$

where $\theta_{it} \sim N(0, \sigma^2_\varepsilon)$ and iid across firms. The shock θ_{it} captures a firm's idiosyncratic preferences for one currency over the other. Introducing some noise in the choice is convenient from a numerical standpoint, as it ensures that currency choices are interior.

Given that the expected payoff of seeking dollar funding is increasing in θ_{it}, we look for monotone strategies in which firms adopt the US asset so long as their idiosyncratic shock exceeds a threshold $\bar{\theta}_{jt}$. Thus, the fraction of country j trading firms using US safe assets is

$$X_{jt} \equiv \int_0^1 \mathbb{1}(\theta_{it} >= \bar{\theta}_{jt})di = 1 - \Phi\left(\frac{\bar{\theta}_{jt}}{\sigma_\varepsilon}\right),$$

where $\Phi(\cdot)$ denotes the standard normal CDF.

The equilibrium cutoff $\bar{\theta}_{jt}$ is the value of the idiosyncratic preference shock that leaves a country j trader indifferent between choosing one asset or the other. We focus on symmetric equilibria where $\theta_{jt} = \bar{\theta}$ for all $j \in [0, \mu_{rw}]$ so that the equilibrium cutoff value solves

$$\Pi^{\$}_t - \Pi^{€}_t + \bar{\theta}_t = 0. \tag{13}$$

For simplicity, we exogenously fix the cutoffs of the US and EZ firms, $\bar{\theta}_{us,t} = -\bar{\theta}_{ez,t}$, and calibrate those to match the high domestic currency bias displayed by US and EZ trading firms in the data. This exogenous currency choice is the only way in which US and EZ trading firms differ from firms in RW: otherwise, they face the same choices (e.g. destination country of trading, entry, etc.) and incentives as RW firms.

Firm Formation (Stage 1): Upon entry, firms choose the probability of importing or exporting to each potential trading partner country, and hence in equilibrium are indifferent among all possible trade routes. Finally, given all of the above choices, prospective firms must decide whether or not to pay the fixed cost $\phi P_{jt} > 0$ in order to become operational during the period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi_{jt}^{\$}, \Pi_{jt}^{€}\} - \phi P_{jt} = 0,$$

is satisfied, thus determining the equilibrium size of active trading firms in each country m_{jt}.

Government

We assume that government purchases are zero, and thus governments play a role only in the large countries $j \in \{us, ez\}$, where they issue bonds in fixed supply $\bar{B} = B^{\$} = B^{€}$ and set the level of lump-sum taxes so as to keep their stock of debt constant at $\bar{B}$:

$$\bar{B} = T_{jt} + Q_t^j \bar{B}.$$

The small rest-of-world countries $j \in [0, \mu_{rw}]$ do not issue debt and set $T_{jt} = 0$.

Equilibrium

We focus on the class of symmetric equilibria where the strategies of the ex-ante identical rest-of-world traders and households are the same. Definition 3 provides a formal definition:

Definition 3 (**Equilibrium).** *A symmetric equilibrium is a pair of bond prices* $\{Q_t^{\$}, Q_t^{€}\}$*, and a set of country specific allocations* $\{C_{jt}^{us}, C_{jt}^{ez}, C_{jt}^{rw}, B_{jt}^{\$}, B_j^{€}, \bar{\theta}_{jt}, m_{jt}\}$*, prices* $\{P_{jt}^{us}, P_{jt}^{ez}, P_{jt}^{rw}\}$*, and liquidity premia* $\{\Delta_{jt}^{\$}, \Delta_{jt}^{€}\}$ *for* $j \in \{us, ez, rw\}$ *such that*

1. *The household optimality conditions are satisfied.*

2. *The trading firms optimality conditions are satisfied.*

3. Bond markets clear; for $c \in \{\$, €\}$

$$\bar{B} = \int_{\mu_{rw}} B^c_j dj + \mu_{us} B^c_{us} + \mu_{ez} B^c_{ez}.$$

4. Real goods markets clear; for $j \in \{us, ez, rw\}$

$$\mu_j Y_{jt} = \sum_{j' \in \{us,ez,rw\}} \mu_{j'} C^j_{j',t}. \tag{14}$$

5. The mass of export/import firms is consistent with import/export allocations $C^{j'}_{jt}$

The problems of households and firms, together with some of their key optimality conditions are described above, with the full details given in the appendix. Conditions 3 and 4 are straightforward market clearing conditions, and condition 5 enforces the friction in international trade – all trade flows are intermediated through the export/import sector, and thus must be consistent with the mass of active firms (and implied matching probabilities).

4 Quantitative Results

We calibrate our model in order to examine its quantitative implications. We fix a set of parameters to standard values, then use the remaining parameters to target four steady-state moments. The model is able to exactly replicate our target moments.

Steady State

The exogenously-fixed parameters are listed in Table 2. We parameterize the two big countries symmetrically, and set $\mu_{us} = \mu_{ez} = 0.2$, consistent with the sizes of the US and the EZ in world GDP. We set the currency use in the big countries (X_{us} and X_{ez}) so that 90% of their firms use the domestic currency, to match the evidence of Gopinath (2015). One model period represents a year, hence we set $\beta = 0.96$; we also assume log preferences ($\sigma = 1$). We minimize real trading frictions by using a low value for the elasticity of the trade matching function, $\varepsilon_T = 0.01$, which ensures that firms on the less crowded side of trade markets are virtually guaranteed a match. We fix $\alpha = 0.5$, implying that importers and exporters have equal bargaining power. Finally, we set the currency mismatch cost to be quite small, representing just 1% of the transaction value ($\kappa = 0.01$).

Parameter	Concept	Value
β	Time preference	0.960
$\mu_{us} = \mu_{ez}$	Big country measure	0.200
κ	Mismatch cost	0.010
r	Funding fee	0.005
υ	Exog. velocity	8.000
X_{us}	US dollar share	0.900
X_{ez}	EZ dollar share	0.100
α	Exporters bargaining parameter	0.500
σ	Risk aversion	1.000
ε_T	Elasticity of trade matching function	0.010
σ_ε^2	Variance of idio. shock	1e-06
τ	Portfolio adj. costs	0.040

Table 2: Exogenously Fixed Parameters

To match the observed maturity and cost of a typical letter of credit contract in the data we set $\nu = 8$, implying a funding relationship lasts about 45 days, and $r = 0.005$.[16] We make the currency preference shocks very small (σ_e^2 =1e-06), ensuring the model is numerically smooth but otherwise they play a negligible quantitative role. Lastly, we parameterize the bond adjustment cost so that it has minimal effects on local dynamics: we set $\tau = 0.04$, implying that a 10% change in bond positions incurs a cost of just 20 basis points.

We calibrate the remaining parameters of our model to match a set of target steady-state moments. As foreshadowed by the analytical analysis, the quantitative model has three steady states – dollar and euro dominant ones, and a symmetric one. Since the data has been generated from a dollar-dominant world (our sample is 1984-2017), we match to the moments of the dollar-dominant steady state in our model. Panel (a) of Table 3 summarizes the target moments. They are (1) government debt of 60% of GDP, consistent with the US average since 1984; (2) rest-of-world trade share ($\frac{\text{Imports + Exports}}{\text{GDP}}$) of 55%, consistent with trade data for non-US and non-EZ countries from the World Bank since 1984; (3) RW dollar invoicing share of 80%, consistent with the evidence of Gopinath (2015) on dollar invoicing and with the evidence of BIS (2014) on the fraction of letters of credit denominated in dollars; and (4) import markups of 10%, consistent with micro-level estimates on import markups in Coşar et al. (2018).

We target these four moments with the four remaining free parameters $\{\bar{B}, a^h, \varepsilon^f, \phi\}$.

[16]Letters of credit tend to be for one to two months (BIS (2014)). Their cost includes a substantial fixed component, on average 40 basis points of the principal, plus a spread on top of the LIBOR. See the guidelines by the US Commerce Department: https://acetool.commerce.gov/cost-risk-topic/trade-financing-costs.

Concept	Target Value
Gross debt/GDP	0.60
ROW trade/GDP	0.55
ROW USD invoice shr.	0.80
Import markup	1.10

(a) Calibration Targets

Parameter	Concept	Value
$\bar{B}$	US/EZ asset supply	1.471
a_j^h	Home bias	0.718
ε^f	Funding match. elas.	0.294
ϕ	Fixed cost of entry	0.038

(b) Implied Parameter Values

Table 3: Calibration Strategy

These parameters are (1) the supply of government debt $\bar{B}$; (2) the home bias parameter in consumption preferences a_h which determines the trade share; (3) the elasticity of the funding matching function ε^f which helps determine the equilibrium level of currency coordination; and (4) the fixed cost of entry in the trading sector ϕ which helps determine export markups. We find the model can exactly match the targeted moments, with the implied parameter values given in Panel (b) of Table 3.

Table 4 summarizes several key moments for each of the three steady states in the calibrated economy. The resulting dollar-dominant steady state matches a number of (untargeted) empirical regularities. First, since the RW countries primarily use dollars for trade finance, foreign households hold a substantial portion of US safe assets. This generates a negative net foreign asset position (NFA) for the US of 42% of GDP (close to the actual US position), and a realistically large positive NFA in the rest-of-world. Since a portion of RW firms still seek euro funding, RW households also hold a non-trivial quantity of EZ assets, leading the EZ to have a negative NFA position as well, but much less so than the US.

The table also shows that the US and EZ portfolios display significant home bias, defined as $1 - \frac{\text{share of foreign bonds in HH portfolio}}{\text{share of foreign bonds in world supply}}$, but the bias is especially large for the EZ. Both the level and relative strength of the home bias are realistic. In the model, home bias is positive because each large country primarily uses its own asset for trade financing, creating high domestic demand for the home asset. US home bias is lower than EZ home bias, however, because the dollar's central role in international trade creates stronger external demand for US assets, leading RW households to concentrate their foreign investments in US assets.

Despite the fact that the US has a much less favorable NFA position than the EZ, its steady-state trade surplus is essentially the same.[17] This is a consequence of the "exorbitant

[17] The fact that the model does not generate a trade deficit for the US is simply due to the simplifying assumption that US and EZ assets are the only savings vehicles. In Appendix C, we consider an extension where the RW also issues an asset in positive net supply. This gives US households access to a high-yielding asset they can save in, leading the US to have both a trade and a NFA deficit in steady state.

Moments	USD Coord. US	EZ	RW	Symmetric US	EZ	RW	EUR Coord. US	EZ	RW
Dollar Share	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100\times(i^{\$} - i^{€})$	1.07	-	-	0.00	0.00	-	-	-1.07	-
100×Seignorage/GDP	0.88	0.23	-	0.56	0.56	-	0.23	0.88	-
100×Trade bal./GDP	0.87	0.86	-0.45	1.01	1.01	-0.52	0.86	0.87	-0.45
Gross debt/GDP	0.60	0.59	0.00	0.60	0.60	0.00	0.59	0.60	0.00
NFA/GDP	-0.42	-0.26	0.18	-0.38	-0.38	0.19	-0.26	-0.42	0.18
Home Bias	0.61	0.90	0.00	0.80	0.80	0.00	0.90	0.61	0.00
Consumption	11.278	11.275	12.387	11.250	11.250	12.402	11.275	11.278	12.387

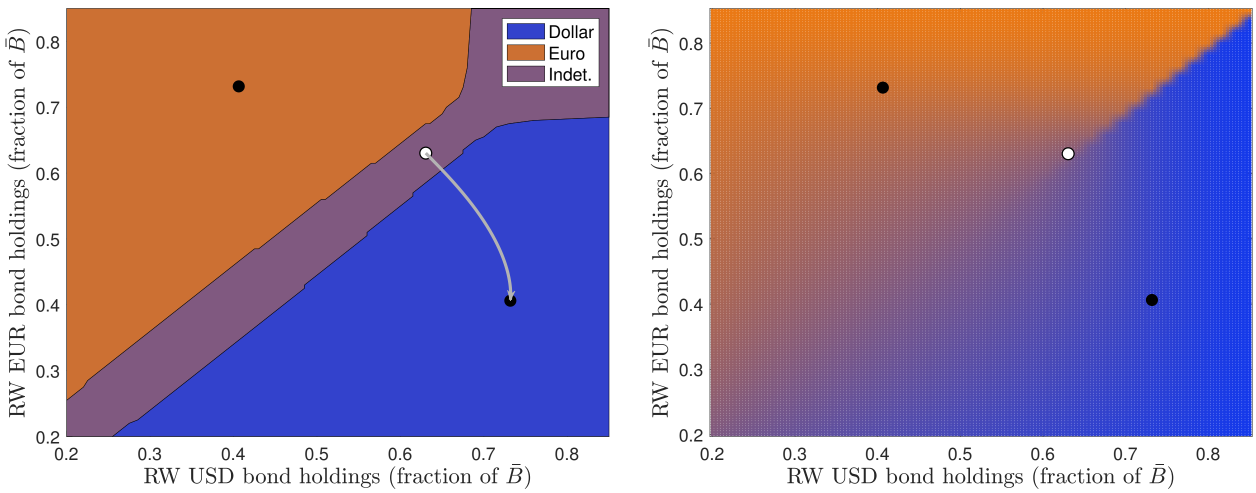
Table 4: Steady-state values for baseline model.

privilege" of the country issuing the dominant medium of exchange: the interest rate the US pays on its obligations is 1.07% lower than the corresponding EZ interest rate. This violation of interest parity earns the US a significant premium on its external position, which helps to fund its worse NFA position. As a result, the US can *indefinitely* support its more negative foreign asset position with virtually the same real trade outlay as the EZ.

The third line of the table (seignorage) provides a standard estimate of the benefit the US receives from the liquidity premia on US assets. We compute it as the additional interest outlay the US would face, if it paid an interest rate equal to the inverse of the time discount, holding everything else constant. Essentially, this is the seignorage the US earns from being the central country, and our model estimated this to be substantial – equal to 0.88% of GDP.

However, this is an *incomplete* measure of welfare, because it takes asset positions as given. A key insight of our theory is that widespread holdings of a country's assets begets key currency status, and such external demand for assets leads to negative steady-state NFA. Thus, the interest rate benefits of being dominant are at least partially offset by the need to support a more negative NFA position in perpetuity. Indeed, the last row of the table shows that steady-state US consumption is only 0.03% higher than EZ consumption. Essentially, even though the US collects significant seignorage, this is offset by its negative NFA position, leaving US households with a similar permanent consumption level to that of EZ households.

Moreover, for similar reasons, while the dollar is indeed stronger than the euro at the dollar-dominant steady state, the real exchange rate $\frac{P_{ez}}{P_{us}}$ is almost perfectly balanced and equals unity up to three decimals. This result contradicts a common view that the central country in the international monetary system must have an appreciated real exchange rate.



(a) Attraction regions, baseline

(b) Dollar usage, baseline

Figure 2: Attraction regions and asset usage in the dynamic model.

Yet, the result broadly matches the empirical evidence: in real terms, the dollar has not been consistently overvalued relative to other advanced currencies. Overall, we conclude that the benefits of currency dominance are much more nuanced than commonly perceived.

Dynamic Stability and Regions of Attraction

Next, we consider dynamics out of steady state. We are particularly interested in determining whether dominant currency steady-states are dynamically stable, and thus if a dominant-currency regime is likely to persist.

Panel (a) of the Figure plots the respective attraction regions of the models' three steady states. We compute these regions by first defining a fine grid of initial asset positions covering the axes depicted in the picture.[18] For each grid point, we then solve a perfect foresight version of the economy, attempting to shoot from the initial vector of states to a candidate steady state. For points in the blue region, we found that the only solutions that exist converge to the dollar-dominant steady state, while for points in the orange region the only feasible outcome is the euro-dominant steady state. Finally, the purple region corresponds to points where we found perfect foresight paths that arrive at both coordinated steady states.

A first observation from the Figure is that only the coordinated steady states are dynamically stable, i.e. dynamic paths that are initialized away from the symmetric steady state

[18]Note that there are four state variables: RW holding of US and EZ bonds, US holdings of US bonds, and EZ holdings of EZ bonds (with US and EZ foreign holdings determined by market clearing.) To display the Figure 2 in 2D, we initialize US and EZ portfolios shares at their symmetric steady-state level.

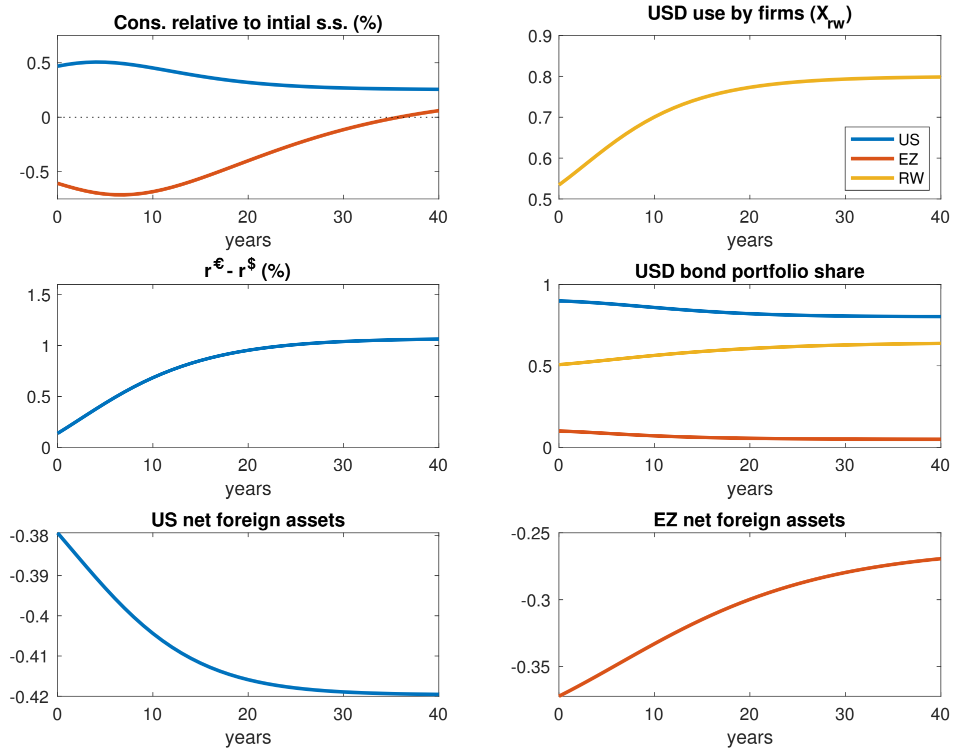


Figure 3: Transition from the symmetric steady state to dollar-dominant steady state.

never converge there.[19] Moreover, both dominant steady states are contained within large regions from which the economy uniquely converges to them. For example, whenever the RW households' initial portfolio positions are sufficiently biased towards US assets, the unique equilibrium path converges to the dollar-dominant steady state. In this respect, the model implies that currency regimes are endogenously persistent and sustainable indefinitely, so long as no large shocks push the economy out of the respective regions of attraction.

Nevertheless, we also find there is a portion of the state-space where the eventual steady-state currency is not uniquely determined by the initial asset allocations. In this region, it is possible for sunspots or short-term policy choices to have long-run effects on the currency regime. We illustrate this possibility when we consider a temporary trade war in Section 4.1.

While attraction regions tell us to where the economy *eventually* converges, these results do not speak directly to the intensity of dollar usage at any given point of the plotted state space. To address that, Panel (b) of Figure 2 presents a heat map of the contemporaneous equilibrium usage of dollars, with darker blue representing stronger dollar usage (values of $\bar{X}_t$ closer to one), orange representing string euro usage, and purple representing cases where

[19] We have also confirmed via linearization that the dominant equilibria are locally-stable, while a locally-stable equilibrium does not exist for the symmetric steady state.

	US	EZ	RW
steady-state	0.25%	0.22%	-0.12%
incl. transition	0.37%	-0.37%	-0.00%

Table 5: Gain/loss as percentage of symmetric steady state consumption.

both currencies are in relatively equal use. As one might expect, dollars tend to dominate in the bottom right corner of the Figure, in the attraction region of the dollar-dominant steady state where RW asset holdings are concentrated in US bonds, and vice versa.

To shed light on the dynamics of the model, Figure 3 plots the transition paths of several endogenous variables as the economy starts at the symmetric steady state and converges to the dollar-dominant steady state. The top right panel shows the evolutions of currency use, which starts close to equally balanced and then gradually converges to dominant dollar use over the subsequent 15-20 years. Along this transition path, the exorbitant privilege of the US gradually builds as RW's household portfolios (right column, middle plot and also the gray line in Panel (a) of Figure 2) shift towards US assets. The shift towards US assets, in turn, drives down the US net foreign asset position.

Perhaps most importantly, the top-left panel plots the paths of US and EZ consumption, showing that during this transition period US consumption is elevated for an extended period of time. This is a result of the *increasing* foreign demand for US assets and their concomitant liquidity services, which allows the US to steadily increase its borrowing from the rest of the world. Meanwhile, EZ consumption is significantly depressed, as the EZ increases savings in order to repatriate some of its assets, which are no longer in high demand externally. Thus, while consumption in the US and the EZ eventually converges to roughly the same steady-state level, the two countries' experiences are quite different during the transition.

Table 5 summarizes the welfare effects of this transition. The top line shows that, at the dollar-dominant steady state itself, the US and the EZ are similarly well off, and both are better off as compared to being at the symmetric steady state. Naturally, the US benefits from a significant seignorage it earns on the external holdings of its safe assets. But at the same time, the EZ enjoys higher consumption because of a much more favorable net foreign asset position, since at the symmetric steady state it also has a low NFA due to high external asset demand. Thus, a comparison of the eventual steady-state outcome alone suggests both US and EZ should be roughly indifferent about which country's asset is dominant.

However, while in the long-run there seems to be no harm of dollar dominance for the EZ, incorporating the transition dynamics from the symmetric steady state to the dollar-dominant steady state completely reverses this conclusion. Taking into account the transition period, in which the US is able to maintain consumption significantly above the eventual steady-state level for an extended period of time, the US permanent consumption equivalent is 0.75% higher than that of the EZ. This is more than an order of magnitude larger than the 0.03% welfare benefit implied by steady-state consumption difference alone. Our subsequent results further reinforce the conclusion that considering transition dynamics is crucial for properly assessing the welfare implications of owning the dominant currency.

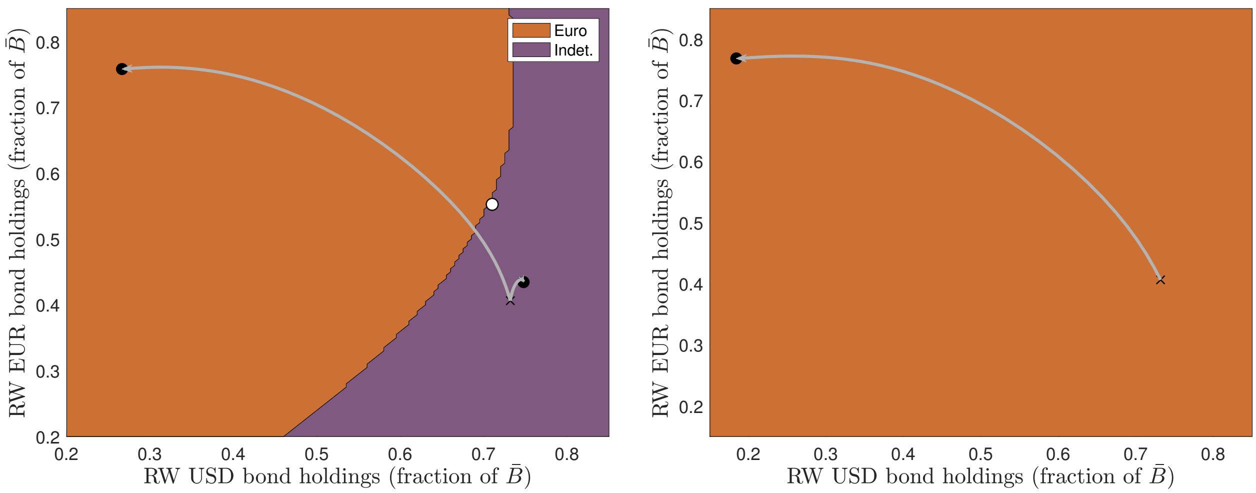
4.1 Consequences of Trade Barriers

Since our main mechanism operates through trade flows, a natural question is whether trade barriers can affect the currency regime or the benefits of issuing the dominant currency. In this section, we consider two scenarios motivated by recent events in US trade policy. In the first scenario, which we call a moderate trade war, the US permanently raises tariffs on all imports by 15% and the EZ and the RW respond in kind. In the second scenario, which we refer to as an acute trade war, we assume that tariffs rise to 30%, and we consider the effects of both a temporary and permanent imposition of these larger tariffs.

Moderate Trade War

Figure 4, Panel (a) depicts the consequences of a permanent trade war between the US and the EZ/RW with tariffs of 15%. The tariffs change the position of the steady states and also their respective attraction regions (old steady states are marked with a $\times$; the new steady states with a dot). In particular, the figure shows that the region of unique attraction to the dollar-dominant steady state is eliminated, and both the old and new dollar-dominant steady states lie within the region of equilibrium indeterminacy. Hence, even a moderate trade war, as long as it is *permanent*, potentially endangers the position of the dollar. Still not *any* permanent trade war would do so. Dominance is only threatened when the tariffs are high enough – for example 10% tariffs leave the old dollar steady state just inside a shrinking, but still existing, unique attraction region to the new dollar-dominant steady state.

The trade war weakens dollar dominance for two reasons. First, it diverts RW trade away from the US towards the EZ. Since EZ firms are far more likely to use EZ assets in their trade, RW firms become more likely to encounter euro trading partners and, hence, to prefer euros. Second, the overall world trade level falls, decreasing trade quantities relative to total asset supply. As a result, the equilibrium anchoring effects of asset availability become weaker, and multiplicity becomes more likely, both in the static sense of Definition



(a) Attraction regions, moderate trade war (b) Attraction regions, acute trade war

Figure 4: Attraction regions under trade ware scenarios.

2 and dynamically, as a smaller conjectured shift in asset holdings can switch trading firms' choices. Hence, the purple multiplicity region grows as well.

Table 6, Panel (a) summarizes the welfare implications of the moderate trade war, *assuming the dollar remains dominant.* The US is disproportionately hurt by the trade war for two reasons. First, the distortions created by the tariffs hit all of its exports, while the EZ and RW face tariffs only on their direct trade with the US (it is not an all around trade war, but one centered at the US). Second, as world trade levels fall, the size of seignorage revenue decreases as fewer firms overall require liquidity and a smaller portion of firms choose dollars. Quantitatively, however, the size of the seignorage effect is modest, with the US "privilege" falling from 1.07% in our baseline to 0.99% with a moderate trade war. Moreover, the effect of factoring-in the transition in this case is negligible, which is not surprising given the very short distance of the transition, depicted by the shorter gray line in Panel (a) of Figure 4.

Table 6, Panel (b) summarizes the welfare implications of the same trade war scenario, but *assuming the dollar loses dominance,* and the economy transitions to the euro steady state. In this case, the US is hurt by the trade war while the EZ is substantially helped. On the one hand, at the new steady state the US loses all of its seignorage, as the world currency use and the resulting exorbitant privilege shift to the EZ. On the other hand, the transition to this new steady state itself, depicted by the long gray line in Panel (a) of Figure 4, further hurts the US (by an additional permanent consumption loss of more than 1%) because along the transition path it runs significant trade surpluses to reduce its NFA position as external demand for dollars dries up. Overall, the US loses 2.13% of permanent consumption.

	US	EZ	RW
steady-state	-1.42%	-0.01%	0.13%
incl. transition	-1.34%	0.07%	0.09%

(a) Moderate trade war, USD remains dominant

	US	EZ	RW
steady-state	-1.06%	0.21%	-0.03%
incl. transition	-2.13%	0.79%	0.10%

(b) Moderate trade war, USD loses dominance

Table 6: Gain/loss as percentage of dollar-dominant steady state consumption.

Acute Trade War

Panel (b) of Figure 4 depicts the implications of a permanent 30% tariff between the US and EZ/RW, a scenario we call an "acute" trade war. In this case, the effects of the trade barriers are strong enough to eliminate both the symmetric and the dollar-dominant steady state, thereby guaranteeing a transition to the now unique euro-dominant steady state.

This is a strong implication, but a *permanent* 30% trade war is (we hope) implausible. Hence, we also consider the effects of a temporary trade war of the same magnitude. Figure 5 depicts the welfare cost for different possible durations, ranging from one to 40 years. The figure shows that for trade wars lasting under 10 years, the economy cannot transition to the euro steady state absent other shocks. Within this range, longer trade wars are worse, but not discretely so, and showcases the general stability of the model – even 10 years of a very acute trade war is not enough to shift the currency regime.

For trade wars lasting more than 10 years, however, transition to the euro-dominant steady state becomes possible as in this case we enter the indeterminacy region. If a transition occurs, the US is discretely worse off, and suffers an additional loss of 1% of permanent consumption. If the trade war lasts longer than 28 years, the *unique* outcome is a transition to the euro-dominant steady state, despite the fact that tariffs return to zero in the long-run. Thus, certain temporary shocks, could have *permanent* effects.

4.2 Path Dependence: Historical context of dollar dominance

What can our model tells us about the historical emergence and subsequent durability of the US dollar dominance? According to our calibration, both initial conditions and path dependence play important roles.

First, our calibration exercise explains why a multi-polar world with equal use of dollars and euros is an unlikely outcome: the empirically-relevant parameters imply that the multi-polar steady state is repulsive, and not attractive. This is a result of the model, not an

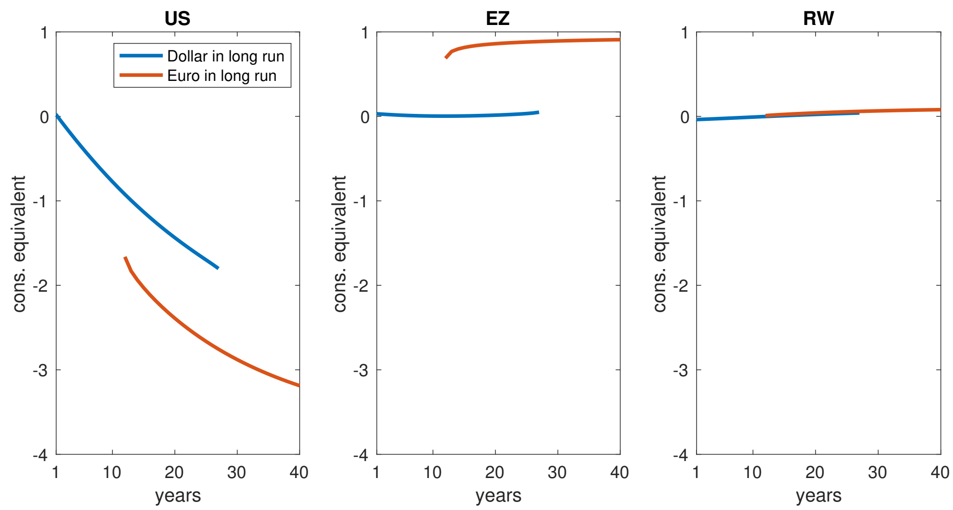


Figure 5: Welfare costs of an acute trade war as a function of duration.

assumption, and can be understood using Proposition 3. Essentially, the model implies that the need for liquid assets as medium of exchange is low relative to the total supply of each safe asset $\bar{B}$, making the κ-threshold above which the coordinated steady states are the stable ones relatively low. The key calibration target moments that inform this are (i) gross debt-to-GDP and (ii) RW trade-to-GDP, respectively 60% and 55% in the data. Through additional experimentation, we have also confirmed that decreasing the debt-to-GDP target or increasing the trade-to-GDP target sufficiently would indeed result in a situation where only the symmetric steady state is stable. Thus, through the lens of our model, we live in a dominant-currency world because there exists (at least) one safe asset with sufficient supply, to be able to intermediate the bulk of trade.

But, if the supply of both dollar and euro safe assets are large enough for either one to play this role, why do we find ourselves near the dollar steady state? The answer lies with the endogenous path dependence of the model, and can be understood in the context of the global macroeconomic history over the past century. Before World War I, the international monetary system was dominated by the British pound and pound-denominated safe assets (Chinn and Frankel (2007)). By the 1870s, the US economy was actually already twice as big as the UK economy. However, prior to the Federal Reserve Act of 1913, the US had a closed capital account and banks were generally prohibited from direct cross-border banking operations. Thus, while the US was a large economy both in terms of GDP and trade, virtually no trade finance was offered in USD, and US exporters and importers had to operate through London, using pounds (Eichengreen and Flandreau (2012), henceforth EF).

As discussed in detail by EF, one of the tasks assigned to the Federal Reserve upon its creation in 1913 was to jump-start a dollar-based trade finance market. This opened the door for the USD to play an international role, and as shown by EF the dollar quickly gained a foothold and came to equal the pound in overall international use in the 1920s and 30s.

Importantly, the dollar did not usurp the pound in the interwar period, but the two existed *as equals*. This period contrasts with our recent experience of a single dominant international currency, and is a puzzle to standard currency competition models based on strong strategic complementarities. The observation aligns well with our model, however, in which the complementarity between traders interacts with households' portfolio holdings. An important consideration, from the view point of our mechanism, is that the interwar period was characterized by strong capital flow restrictions around the world, and thus relatively low cross-border holdings of assets (Ghosh and Qureshi (2016)). Comprehensive data on total cross-border holdings from the early 20th century is not available, but using official foreign reserves as proxy for total holdings suggests that foreign asset holdings grew an order of magnitude from 1928 to 2016. In 1928, total foreign reserves amounted to only 1.2% of world GDP, while in 2016 that number reached 13%. By comparison, world trade-to-GDP in 1928, was already 70% of its 2016 level.[20]

Through the lens of our model, restrictions on foreign asset supply during the 1920's and 30s actually explain why we observed of a multipolar system over that period: When the availability of safe assets for the rest-of-world traders is low, the only stable steady state is the symmetric one, where both safe assets play an equally important role in the international monetary system. This characterizes the interwar experience very well – a situation of low availability of cross-border financing, and an equal use of dollars and pounds internationally.

After World War II the world did not return to a multipolar system, but instead the dollar quickly gained dominance (Eichengreen et al. (2016)). This development is also predicted by our model, given the particular structure of the post-war global economy. The Bretton Woods agreement codified the US as the central country of the eponymous post-war exchange rate system, and as a result was also the *only* country with virtually no capital controls and thus free international access to its (abundant) supply of safe assets.[21] Second, the Marshall plan effectively anchored a large portion of the post-war reconstruction spending to international trade sourced by the US and financed by US dollars. Hence, the abundant access to dollar safe assets opened the door for the dollar-dominant steady state to be a stable outcome, and

[20]Data from IMF COFER database, Eichengreen and Flandreau (2012) and Jordà et al. (2017).

[21]See Ghosh-Qureshi(2016) for a detailed description of the evidence on capital controls. They also note that the US Treasury secretary at the time was well aware of the benefit of an open US capital account, having stated in his 1948 testimony to Congress that controlling capital inflows would require exchange controls that "would do maximum violence to our position as a world financial center."

the Marshall plan actually pushed the world economy there.

Once the Bretton-Woods agreement fell apart in 1971, and capital controls were mostly lifted by the advanced economies in the 1970s, international traders could freely access alternative currencies and associated safe assets. However, our model also explains why the availability of other options does not guarantee they will be adopted. Recall that our parameter estimates given the data of the last few decades imply that the dollar-dominant steady state is dynamically stable, hence leaving its vicinity (where the world found itself at the end of the Bretton-Woods era) would require a sufficiently large shock.

One possible reason that such a change has not occurred is that, until recently, the world simply lacked another economy with sufficient size, capital mobility and ability to generate safe assets to take over the role of the dollar. With the introduction of the euro, however, many policy makers and academics speculated that the euro could "unseat" the dollar as the sole international currency (e.g. Chinn and Frankel (2007)). Yet, our experience since shows that the introduction of the euro alone was not sufficient to drive such a transition.

To understand why the creation of the Eurozone was not enough to shift the currency regime, we use our quantitative model to simulate two alternative scenarios around the introduction of the euro. In both cases we consider the starting point to be a situation where the supply of the alternative safe asset is 60% of the size of the dollar safe asset supply: $\bar{B}^{€} = 0.6\bar{B}^{\$}$. Essentially, we assume that before the Eurozone was created the only European safe assets were generated by the two central countries Germany and France, and was thus in much smaller supply. We then model the introduction of the Eurozone as a gradual increase in $\bar{B}^{€}$ over time.[22]

Initially, when the supply of EZ assets is only 60% of the supply of US assets, the model has a unique, dollar-dominant steady state. We initialize the economy at this steady state, and consider the resulting transition path as the supply of EZ assets grows.

The results of this excise are displayed Figure 6. In the first scenario (blue line), we assume that the total supply of EZ safe assets converges to the same level as that of the US asset over a 10 year period (i.e. $\bar{B}^{€} = \bar{B}^{\$}$). We find that the unique equilibrium path transitioning from the initial steady state still converges to the new dollar steady state. Along the transition, dollar use, the interest rate differential, and the US net foreign asset position are essentially unchanged, rather consistent with the continued dominance of the dollar since 2000. Thus, the creation of the Euro by itself is not enough to shift the long-run

[22] The gradual increase is motivated by the fact that interest rates on euro-area sovereigns did not collapse to the interest rate of the German Bund at the moment of the introduction of the euro, but took several years to converge. This suggests markets only gradually accepted euro bonds as a homogeneous safe asset.

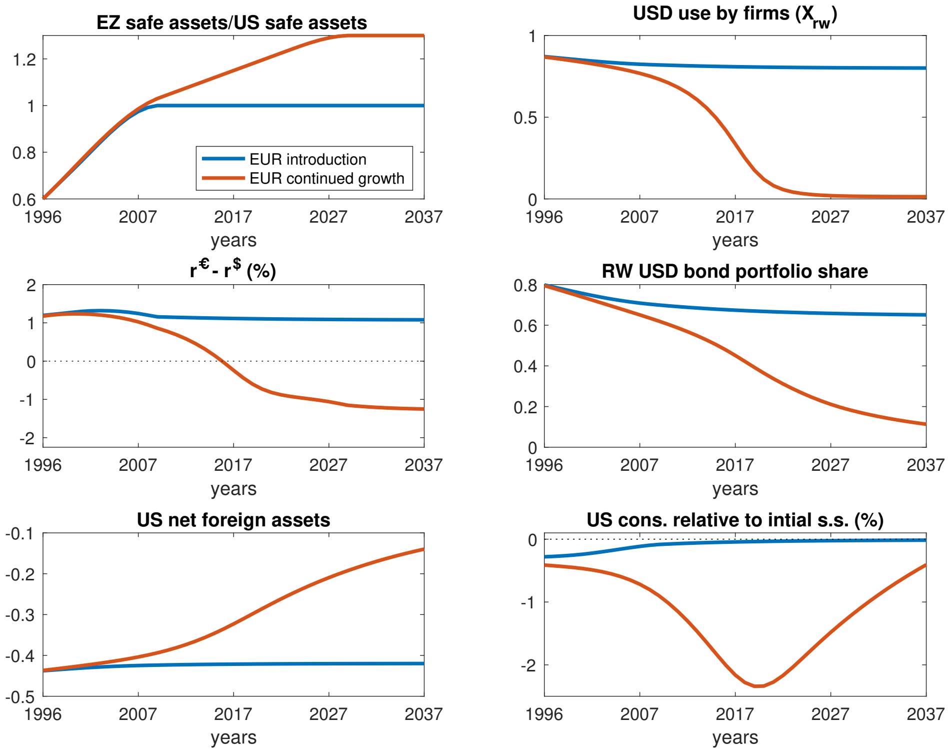


Figure 6: Introduction of euro

equilibrium, as was the case in reality, and the reason is the endogenous path dependence of the model. The introduction of the euro does create a second stable steady state, where the euro is dominant, however the world starts near the vicinity of the dollar steady state, and hence converges towards it.

In the second scenario (red line), we consider the counter-factual possibility that the EZ grew larger than the US over time. For example, it was initially anticipated that eventually Sweden, the UK and new eastern European members would eventually join the monetary union. For our particular counter-factual, we looked for the threshold value for $\bar{B}^{€}$ such that the EZ asset becomes so large and ubiquitous, that the unique stable steady state is the euro-dominant one. It turned out that for this to happen, the supply of EZ safe assets must exceed the US one by at least 30% in the long-run. We show this case with the red line in Figure 6. This scenario would have been quite plausible if the UK had indeed joined the EZ.

Along the path to the new, uniquely-stable euro-dominant steady state, the US net foreign asset position shrinks towards zero, and the US's exorbitant privilege benefits disappears as the US asset leaves international markets and return to US portfolios. The figure shows that anticipation effects are important, as much of the shift in portfolios and usage occurs before the end of the rise in EZ asset supply. The welfare impact in this case is also considerable:

the US loses 0.77%, while the EZ gains 0.66% in permanent consumption.

Thus, we conclude that our model can accounts for the evolution of the international monetary system over the last century, and highlights in particular the important role played by inertia and the changing availability of safe assets internationally. In the current situation, merely introducing a rival safe asset that is equivalent in fundamentals to the dollar is not enough to shift the currency regime – for that to happen, the alternative safe asset needs to offer a substantially bigger base than the dollar. Yet the 30% threshold we found is not necessarily unsurmountable, and could potentially be achieved by either the EZ or the Chinese renminbi in the future. However, the model does caution that the Chinese renminbi would not play a significant role until Chinese capital markets are sufficiently liberalized.

5 Conclusions

This paper presents a new theory describing the emergence of dominant international currencies. Our model is quantitatively realistic and tractable enough to use for standard macroeconomic analysis. Throughout, we have abstracted from risk: both the potential for short run shocks that perturb the economy around a given steady state and possible longer-run stochastic transitions between currency regimes. Both of these extensions are rather straightforward: Business-cycle analysis can be conducted using policy functions approximated locally around a given steady state, or via global solution techniques, such as the ones suggested by Richter et al. (2013). Such extensions could help the model address the observation of Gourinchas et al. (2017) that an "exorbitant duty" coincides with the privilege of being the dominant currency. We leave exploration of this issue to future work.

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Appendix

A Proofs

Proof of Lemma 1. We first prove the Lemma when country-j bond holdings are positive, then consider cases where holdings of one of the bonds might are zero.

For the case of $B^{\$} > 0$ and $B^{€} > 0$, we look for the value of average dollar use in country j, X_j, such that all country j traders' optimality conditions are satisfied. Each trader faces the binary choice of either seeking 1 unit of dollars or 1 unit of euros. So If $X_j^{(i)}$ is the dollar use of trader i in country j, then $X_j^{(i)} \in \{0, 1\}$. Naturally, the average dollar use in country j is bounded between 0 and 1: $X_j = \int X_j^{(i)} di \in [0, 1]$.

Moreover, since all traders are identical, $X_j \in (0, 1)$, only if all traders are indifferent between seeking dollar and euro collateral. This is the case if the net payoff of using dollars is equal to zero. From equation (2):

$$V^{\$}(X_j) = \frac{B_j^{\$}}{B_j^{\$} + X_j}\left[\pi - r - \kappa(1 - \bar{X})\right] - \frac{B_j^{€}}{B_j^{€} + 1 - X_j}\left[\pi - r - \kappa\bar{X}\right] = 0.$$

The solution of this equation is given by:

$$X_j^* \equiv \frac{B_j^{\$}\left[\pi - r - \kappa\left(B_j^{€} + 1 - \bar{X}(2B_j^{€} + 1)\right)\right]}{\left(B_j^{\$} + B_j^{€}\right)(\pi - r) + \kappa\left(\bar{X}(B_j^{\$} - B_j^{€}) - B_j^{\$}\right)}. \tag{15}$$

Since $X_j \in [0, 1]$, equation (15) describes the optimal use of dollar collateral in country j only if $X_j^* \in [0, 1]$.

For cases where X_j^* is outside of $[0, 1]$, notice that

$$\frac{\partial V^{\$}(X_j)}{\partial X_j} = -\frac{B_j^{\$}}{(B_j^{\$} + X_j)^2}\left[\pi - r - \kappa(1 - \bar{X})\right] - \frac{B_j^{€}}{(B_j^{€} + 1 - X_j)^2}\left[\pi - r - \kappa\bar{X}\right] < 0.$$

It follows that if $X_j^* > 1$ then $V^{\$}(1) > 0$, and the optimal currency choice for traders in country j is $X_j = 1$ – i.e. dollars strictly dominate euros. A similar analysis establishes that whenever $X_j^* < 0$, the optimal choice $X_j = 0$.

Thus, we conclude that whenever bond holdings are strictly positive the optimal currency choice by country j traders satisfies

$$X_j = \max\{\min\{X_j^*, 0\}, 1\}.$$

Suppose now that $B^{€} = 0$. Note that

$$V^{\$}(X_j) = \frac{B_j^{\$}}{B_j^{\$} + X_j} \left[\pi - r - \kappa(1 - \bar{X})\right] > 0$$

so $X_j = 1$ is optimal. At the same time, substituting in $B^{€} = 0$ into the expression for X_j^* we obtain $X_j^* = 1$. Thus, whenever $B^{€} = 0 \Rightarrow X_j = X_j^*$. Similar arguments can show that whenever $B^{\$} = 0$, then the optimal $X_j = X_j^* = 0$.

Thus, we conclude that for any household bond holdings $B^{\$}$ and $B^{€}$, and given rest-of-world currency use $\bar{X}$, the optimal currency use in country j is given by

$$X_j = \max\{\min\{X_j^*, 0\}, 1\}.$$

■

Proof of Lemma 2. Accounting explicitly for the inequality constraint in bonds, the steady-state Euler equations (5) for dollars can be written

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta_j^{\$}} + \lambda_j^{\$} = \frac{1}{Q^{\$} - \Delta_{us}^{\$}} + \lambda_{us}^{\$} = \frac{1}{Q^{\$} - \Delta_{eu}^{\$}} + \lambda_{eu}^{\$}, \tag{16}$$

where the weakly positive λ's are appropriately-scaled Lagrange multipliers and the complementarity slackness conditions

$$\lambda_j B_j^{\$} = \lambda_{us} B_{us}^{\$} = \lambda_{eu} B_{eu}^{\$} = 0 \tag{17}$$

must hold for all countries.

We being by proving the following Lemma:

Lemma 3. *$B_j^{\$} = 0$ if and only if $X_j = 0$. Similarly, $B_j^{€} = 0$ if and only if $X_j = 1$.*

We prove the statement for dollar bond holdings, the statement for Euro holdings follows by a parallel argument.

If: If $X_j = X_{eu} = 0$ the premia $\Delta_j^{\$} = \Delta_{eu}^{\$} = 0$ by equation (3), while $\Delta_{us}^{\$} > 0$. Hence equation (16) reduces to

$$\frac{1}{Q^{\$}} + \lambda_j^{\$} = \frac{1}{Q^{\$}} + \lambda_{eu}^{\$} = \frac{1}{Q^{\$} - \Delta_{us}^{\$}} + \lambda_{us}^{\$}. \tag{18}$$

But this equation shows that $\lambda_j > \lambda_{us}$ and $\lambda_{eu} > \lambda_{us}$. Since $\lambda_{us} \geq 0$, we know that $\lambda_j > 0$ and $\lambda_{eu} > 0$ and, from complementary slackness, that $B_j^{\$} = B_{eu}^{\$} = 0$.

Only If: To find a contradiction, suppose that $X_j > 0$ but $B_j^{\$} = 0$. Since $B_j^{\$} = 0$, there exists some other country j' (it could be the US or it could be another small country) which holds positive bonds and pays premium $\Delta^{\$}_{j'} < r$. In this case, equation (16) implies

$$\frac{1}{Q^{\$} - \Delta^{\$}_{j'}} = \frac{1}{Q^{\$} - r} + \lambda^{\$}_{j}. \tag{19}$$

which cannot be true since $\lambda^{\$}_{j} \geq 0$. Hence, $B^{\$}_{j} > 0$. And the Lemma is proved.

An implication of the proof above is that $B^{\$}_{eu} = B^{€}_{us} = 0$. We can now use the equality of the remaining premia along with market clearing conditions to compute the expressions in the text. For example, $\Delta^{\$}_{us} = \Delta^{\$}_{j}$ implies that

$$\frac{X_j}{B^{\$}_{j} + X_j} = \frac{X_{us}}{B^{\$}_{us} + X_{us}},$$

which simplifies to

$$B^{\$}_{us} = B^{\$}_{j} \frac{X_{us}}{X_j}.$$

Using this expression and the fact that $B_{eu} = 0$, $B^{\$}_{us}$ and $B^{\$}_{eu}$ can be eliminated in the market clearing condition for dollar bonds.

$$\bar{B} = \int_{\mu_{rw}} \frac{B^{\$}_{us}}{X_{us}} X_j dj + \mu_{us} B^{\$}_{us} \tag{20}$$

$$\bar{B} = \frac{B^{\$}_{j}}{X_j} \int_{\mu_{rw}} X_j dj + \mu_{us} \frac{B^{\$}_{j}}{X_j} X_{us}. \tag{21}$$

Solving expression (21) for $B^{\$}_{j}$ gives equation (6) in the Lemma 2. The same steps for Euro bonds imply equation (7).

■

Proof of Proposition 1. To show that the dollar-dominant steady state exists, conjecture that the RW traders all use dollars and thus $X = 1$. Using Lemma 2, the optimal bond holdings of the RW households are then

$$B^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}}$$
$$B^{€} = 0.$$

Plugging those expression into the relative payoff of seeking dollar vs euro funding for a RW trader ($V^{\$}$), we have:

$$V^{\$} = \frac{\bar{B}}{\bar{B} + \mu_{rw} + \mu_{us}} \left(\pi - r - \kappa(1 - \mu_{rw} - \mu_{us})\right) > 0.$$

Thus, using dollars is strictly preferred by any given RW trader, and the dollar dominant steady state where $X = 1$ is indeed sustained. Conjecturing $X = 0$, instead, and following a similar argument shows that the euro-dominant steady state $X = 0$ also exists.

Lastly, we look for interior equilibria where $X \in (0, 1)$. In that, case the optimal bond holdings for the RW households are given by:

$$B^{\$} = \bar{B}\frac{X}{\mu_{rw}X + \mu_{us}}$$
$$B^{€} = \bar{B}\frac{1 - X}{\mu_{rw}(1 - X) + \mu_{ez}}$$

Substituting in those expressions for bond holdings in the value of seeking dollar collateral relative to euro collateral for a RW trader, we have

$$V^{\$}(X) = \frac{\bar{B}}{\bar{B} + \mu_{rw}X + \mu_{us}}\left[\pi - r - \kappa(1 - X\mu_{rw} - \mu_{us})\right] - \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - X) + \mu_{ez}}\left[\pi - r - \kappa(X\mu_{rw} + \mu_{us})\right]. \tag{22}$$

Any interior equilibrium must satisfy $V(X) = 0$ – these are the points at which the traders are indifferent between seeking dollar and euro financing. To find the zeros of $V^{\$}(X)$, we set (22) equal to 0, and multiply through with $(\bar{B} + \mu_{rw}X + \mu_{us})(\bar{B} + \mu_{rw}(1 - X) + \mu_{ez})$. Then further dividing by $\bar{B}$, gives us the condition

$$(\bar{B}+\mu_{rw}(1-X)+\mu_{ez})\left[\pi - r - \kappa(1 - X\mu_{rw} - \mu_{us})\right] - (\bar{B}+\mu_{rw}X+\mu_{us})\left[\pi - r - \kappa(X\mu_{rw} + \mu_{us})\right] = 0$$

Using the fact that $\mu_{us} = \mu_{ez}$ and $\mu_{us} + \mu_{eu} + \mu_{rw} = 1$, this equation simplifies to

$$\mu_{rw}(\kappa(\bar{B} + 1) - \pi - r)(2X - 1) = 0.$$

This linear equation has the unique solution $X = \frac{1}{2}$ when $\kappa \neq \frac{\pi - r}{\bar{B}+1}$, and admits any $X \in [0, 1]$ as a solution in the knife edge case $\kappa = \frac{\pi - r}{\bar{B}+1}$. Thus, for any $\kappa \geq 0$ that is different from $\frac{\pi - r}{\bar{B}+1}$ there are three steady states, $X \in \{0, \frac{1}{2}, 1\}$, and when $\kappa = \frac{\pi - r}{\bar{B}+1}$ there is a continuum of steady states $X \in [0, 1]$. ■

Proof of Proposition 2. We prove the two parts of the proposition in order.

Uniqueness:

We first show uniqueness if one bond holdings of one of the assets are zero. If $B^{€} = 0$ then

$$V^{\$}(X) = \frac{B^{\$}}{B^{\$} + X}\left(\pi - r - \kappa(1 - X\mu_{rw} - \mu_{us})\right) > 0$$

since $\kappa < \pi$. Thus, the only quasi-equilibrium in currency markets is $X = 1$. Similarly, if $B^{\$} = 0$, the only quasi-equilibrium is $X = 0$.

We now aim to show that, when $B^{\$} > 0$ and $B^{€} > 0$, the net payoff of using dollars crosses zero exactly once. Using $\mu_{us} = \mu_{eu} = \frac{1-\mu_{rw}}{2}$, equation (2) can evaluated:

$$V^{\$}(1) = -\frac{\pi - r}{B^{\$}+1} + \kappa\frac{\mu_{rw}(B^{\$}+\frac{1}{2})+\frac{1}{2}}{B^{\$}+1}$$

This is strictly negative (and thus $X = 1$ is not a quasi-equilibrium) if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{\$}+\frac{1}{2})+\frac{1}{2}}.$$

A similar argument implies that $V^{\$}(0) > 0$ if and only if

$$\kappa < \frac{\pi - r}{\mu_{rw}(B^{€}+\frac{1}{2})+\frac{1}{2}}.$$

Hence, under the condition stated in the Proposition, at most one of the two fully-coordinated quasi-equilibria, $X = 0$ or $X = 1$, can exist. Below we consider first the case of $\kappa < \frac{\pi-r}{\mu_{rw}(\max\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}$, then the case of $\kappa \in \left[\frac{\pi-r}{\mu_{rw}(\min\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}, \frac{\pi-r}{\mu_{rw}(\max\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}\right)$.

If $\kappa < \frac{\pi-r}{\mu_{rw}(\min\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}$, the above results on $V^{\$}(1)$ and $V^{\$}(0)$ show that any quasi-equilibrium must be interior. Evaluating equation (2) gives:

$$V^{\$}(X) = \frac{B^{\$}}{B^{\$}+X}\left[\pi - r - \kappa(1 - X\mu_{rw} - \mu_{us})\right] - \frac{B^{€}}{B^{€}+(1-X)}\left[\pi - r - \kappa(X\mu_{rw} + \mu_{ez})\right].$$

Setting the above expression equal to zero and multiplying by $\frac{1}{\kappa}(B^{\$}+X)(B^{€}+(1-X))$ results in a quadratic equation in X. Simplifying further, and dividing through by κ, allows us to express the resulting quadratic polynomial as $P(X)$:

$$\begin{aligned}P(X) = (B^{€} - B^{\$})\mu_{rw}X^2 + \left(B^{\$}\left(\frac{1}{2}+\frac{3}{2}\mu_{rw} - 2\frac{\pi-r}{\kappa}\right) + B^{€}\left(\frac{1}{2} - \frac{\mu_{rw}}{2} + 2B^{\$}\mu_{rw} - \frac{\pi-r}{\kappa}\right)\right)X \\ + B^{\$}\left(\frac{\pi-r}{\kappa} - \left(\frac{1}{2}+\mu_{rw}(B^{€}+1)\right)\right)\end{aligned}$$

Since $P(X)$ has different signs at $P(0)$ and $P(1)$ and is also quadratic, it can only have a single crossing in the range $X \in (0,1)$ and thus there is a unique quasi-equilibrium.

In this case, let $X^* \in (0,1)$ be the unique quasi-equilibrium value that satisfied $P(X^*) = 0$. To see that $X^* > \frac{B^{\$}}{B^{\$}+B^{€}}$, and hence greater than 1/2 whenever $B^{\$} > B^{€}$, evaluate

$$P(X = \frac{B^{\$}}{B^{\$}+B^{€}}) = \frac{B^{\$}B^{€}\mu_{rw}\left(B^{\$}(1+B^{\$}) - B^{€}(1+B^{€})\right)}{(B^{€}+B^{\$})^2} > 0.$$

Hence the zero of the quadratic polynomial $P(X)$ must be bigger than $\frac{B^{\$}}{B^{\$}+B^{€}}$ (recall that $P(1) < 0$).

Now consider the case of $\kappa \in \left[\frac{\pi-r}{\mu_{rw}(\min\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}, \frac{\pi-r}{\mu_{rw}(\max\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}\right)$. We show that there are no interior quasi-equilibria, and thus the single existing coordinated quasi-equilibrium is also the only quasi-equilibrium. We will explicitly prove the case where $B^{\$} > B^{€}$, with the mirror case being a perfectly symmetric argument.

When $B^{\$} > B^{€}$, the coordinated quasi-equilibrium that exists is $X = 1$. Thus in this case both $P(0) > 0$. and $P(1) > 0$. However, since $B^{\$} > B^{€}$, $P(X)$ is a concave polynomial and hence there must be no zero-crossing for $X \in (0, 1)$, and hence there is no interior quasi-equilibrium. Moreover, the coordinated equilibrium clearly satisfies $X > \frac{1}{2}$.

Reinforcing effect of κ:

We now want to show that when the quasi-equilibrium is unique and $B^{\$} > B^{€} > 0$, increasing κ strengthens coordination on the dollar – i.e. $X(B^{\$}, B^{€})$ is increasing in κ.

For the case of $\kappa < \frac{\pi-r}{\mu_{rw}(\min\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}$, observe that

$$\frac{\partial P(X)}{\partial \kappa} = \frac{(\pi-r)((B^{€}+B^{\$})X - B^{\$})}{\kappa^2} > 0 \text{ if } X > \frac{B^{\$}}{B^{\$}+B^{€}}$$

But since zero-crossing value us indeed such that $X^* > \frac{B^{\$}}{B^{\$}+B^{€}}$, the polynomial is increasing in κ: $\frac{\partial P(X^*)}{\partial \kappa} > 0$. Therefore, the value of X at which the polynomial crosses zero is also increasing in κ, and hence we have shown

$$\frac{\partial X(B^{\$}, B^{€})}{\partial \kappa} > 0.$$

Finally, when $\kappa \in \left[\frac{\pi-r}{\mu_{rw}(\min\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}, \frac{\pi-r}{\mu_{rw}(\max\{B^{\$},B^{€}\}+\frac{1}{2})+\frac{1}{2}}\right)$, the unique quasi-equilibrium is perfectly coordinated and the derivative with respect to κ is exactly zero.

■

Proof of Proposition 3. To prove local stability of a given steady state, we need to show that the best-response functions define a contraction in the neighborhood of that steady state. Define the vector of best response functions of trading firms and households in country j, given the actions of all other firms, X, and households in the rest of the world $B^{\$}$ and $B^{€}$:

$$\varphi_X(X, B^{\$}, B^{€}) = \frac{B^{\$}(\pi - \kappa(B^{€}+1) + \kappa(\mu_{rw}X + \mu_{us})(2B^{€}+1))}{(B^{\$}+B^{€})\pi + \kappa((\mu_{rw}X + \mu_{us})(B^{\$}-B^{€}) - B^{\$})}$$
$$\varphi_{B^{\$}}(X, B^{\$}, B^{€}) = \bar{B}\frac{X}{\mu_{rw}X + \mu_{us}}$$
$$\varphi_{B^{€}}(X, B^{\$}, B^{€}) = \bar{B}\frac{1-X}{\mu_{rw}(1-X) + \mu_{ez}}$$

Stacking these in the vector $\Phi \equiv [\varphi_X, \varphi_{B^{\$}}, \varphi_{B^{€}}]$, we want to show that Φ is a local contraction map, which is the case whenever the eigenvalues of the Jacobian $\nabla\Phi$ lie inside the unit circle.

The Jacobian has the form

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\varphi_X}{\partial X} & \frac{\partial\varphi_X}{\partial B^{\$}} & \frac{\partial\varphi_X}{\partial B^{€}} \\ \frac{\partial\varphi_{B^{\$}}}{\partial X} & 0 & 0 \\ \frac{\partial\varphi_{B^{€}}}{\partial X} & 0 & 0 \end{bmatrix}$$

hence its eigenvalues are given by the roots of the characteristic polynomial

$$\lambda\left(\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - \frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} - \frac{\partial\varphi_X}{\partial B^{€}}\frac{\partial\varphi_{B^{€}}}{\partial X}\right) = 0.$$

Clearly, one of the solutions is $\lambda = 0$, so we just need to ensure that the roots of the quadratic expression in the parenthesis are inside the unit circle We proceed to check this condition for each steady state.

Case I: Symmetric Steady State

At the symmetric steady state we have that $\frac{\partial\varphi_X}{\partial B^{\$}} = -\frac{\partial\varphi_X}{\partial B^{€}}$ and $\frac{\partial\varphi_{B^{\$}}}{\partial X} = -\frac{\partial\varphi_{B^{€}}}{\partial X}$. Hence, the relevant condition for the eigenvalues reduces to

$$\lambda^2 - \lambda\frac{\partial\varphi_X}{\partial X} - 2\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} = 0$$

with roots

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial X} \pm \sqrt{\left(\frac{\partial\varphi_X}{\partial X}\right)^2 + 8\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X}}\right).$$

At the symmetric steady state.

$$\frac{\partial\varphi_X}{\partial X} = \frac{(1+2\bar{B})\kappa\mu_{rw}}{2\pi - \kappa} > 0$$

since $\kappa < \pi$. Hence, the bigger root (in absolute value) is

$$\lambda^* = \frac{1}{2}\left(\frac{\partial\varphi_X}{\partial X} + \sqrt{\left(\frac{\partial\varphi_X}{\partial X}\right)^2 + 8\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X}}\right).$$

Lastly, since we also have that

$$\frac{\partial\frac{\partial\varphi_{B^{\$}}}{\partial X}}{\partial\kappa} = \frac{\partial\frac{\partial\varphi_X}{\partial B^{\$}}}{\partial\kappa} = 0,$$

the root is growing in κ. The threshold $\bar{\kappa}$ that ensures the root is within the unit circle solves $\lambda^* = 1$, which after some re-arranging results in:

$$1 - \frac{\partial\varphi_X}{\partial X} - 2\frac{\partial\varphi_X}{\partial B^{\$}}\frac{\partial\varphi_{B^{\$}}}{\partial X} = 0.$$

Solving for the threshold κ, we obtain

$$\bar{\kappa} = \frac{\pi - r}{\bar{B} + 1}$$

Hence, in the neighborhood of the symmetric steady state, the roots of the characteristic polynomial are inside the unit circle so long as $\kappa < \bar{\kappa}$.

Case II: Dollar-dominant Steady State

At the dollar dominant steady state $\bar{X} = 1$ and

$$\frac{\partial \varphi_X}{\partial X} = \frac{\partial \varphi_X}{\partial B^{\$}} = 0.$$

Hence the roots λ are given by

$$\lambda^2 = \frac{\partial \varphi_{B^{€}}}{\partial X} \frac{\partial \varphi_X}{\partial B^{€}}$$

where

$$\frac{\partial \varphi_{B^{€}}}{\partial X} \frac{\partial \varphi_X}{\partial B^{€}} = \frac{(1 + \mu_{rw})(\pi - r) - \kappa(\frac{1}{2} + \mu_{rw}(1 + 2\bar{B} + \frac{\mu_{rw}}{2}))}{(1 - \mu_{rw})(\pi - r - \frac{\kappa}{2}(1 - \mu_{rw}))}.$$

If $\kappa < \frac{(\pi - r)(1+\mu_{rw})}{\frac{1}{2} + \mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ then the above expression is positive. In that case, $\lambda < 1$ if

$$\kappa > \frac{\pi - r}{\bar{B} + 1}$$

which is equal to $\bar{\kappa}$ when $X^* = \frac{1}{2}$.

On the other hand, if $\kappa > \frac{(\pi - r)(1+\mu_{rw})}{\frac{1}{2} + \mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ the best response function φ_X hits is upper bound of 1. In particular, in that case $\frac{\partial \varphi_X}{\partial B^{€}} > 0$ in the neighborhood of the dollar-dominant steady state. This implies that starting at the dollar-dominant steady state, a small increase in $B^{€}$ will increase φ_X even further, going over 1. However, $X = 1$ is the upperbar, and enforcing this, means that for $\kappa > \frac{(\pi - r)(1+\mu_{rw})}{\frac{1}{2} + \mu_{rw}(1+2\bar{B}+\frac{\mu_{rw}}{2})}$ effectively $\frac{\partial \varphi_X}{\partial B^{€}} = 0$ and thus $\lambda^2 = 0$. Thus, all eigenvalues of $\nabla\Phi$ are zero, and the system is especially (locally) stable.

Thus, the dollar-dominant steady state is stable for any $\kappa > \bar{\kappa} = \frac{\pi - r}{\bar{B}+1}$.

Case III: Euro-dominant Steady State

Can be proven with identical steps to Case II.

■

Appendix for Online Publication

B Model Details

B.1 Households

For $j \in \{us, ez\}$, foreign imports consist of the good of the other big country and an aggregate of rest-of-the world goods. Hence, the big country consumption aggregator is

$$C_{jt} = (C_{jt}^{j})^{a_h} \left((C_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (C_{jt}^{rw})^{(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}})} \right)^{1-a_h}, \tag{23}$$

where j' is the complement of j and $C_{jt}^{j'}$ is the consumption in country j of the good of country j', and a_h controls the degree of home bias in consumption. Rest-of-world consumption goods are aggregated according to $C_{jt}^{rw} = (\int (C_{jt}^{i})^{\frac{\eta-1}{\eta}} di)^{\frac{\eta}{\eta-1}}$. The corresponding aggregate consumption price index is

$$P_{jt} = \frac{1}{K} (P_{jt}^{j})^{a_h} \left((P_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}} \right)^{1-a_h}. \tag{24}$$

where $K \equiv a_h^{a_h} (1-a_h)^{1-a_h} \left(\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}} \right)^{(1-a_h)\frac{\mu_{j'}}{\mu_{j'}+\mu_{rw}}} \left(\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}} \right)^{(1-a_h)\frac{\mu_{rw}}{\mu_{j'}+\mu_{rw}}}$.

For small countries $j \in [0, \mu_{rw}]$, the consumption basket includes imports from both big countries and all other rest-of-world small countries:

$$C_{jt} = C_{jt}^{j\,a_h} \left((C_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (C_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (C_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} \right)^{1-a_h}. \tag{25}$$

The associated price index is

$$P_{jt} = \frac{1}{K_{rw}} (P_{jt}^{j})^{a_h} \left((P_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (P_{jt}^{ez})^{\frac{\mu_{ez}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us}+\mu_{ez}+\mu_{rw}}} \right)^{1-a_h}. \tag{26}$$

where $K_{rw}$is defined analogously to K above.

B.2 The Import-Export Sector

The following subsection provide additional details on each stage of the trading firm game.

Stage 3: Trading Round and Profits

We solve the problem of the trading firms starting with stage three and working backwards. In the final stage, firms discover whether they are importing or exporting this period and with what country, and then search for an appropriate foreign counterpart. For each sub-market, we again assume that the total number of successful matches is given by the den Haan et al. (2000) matching function $M^t(u,v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}}+v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$ with elasticity parameter ε_T which may in general be different that ε_f.

Let $c = (j, j')$ be a double index, capturing an arbitrary country pair, and let $\tilde{m}^{im}_{ct}$ be the mass of *funded* importing firms in country j seeking trade with funded exporting firms in country j' at time t. Then the probability of a country j importer matching with a country j' exporter is

$$p^{ie}_{ct} = \frac{\tilde{m}^{ex}_{c',t}}{\left[(\tilde{m}^{ex}_{c',t})^{1/\xi_t} + (\tilde{m}^{im}_{ct})^{1/\xi_t}\right]^{\xi_t}},$$

where $c' \equiv (j', j)$. Using analogous definitions, the probability of a country j exporter matching with a country j' importer is

$$p^{ei}_{ct} = \frac{\tilde{m}^{im}_{c',t}}{\left[(\tilde{m}^{im}_{c',t})^{1/\xi_t} + (\tilde{m}^{ex}_{ct})^{1/\xi_t}\right]^{\xi_t}}.$$

Given a successful match, the two parties split the surplus that emerges from their trade. Here, we compute this value in the case of a successful match between a country j importer and a country j' exporter; the remaining possibilities can be computed in parallel. For each good that the two firms exchange, they earn a surplus that is equal to the difference in the price of the j' good in its origin country ($P^{j'}_{j',t}$) and its price in the destination country j ($P^{j'}_{j,t}$). If there is a currency mismatch between the two counter-parties, however, the trading surplus is reduced by an additional fraction $\kappa > 0$ of the transaction price.

We assume that the resulting surplus is split via Nash bargaining, with a weight α for the importer. The effective transaction price is thus:

$$P^{whol}_{c',t} = P^{j'}_{j',t} + (1-\alpha)(P^{j'}_{j,t} - P^{j'}_{j',t})$$

This is the wholesale price of country j' exports – the price at which the country-j importer purchases the good from the country-j' exporter. In turn, the country-j importer sells the good at its equilibrium j retail price $P^{j'}_{j,t}$. In equilibrium, $P^{j'}_{j,t} > P^{whol}_{c',t}$ and hence there is a markup and an associated positive surplus to sustain trading.

Let $\tilde{X}_{jt}$ be the fraction of funded country j firms who hold dollar collateral. Then the expected profits of country-j importer importing from j' who hold dollars is given by

$$\pi^{\$,im}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c',t}}\left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}(1-\tilde{X}_{jt})\right],$$

while if it hold euros, expected profits are

$$\pi^{€,im}_{c,t} = \frac{(1-\alpha)}{P^{whol}_{c',t}}\left[P^{j'}_{j,t} - P^{j'}_{j',t} - \kappa P^{whol}_{c',t}\tilde{X}_{jt}\right].$$

Similar expressions hold for exporters:

$$\pi_{c,t}^{\$,ex} = \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[P_{j',t}^{j} - P_{j,t}^{j} - \kappa P_{c,t}^{whol} (1 - \tilde{X}_{j',t}) \right]$$
$$\pi_{c,t}^{€,ex} = \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[P_{j',t}^{j} - P_{j,t}^{j} - \kappa P_{c,t}^{whol} \tilde{X}_{j',t} \right].$$

Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. We refer to funding with US safe assets as "dollar" funding, and funding with EZ safe assets as "euro" funding. Firms seek their funding in matching markets, and not all firms succeed in finding funding each period. Supply in each of these markets is furnished by the domestic household, which lends its bond holdings of a particular currency. On the demand side are the domestic firms who choose to search for that currency.

In order to make their currency choice, firms compare their expected profits conditional on either being funded with dollars or euros. At this stage, they do not yet know whether they will be importers or exporters, with what country they may trade, or whether they will be able to find a successful trading matches in the next stage. Hence, they form expectations over the trading profits that they would receive, conditional on choosing one type of funding over the other.

The expected profit of a country-j trading firm funded with US assets is

$$\tilde{\Pi}_{jt}^{\$} = \sum_{c} p_{ct}^{im} p_{ct}^{ie} \pi_{ct}^{\$,im} + \sum_{c} p_{ct}^{ex} p_{ct}^{ei} \pi_{ct}^{\$,ex},$$

where p_{ct}^{im} is the probability the firm from country j seeks to import from an exporter in country j'.

The first of the two terms in the above sum equals the expected profit of being a dollar-funded importer. It equals the probability of being an importer from country j' times the probability of finding a successful match with a foreign exporter from j', times the resulting profits from that match. The second component is the expected profit of being a dollar-funded exporter. The corresponding expected profits of a country j trading firm funded with EZ assets instead is:

$$\tilde{\Pi}_{jt}^{€} = \sum_{c} p_{ct}^{im} p_{ct}^{ie} \pi_{ct}^{€,im} + \sum_{c} p_{ct}^{ex} p_{ct}^{ei} \pi_{ct}^{€,ex}.$$

We now compute the probability that firms find the funding they seek. Bonds promise payment of one unit of the issuing country's tradable good. Thus, the total value of the US bonds available for lending in country j at time t is given by $P_{us,t}^{us} B_{jt}^{\$} Q_{t}^{\$}$, where $B_{jt}^{\$}$ are the holdings of US bonds in the country j household's portfolio, $P_{us,t}^{us}$ is the price of the US tradable good, and $Q_{t}^{\$}$ is the real price of the bond that pays off one unit of US consumption tomorrow.

Let m_{jt} be mass of trading firms operating in country j, and let X_{jt} be the fraction of country-j trading firms choosing to seek US bonds. Then, the total mass of country-j trading firms searching the domestic US bond market is $m_{jt}X_{jt}$. The total mass of US funds available to intermediate trade is $P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_{t}$. Moreover, we incorporate a parameter, ν, that describes the maximum number of times a given bond could intermediate trade per period of time. This parameter allows us to capture an exogenous notion of velocity for assets lent for trade finance, making it possible to consider an annual calibration in our quantitative results without unduly limiting the availability of trade funding. In our quantitative results, we fix $\nu = 8$ throughout.

The probability that a country j trading firm seeking US bonds finds a supplier is

$$p^{\$}_{jt} = \frac{M^f\left(m_{jt}X_{jt}, \nu P^{us}_{us,t}B^{\$}_{jt}Q^{\$}_{t}\right)}{m_{jt}X_{jt}}.$$

Similarly, the probability that a country j trading firm seeking EZ bonds finds a match is

$$p^{€}_{jt} = \frac{M^f\left(m_{jt}(1-X_{jt}), \nu P^{ez}_{ez,t}B^{€}_{jt}Q^{€}_{t}\right)}{m_{jt}(1-X_{jt})}.$$

It is now straightforward to evaluate

$$\tilde{X}_{jt} = \frac{p^{\$}_{jt}X_{jt}}{p^{\$}_{jt}X_{jt} + p^{€}_{jt}(1-X_{jt})}.$$

In the event that the trading firm finds the funding it seeks, it pays a fee r for the funding services of dollars or euros. Thus, the expected profit of a country-j firm seeking dollar funding is given by

$$\Pi^{\$}_{jt} = p^{\$}_{jt}(\tilde{\Pi}^{\$}_{jt} - r), \tag{27}$$

which is simply the probability of obtaining dollar funding, $p^{\$}_{jt}$, times the expected profit net of the dollar funding costs. Similarly, we can compute the expected profit of a country-j firm seeking Euro funding:

$$\Pi^{€}_{jt} = p^{€}_{jt}(\tilde{\Pi}^{€}_{jt} - r). \tag{28}$$

The only equilibrium requirement for the funding fee r is that it leaves firms with a positive surplus relative to the alternative of declining funding and doing no trade. In parallel with the labor match and searching literature, these prices can be fixed exogenous parameters — so long as they fall within the surplus range of the trading firms — or they could be endogenously determined by assuming some bargaining paradigm, like Nash bargaining. For simplicity, we follow the first of these paths and fix the funding prices to a common value.

Stage 1: Firm Formation

Equilibrium with interior p^{im}_{ct} and p^{ex}_{ct} requires that, prior to learning their private currency choice, firms are ex post indifferent between importing and exporting to the various countries.

Moments	USD Coord. US	EZ	RW	Symmetric US	EZ	RW	EUR Coord. US	EZ	RW
Dollar Share	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100\times(i^{\$}-i^{€})$	1.03	-	-	0.00	0.00	-	-	-1.03	-
100×Seignorage/GDP	0.85	0.22	-	0.54	0.54	-	0.22	0.85	-
100×Trade bal./GDP	-0.25	-0.28	0.14	-0.12	-0.12	0.07	-0.28	-0.25	0.14
Gross debt/GDP	0.60	0.59	0.40	0.60	0.60	0.40	0.59	0.60	0.40
NFA/GDP	-0.14	0.01	0.03	-0.10	-0.10	0.05	0.01	-0.14	0.03
Home Bias	0.12	0.39	0.06	0.23	0.23	0.00	0.39	0.12	0.06
Consumption	11.492	11.492	12.264	11.466	11.466	12.279	11.492	11.492	12.264

Table 7: Steady-state values for baseline model.

Hence, for example in the US, we must have

$$X_{us}p^{ie}_{(us,j),t}\pi^{\$,im}_{(us,j),t}+(1-X_{us})p^{ie}_{(us,j),t}\pi^{€,im}_{(us,j),t}=X_{us}p^{ie}_{(us,j'),t}\pi^{\$,im}_{(us,j'),t}+(1-X_{us})p^{ie}_{(us,j'),t}\pi^{€,im}_{(us,j'),t}$$

for all US potential trading partners j and j'. Similarly,

$$X_{us}p^{ie}_{(us,j),t}\pi^{\$,im}_{(us,j),t}+(1-X_{us})p^{ie}_{(us,j),t}\pi^{€,im}_{(us,j),t}=X_{us}p^{ei}_{(us,j),t}\pi^{\$,ex}_{(us,j),t}+(1-X_{us})p^{ei}_{(us,j),t}\pi^{€,ex}_{(us,j),t}.$$

The above equations are sufficient to pin down the equilibrium probabilities for importing and exporting to and from each country pair.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost $\phi > 0$ in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi^{\$}_{jt},\Pi^{€}_{jt}\}-\phi P_{jt}=0,$$

is satisfied. The entry condition thus determine the equilibrium size, m_{jt}, of the import-export sector in each country.

C Rest-of-World Asset Supply

Our baseline economy abstracts from the presence of any savings vehicle issued by the rest of the world. This is in part because, absent a liquidity premium term, adding such an asset would create an indeterminacy in long-run wealth levels. The same sort of indeterminacy is pervasive in open economy models with incomplete asset markets.

As simple way to include a rest-of-world asset market is to assume there exists an exogenous liquidity demand z_j for the rest of world asset. Though we don't model this role explicitly, we assume it is proportional to the measure of firms in the economy, so that the liquidity wedge for the RW asset is given by

$$\Delta_{jt}^{RW} = \frac{M^f\left(m_{jt}z_j, \nu P_{rw,t}^{rw} B_{jt}^{RW} Q_t^{\$}\right)}{\nu P_{rw,t}^{rw} B_{jt}^{RW} Q_t^{\$}} r.$$

The household euler equation for the RW bond is

$$1 = \beta E_t\left[\left(\frac{C_{jt+1}}{C_{jt}}\right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{rw,t+1}^{rw}}{P_{rw,t}^{rw}} \frac{1}{Q_t^{RW}(1-\Delta_{jt}^{RW}+\tau'(B_{j,t}^{RW}, B_{j,t-1}^{RW})}\right].$$

A desirable feature of this approach to determining holding of the RW bond is that the steady state portfolio allocations are independent of a scale shift in the z_j.

Table 7 reproduces the moments for a calibration targeted to the moments in Panel (a) of Table 2, except that the share of RW assets is increased from zero to 40% of rest-of-world GDP and $z_j = 0.10$. Consumption for the US and EZ remain extremely close to each other, but two other differences stand out. First, the EZ NFA position is now roughly zero, while the US NFA position is reduced to -14%, more consistent with its average value over the past 40 years. Second, as a result of the more moderate NFA position, the US runs a permanent trade deficit, despite its negative NFA position.