Currency Wars, Trade Wars and Global Demand

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Abstract

I present a tractable model of a global economy with downward nominal wage rigidity in which national social planners have access to various policy instruments—the nominal interest rate, taxes on imports and exports, and taxes on capital flows or foreign exchange interventions. The welfare costs of trade and currency wars depend on the state of global demand and on the policy instruments. If global demand is low, trade wars aggravate unemployment and lead to large welfare losses if they involve tariffs on imports, not if they involve export subsidies. Capital wars may lead to endogenous symmetry breaking, with a fraction of countries competitively devaluing their currencies to achieve full employment.

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1 Introduction

Countries have regularly accused each other of being aggressors in a currency war since the global financial crisis. Guido Mantega, Brazil's finance minister, in 2010 blamed the US for launching a "currency war" through quantitative easing and a lower dollar. "We're in the midst of an international currency war, a general weakening of currency. This threatens us because it takes away our competitiveness."[1] At the time Brazil itself was trying to hold its currency down by accumulating reserves and by imposing a tax on capital inflows. Many countries, including advanced economies such as Switzerland, have depreciated or resisted the appreciation of their currency by resorting to foreign exchange interventions. The term "currency war" was again used when the Japanese yen depreciated in 2013 after the Bank of Japan increased its inflation target (and more recently when it reduced the interest rate to a negative level). Bergsten and Gagnon (2012) proposed that the US undertake countervailing currency intervention against countries that manipulate their currencies, or tax the earnings on the dollar assets of these countries. The actions of the Trump administration added to these concerns that of a tariff war.

While G20 countries have regularly renewed their pledge to avoid depreciating their currencies to gain a competitive trading advantage, they have also implemented stimulatives policies that often led to depreciation. Bernanke (2015) argues that this situation should not raise concerns about currency wars as long as the depreciations are the by-product, rather than the main objective, of monetary stimulus (see also Blanchard (2016)). Mishra and Rajan (2016) find the international spillovers from monetary and exchange rate policies less benign and advocate enhanced international coordination to limit the effects of these spillovers.

The concepts of currency war and trade war are old ones but we do not have many models to analyze these wars, separately or as concurrent phenomena (more on this in the discussion of the literature below). In this paper I present a simple model in which an open economy can increase its employment and welfare through the traditional expenditure-switching effect. I consider a symmetric world with many identical countries, each one producing its own good like in Gali and Monacelli (2005). There is downward

[1] Reported by Martin Wolf in "Currencies Clash in New Age of Beggar-thy-Neighbor," Financial Times September 28, 2010.

nominal stickiness in wages like in Schmitt-Grohé and Uribe (2016). Each country can use four policy instruments: the nominal interest rate, a tax on capital flows, a tariff on imports and a subsidy on exports. The tax on capital flows can be interpreted as foreign exchange intervention, which in the real world is used by most emerging market economies as an unconventional monetary policy instrument.

The main qualities that I look for in the model are tractability and analytical transparency. This is achieved by assuming that preferences are intertemporally quasi-linear, i.e., that preferences are linear in the home and foreign goods in an arbitrarily distant period in the future. This makes it possible to derive closed-form expressions for the Nash equilibrium under various assumptions about the available policy instruments. The model is a good approximation to a non-quasi-linear infinite-time model, and can also be used to quantify the size of the effects, in particular the welfare cost of currency and trade wars.

I first take the perspective of a small open economy and derive some equivalence results between the policy instruments. I show that in partial equilibrium one of the four policy instruments is redundant and that exchange rate policy is equivalent to trade policy under some (fairly restrictive) conditions. However, the international spillovers associated with the different policy instruments are very diferent.

The Nash equilibrium between national social planners is especially interesting when the global economy falls in a liquidity trap with a zero nominal interest rate and a positive level of unemployment in all countries. Each country is then tempted to boost its own employment by increasing its share in global demand but the collective implications of such beggar-thy-neighbor policies crucially depend on which policy instrument is used.

There is no benefit from international coordination of interest rates in a global liquidity trap. By contrast, the lack of international policy coordination is particularly costly when countries use tariffs on imports. Because the liquidity trap is a transitory state, the tariffs act as an intertemporal tax on consumption which further reduces demand and increases unemployment. The welfare impact of a tariff war can be substantial, possibly doubling the unemployment rate under plausible calibrations of the model. The outcome of a trade war is quite different when countries use a subsidy on exports. An export subsidy acts as an intertemporal subsidy on consumption and so stimulates consumption. In the Nash equilibrium with export subsidies, full employment is achieved and there is no benefit from international coordination.

I also look at the case where countries can depreciate their currencies by restricting capital inflows and accumulating reserves (still in a global liquidity trap), a situation that could be described as a "capital war." Setting a tax on capital flows is a zero-sum game that simply transfers welfare from the rest of the world to the country imposing capital controls. Thus a capital wars leading to a symmetric Nash equilibrium leaves welfare unchanged and there is no need for international coordination. However if the elasticity of substitution between exported goods is large enough (but within the range of a plausible calibration), a capital war may lead to endogenous symmetry-breaking. A fraction of countries accumulate foreign assets to achieve a trade surplus and full employment, whereas the other countries accept a trade deficit and less than full employment. Somewhat surprisingly, such a capital war increases the welfare of all countries.

Literature. There is a long line of literature on international monetary coordination—see e.g. Engel (2016) for a review. The case for international monetary cooperation in New Open Macro models was studied by Obstfeld and Rogoff (2002), Benigno and Benigno (2006), Canzoneri, Cumby and Diba (2005) among others. Obstfeld and Rogoff (2002) concluded that the welfare gains from international coordination of monetary policy tended to be small. There are several important differences between that literature and the model considered here, most notably the fact that I do not assume that national policymakers can commit to policy rules. Discretion seems a more natural assumption than commitment to study the behavior of governments in currency or trade wars.[2]

A more recent group of papers has explored the international spillovers associated with monetary policy when low natural rates of interest lead to insufficient global demand and liquidity traps including Eggertsson et al. (2016), Caballero, Farhi and Gourinchas (2015), Fujiwara et al. (2013), Devereux and Yetman (2014), Cook and Devereux (2013), and Acharya and Bengui (2016). This paper shares some themes with that literature, in particular the fact that economic integration may propagate a liquidity trap across countries. Eggertsson et al. (2016) and Caballero, Farhi and Gour-

[2]In a more recent contribution, Korinek (2016) gives a set of conditions under which international spillovers are efficient and policy coordination is uncalled for. The model in this paper does not satisfy these conditions—in particular the fact that countries do not have monopoly power.

inchas (2015) study the international transmission of liquidity traps using a model that shares several features with this paper, in particular the downward nominal stickiness a la Schmitt-Grohé and Uribe (2016). Those papers assume Taylor rules for monetary policy and do not incorporate trade taxes to the analysis.[3] Fornaro and Romei (2018) present a model in which macroprudential policy has a negative effect on global demand when the monetary policy is at the zero lower bound.

Other papers have explored whether the constraints on monetary policy resulting from a fixed exchange rate or the ZLB can be circumvented with fiscal instruments (Farhi, Gopinath and Itskhoki, 2014; Correia et al., 2013). Farhi, Gopinath and Itskhoki (2014) show that value added and payroll taxes used jointly with trade taxes can replicate the effects of nominal exchange rate devaluations across a range of model specifications. Correia et al. (2013) study how fiscal instruments can be used to achieve the same allocations as if there were no ZLB on the nominal interest rate in a closed economy. By contrast, the model presented here assumes that the set of policy instruments is more limited.

This paper is related to the recent literature that looks at the macroeconomic impact of trade policy. Barbiero et al. (2017) study the macroeconomic consequences of a border adjustment tax in the context of a dynamic general equilibrium model with monetary policy conducted according to a conventional Taylor rule. Lindé and Pescatori (2017) study the robustness of the Lerner symmetry result in an open economy New Keynesian model and find that the macroeconomic costs of a trade war can be substantial. Erceg, Prestipino and Raffo (2017) study the short-run macroeconomic effects of trade policies in a dynamic New Keynesian open-economy framework. One difference between my paper and these contributions is that I solve for the optimal discretionary policies, including for the interest rate.

The paper is also related to the literature that has quantified the welfare cost of trade wars in general equilibrium (Ossa, 2014). In this type of framework, Amiti, Redding and Weinstein (2019) and Fajgelbaum et al. (2019) find that the welfare cost of the 2018 trade war is moderate (less than 0.1 % of US GDP) but these papers do not take into account the global demand effects that I focus on in this paper.[4]

[3] Bénassy-Quéré, Bussière and Wibaux (2018) consider a model in which countries are more likely to resort to tariffs at the ZLB.

[4] Freund et al. (2018) find a larger cost for the US-China trade war in a scenario where the trade war depresses investment.

The presentation is structured as follows. A large part of the analysis is presented in the context of a two-period model. The two-period framework is convenient to highlight the basic logic behind the analysis of the international spillovers and the benefits of international cooperation. The assumptions of this model are presented in section 2. We analyze the optimal policies from a small open economy perspective in section 3 and the benefits of international coordination in section 4.

Most results however can be extended to more dynamic frameworks, including in infinite time, as shown in section 5. The T-period model inherits the exact properties of the 2-period model as long as it is intertemporally quasi-linear, i.e., utility is linear in the last period. We show that intertemporal quasi-linear models provide a good approximation to infinite-time models that do not have this property. The dynamic model lends itself to quantitative simulations that are explored in section 5.

2 A Two-period Model

The model represents a world composed of a continuum of atomistic countries indexed by $j \in (0,1)$ over two periods $t = 1, 2$. The goods structure is similar to Gali and Monacelli (2005). Each country produces its own good and has its own currency. The nominal wage is rigid downwards as in Schmitt-Grohé and Uribe (2016). There is no uncertainty. The country index j is omitted to alleviate notations until section 4 (when global equilibria are considered).

Preferences. Each country is populated by a mass of identical consumers. The utility of the representative consumer depends on his consumption and real money balances,

$$U_1 = u\left(C_1\right) + v\left(\frac{M_1}{P_{H1}}\right) + \beta\left[C_2 + v\left(\frac{M_2}{P_{H2}}\right)\right], \tag{1}$$

where P_{Ht} is the nominal price of the home good.[5] the utility function has a constant elasticity of intertemporal substitution σ,

$$u\left(C\right) = C^{1-1/\sigma} / \left(1 - 1/\sigma\right).$$

[5]The analysis would be the same if real money balances were defined in terms of the foreign good or the aggregate consumption index.

The consumer consumes the good that is produced domestically (the home good) as well as a basket of foreign goods. In period 1 the consumer cares about the Cobb-Douglas index,

$$C_1 = \left(\frac{C_{H1}}{\alpha_H}\right)^{\alpha_H} \left(\frac{C_{F1}}{\alpha_F}\right)^{\alpha_F}, \tag{2}$$

(with $\alpha_H + \alpha_F = 1$) where C_{H1} is the consumption of home good, and C_{F1} is the consumption of foreign good.

Consumption in the second period is linear in the consumption of the two goods,

$$C_2 = C_{H2} + C_{F2}. \tag{3}$$

This specification implies that the period-2 terms of trade are equal to 1 independently of the country's net foreign assets, which makes the model quite tractable. Without this assumption, the final terms of trade would be an increasing function of the country's foreign assets. As we show in section 5 this effect is very small and does not significantly affect the qualitative or quantitative properties of the model.

The consumption of foreign good is a CES index of the goods produced in all the countries,

$$C_F = \left[\int_0^1 C_k^{(\gamma-1)/\gamma} dk\right]^{\gamma/(\gamma-1)}.$$

The composite good defined by this index will be called the "global good" in the following. The elasticity of substitution between foreign goods is assumed to be larger than one, $\gamma > 1$.

Budget constraints. The consumers trade one-period bonds denominated in the global good. The budget constraints of the representative consumer in periods 1 and 2 are given by

$$P_{F1}\frac{B}{R\left(1+\tau^b\right)} + M_1 + P_{H1}C_{H1} + \left(1+\tau^m\right)P_{F1}C_{F1} = W_1L_1 + Z_1, \tag{4}$$

$$P_{H2}C_{H2} + P_{F2}C_{F2} + M_2 = W_2L_2 + P_{F2}B + M_1 + Z_2, \tag{5}$$

where P_{Ft} denotes the offshore domestic-currency price of the global good, τ^m is a tax on imports, τ^b is a tax on foreign borrowing, B is the quantity of real bonds accumulated by the representative consumer in period 1, R is the offshore gross real interest rate in terms of the global good and Z_t is a lump-sum transfer from the government that is used to rebate the taxes and inject money. Home currency nominal bonds can be traded but the supply of these bonds is zero in equilibrium and they have not been included in the budget constraints.[6]

Production and labor market. The home good is produced with a linear production function that transforms one unit of labor into one unit of good, $Y = L$. The representative consumer is endowed with a fixed quantity of labor $\overline{L}$ and the quantity of employed labor satisfies

$$L \leq \overline{L}. \tag{6}$$

There is full employment if this constraint is satisfied as an equality. We normalize $\overline{L}$ to 1.

The nominal wage in period $t = 1, 2$ is denoted by W_t and the wage inflation rate is denoted by π_t,

$$1 + \pi_t = \frac{W_t}{W_{t-1}},$$

where W_0 is exogenous and inherited from the past. The linearity in production implies $P_{Ht} = W_t$ so that π_t is the inflation rate in the price of the home good.

We assume that the nominal wage is sticky in period 1. Like in Schmitt-Grohé and Uribe (2016) or Eggertsson et al. (2016), downward nominal stickiness in the wage is captured by the constraint,

$$\pi_1 \geq \underline{\pi}, \tag{7}$$

where the lower bound on the inflation rate $\underline{\pi}$ is nonpositive. The economy can be in two regimes: full employment ($L_1 = \overline{L}$), or less than full employment, in which case wage inflation is at its lower bound ($L_1 < \overline{L}$ and $\pi_1 = \underline{\pi}$). This leads to a L-shaped Phillips curve where inflation can be set independently of employment once there is full employment. The constraints on the labor market can be summarized by (6), (7) and

$$\left(\overline{L} - L_1\right)(\pi_1 - \underline{\pi}) = 0. \tag{8}$$

[6]Foreign investors do not hold domestic currency bond or cash (or if they do, the tax τ^b applies to these assets).

By contrast, the nominal wage is flexible in period 2 (which is interpreted as the long run), so that $L_2 = \overline{L}$.

Demand for home labor. The period-1 demand for home labor is,

$$L_1 = C_{H1} + \left[(1 + \tau^x) \frac{P_{H1}}{P_{F1}} \right]^{-\gamma} C_{F1}^W, \tag{9}$$

where $C_{F1}^W = \int C_{Fk} dk$ denotes global gross imports and τ^x is the tax on exports. The first term on the right-hand side of (9) is the labor used to serve home demand for the home good and the second term is the labor used to produce exports.

It will be convenient to define three terms of trade,

$$S_1 \equiv \frac{P_{H1}}{P_{F1}}, \; S_1^m \equiv \frac{S_1}{1 + \tau^m} \text{ and } S_1^x \equiv (1 + \tau^x) S_1, \tag{10}$$

where S_1 denotes the period-1 undistorted terms of trade, and S_1^m and S_1^x are the tax-distorted terms of trade that apply to imports and exports respectively.

Given the Cobb-Douglas specification (2) the home demand for the home good and for imports are respectively given by,

$$C_{H1} = \alpha_H (S_1^m)^{-\alpha_F} C_1, \tag{11}$$
$$C_{F1} = \alpha_F (S_1^m)^{\alpha_H} C_1. \tag{12}$$

The demand for home labor (9) can thus be re-written as a function of the terms of trade,

$$L_1 = \alpha_H (S_1^m)^{-\alpha_F} C_1 + (S_1^x)^{-\gamma} C_{F1}^W. \tag{13}$$

The demand for home labor increases with home consumption and global consumptions but is reduced by a loss in domestic competitiveness (an increase in S_1^m or S_1^x).

Balance of payments. Using $Z_1 = \tau^m P_{F1} C_{F1} + \tau^x P_{H1} (L_1 - C_{H1}) - \tau^b P_{F1} B / \left(1 + \tau^b\right) + M_1$, $Z_2 = M_2 - M_1$, equations (9), (10), and (12) to substitute out Z_1, L_1, C_{H1}, C_{F1}, P_{H1} and P_{F1} from the representative consumer's budget constraint (4) gives the balance of payments equation

$$B = RX, \tag{14}$$

where net exports in terms of global good are given by

$$X = (S_1^x)^{1-\gamma} C_{F1}^W - \alpha_F (S_1^m)^{\alpha_H} C_1. \tag{15}$$

Note that the value of net exports in terms of the global good decreases if the country loses competitiveness in export markets (an increase in S^x) because $\gamma > 1$.

3 National Policy-Making

In period 1 the national social planner can use monetary policy, trade policy, and capital account policy.[7] The instruments of trade policy are the taxes on imports and exports, τ^m and τ^x, and the instrument of capital account policy is the tax on capital inflows, τ^b. (As explained at the end of next section capital controls can be re-interpreted as foreign exchange interventions.) Section 3.1 shows that the instrument of monetary policy can be specified as the nominal interest rate i conditional on an inflation target. Section 3.2 compares the impact of the different policy instruments and section 3.3 studies the conditions under which the policy instruments are equivalent.

3.1 Monetary policy

Consider monetary policy in period 2. The first-order condition for money demand is

$$v'\left(\frac{M_2}{P_{H2}}\right) = 1. \tag{16}$$

(the first-order conditions for the consumer's problem are derived in Appendix A). Thus the nominal price of the home good is proportional to money supply. A change in money supply does not affect real variables or welfare in period 2. I assume that the social planner sets the inflation rate to a target,

$$\pi_2 = \pi^*.$$

The target is exogenous for now but we will consider equilibria in which national social planners set the inflation target at the beginning of period 0.

[7] We do not introduce taxes or subsidies on labor, which can be used to ensure full employment in this model.

In period 1 the economy could be or not in the regime where the nominal wage $W_1 = P_{H1}$ is sticky or in the regime where it is flexible. The marginal benefit of real money balances is equal to their opportunity cost, which is the loss of interest rate payment in period 2,

$$v'\left(\frac{M_1}{P_{H1}}\right) = \beta\frac{i}{1+\pi^*}. \tag{17}$$

Arbitrage between real and nominal bonds implies

$$R\left(1+\tau^b\right) = (1+i)\frac{P_{F1}}{P_{F2}}. \tag{18}$$

The left-hand-side is the global good own rate of interest at home, which is equal to the foreign level plus the tax on capital inflows. The right-hand side is the same real interest rate expressed through the Fisher relationship. Using $P_{Ft} = P_{Ht}/S_t$, $S_2 = 1$ and $P_{H2}/P_{H1} = 1+\pi^*$, one obtains an expression for the first period terms of trade,

$$S_1 = \frac{1+i}{R\left(1+\tau^b\right)(1+\pi^*)}. \tag{19}$$

The terms of trade are increased (the currency appreciated in real terms) by an increase in the nominal interest rate or a decrease in the tax on capital flows. The interest rate and capital controls are alternative instruments of exchange rate policy.

The last relevant equilibrium condition is the Euler equation for consumption,

$$u'(C_1)\left(S_1^m\right)^{\alpha_F} = \beta\frac{1+i}{1+\pi^*}. \tag{20}$$

The left-hand side is the marginal utility of increasing the consumption of home good in period 1 and the right-hand side is the marginal utility of purchasing domestic bonds to consume the home good in period 2. Equations (19) and (20) imply that consumption C_1 is a decreasing function of i.[Show it?]

Taking (17), (19) and (20) into account the period-1 equilibrium depends on money supply as shown in Figure 1. There is unemployment if money supply is lower than a threshold. In this range, P_{H1} is fixed and an increase in money supply lowers the nominal interest rate by equation (17). This depreciates the home currency and raises consumption and the demand for home labor by equations (19), (20) and (13). When the demand for home labor reaches $\overline{L}$ the economy transitions to the flexible wage regime where further increases in money supply raise the nominal wage and have no impact on real variables. Like for period 2, I assume that the social planner sets inflation at the target level if the economy is at full employment ,

$$L_1 = \overline{L} \Longrightarrow \pi_1 = \pi^*,$$

which corresponds to point A in Figure 1.

Observe however that it is not always possible to raise L_1 to the full employment level as money supply increases. The nominal interest rate goes to zero as money supply goes to infinity (or reaches the satiation level). It is not always the case that the level of labor demand corresponding to $i = 0$ is larger than labor supply $\overline{L}$. If it is smaller the economy is in a liquidity trap.

Conditional on the inflation target π^*, the instrument of monetary policy can be the nominal interest rate i rather than money supply M_1. Using equations (19), (20), (11), (12) and (13), any policy mix $\left(i, \tau^m, \tau^x, \tau^b\right)$ can be mapped into a real allocation $(C_1, C_{H1}, C_{F1}, L_1)$. The policy is *feasible* if and only if the implied L_1 does not exceed $\overline{L}$. This is how a country's policy is defined in the rest of this paper.

Remark. An alternative interpretation for capital account policy is foreign exchange interventions (Jeanne, 2013). To see this, assume that the capital account is closed, i.e., that the social planner monopolizes financial transactions with the rest of the world and holds all the foreign assets B, which are the country's foreign exchange reserves.[8] Then by setting B the social planner can achieve the same allocations as when when she uses the tax on capital inflows τ^b. An arbitrary level of B is associated with a wedge between the global and home levels of the real rate of interest that corresponds to the level of τ^b implementing the same allocation.

3.2 Comparative statics

The impact of a policy mix $\left(i, \tau^m, \tau^x, \tau^b\right)$ on home welfare can then be assessed using the expression

$$U_1 = u\left(C_1\right) + \beta RX, \tag{21}$$

[8] The assumption that there are no private capital flows is extreme but the insights remain true if frictions prevent economic agents from arbitraging the wedge between onshore and offshore interest rates.

which comes from $U_2 = \overline{L} + B$ and (14). This expression omits unimportant constants and does not take into account the utility from real money balances.[9]

The following proposition explains how a country's employment, trade balance and welfare are affected by domestic policies.

Proposition 1 *Consider a symmetric undistorted allocation with unemployment and assume $\sigma < 1$. Then a country's employment, trade balance and welfare are moved in the same direction by all its policy instruments. The welfare of a given country is increased by (i) a decrease in the nominal interest rate; (ii) an increase in the tariff on imports; (iii) a decrease in the tax on exports; (iv) an increase in the tax on capital inflows:*

$$\frac{\partial U_1}{\partial i} < 0,\ \frac{\partial U_1}{\partial \tau^m} > 0,\ \frac{\partial U_1}{\partial \tau^x} < 0,\ \frac{\partial U_1}{\partial \tau^b} > 0.$$

Proof. See Appendix. ■

The proposition considers an allocation that is symmetric and undistorted in the sense that all countries are identical and set their taxes to zero. Furthermore we assume that all countries have some unemployment, implying that the home nominal wage is sticky.[10] The first important result is that the elasticities of employment and welfare with respect to all policy instruments have the same signs. This means that any policy that raise employment also raises the trade balance and welfare.[11]

A decrease in the nominal interest rate raises home consumption and the trade balance and so, by equation (21), welfare. Raising the tariff on imports increases the trade balance but reduces home consumption since a tariff is an intertemporal taxes on period-1 consumption. Ambiguous impact on labor demand because expenditure-switching effect and expenditure-changing

[9] I will generally omit the utility from real money balances for simplicity but this does not matter for most of the results.

[10] When there is full employment the policy changes considered in Proposition 1 are not always feasible. A change in instrument that increases labor demand must be offset by a change in another instrument (or in inflation) to keep labor demand below $\overline{L}$.

[11] One should not infer from this result that maximizing welfare is always equivalent to reaching full employment because it applies only around a symmetric undistorted allocation. We will indeed see that in a distorted economy welfare-maximizing social planners may not seek full employment.

effects go in opposite directions. Condition $\sigma < 1$ ensures that tariff raises employment and welfare. Since we are interested in tariff wars we assume that this condition is satisfied in the rest of the paper.

A subsidy on exports does not affect home consumption but raises employment and the trade balance, and so welfare by equation (21).

3.3 Equivalence between policy instruments

The equivalence between policy instruments is characterized by the following proposition.

Proposition 2 . *(Instrument equivalence) Any allocation* (C_H, C_F, L, π) *achieved by policy* $(i, \tau^m, \tau^x, \tau^b)$ *can also be achieved by policy* $(i, \widetilde{\tau}^m, \widetilde{\tau}^x, \widetilde{\tau}^b)$ *with*

$$(1+\widetilde{\tau}^m)(1+\widetilde{\tau}^x) = (1+\tau^m)(1+\tau^x), \tag{22}$$

$$(1+\widetilde{\tau}^m)\left(1+\widetilde{\tau}^b\right) = (1+\tau^m)\left(1+\tau^b\right). \tag{23}$$

Proof. See Appendix. ∎

In words, the allocation is unchanged if the social planner shifts the tax on exports to imports and at the same time increase the tax on capital inflows by the same amount as the tax on exports.[12] For the gross exports to be left unchanged, the decrease in the export tax must be perfectly offset by an increase in the terms of trade (a real appreciation). The real appreciation must in turn be offset by an equivalent increase in the tax on imports to keep the domestic price of imports the same. The real appreciation results from a decrease in the tax on capital inflows of the same size as the tax on exports.

A long-standing question in the macroeconomic and trade literature is that of the conditions under which exchange rate manipulation can replicate the impact of tariffs (Meade, 1955). To answer this question, let us denote by E_1 the period-1 nominal exchange rate between the small open economy and a given foreign country, defined as the price of the foreign currency in terms of domestic currency. The law of one price implies

$$P_1 = E_1 P_1^*,$$

[12]The fact that a tax on imports has the same impact as a tax on exports is known as Lerner's symmetry theorem in the trade literature (Lerner, 1936). Costinot and Werning (2017) provide a number of generalizations and qualifications of the Lerner symmetry theorem in a dynamic environment.

where P_1^* is the (offshore) price of the global good in terms of foreign currency. The policies and allocations are consistent with a fixed exchange rate regime if they satisfy $E_1 = \overline{E}$.

Then using Proposition 2 it is easy to see that the allocations achievable with a fixed exchange rate $E_1 = \overline{E}$ and trade taxes τ^m and τ^x can be replicated with a floating exchange rate and zero trade taxes if and only if

$$(1 + \tau^m)(1 + \tau^x) = 1. \tag{24}$$

If condition (24) is satisfied then the allocations achievable with a fixed exchange rate $\overline{E}$ and trade taxes can also be achieved without trade tax by the floating exchange rate

$$\widetilde{E}_1 = (1 + \tau^m)\overline{E},$$

i.e., the social planner depreciates the domestic currency relative to the fixed exchange rate as a substitute to the tariff on imports. Importantly, the social planner must be able to tax capital flows for the equivalence between trade taxes and exchange rate flexibility to hold.[13] The exchange rate adjustments required by the removal of trade taxes are achieved by capital controls rather than the nominal interest rate, which again is pinned down by the domestic Euler equation (20). To summarize, there is an equivalence between exchange rate policy and trade policy provided that (i) trade policy introduces the same terms of trade distortion in domestic and foreign markets; and (ii) the instrument of exchange rate policy is the tax on capital flows rather than the interest rate.

Another implication of Proposition 2 is that some instruments are redundant in the full set $(i, \tau^m, \tau^x, \tau^b)$. Any allocation can be achieved without using the tariff on imports, the subsidy on exports or the tax on capital inflows. This equivalence is in partial equilibrium only. As we will see, instruments that are substitutable at the small country level have very different international spillovers in general equilibrium.

[13] The equivalence between taxes on trade and taxes on capital flows is studied in Costinot, Lorenzoni and Werning (2014).

4 The Benefits of International Policy Coordination

This section looks at the benefits of international coordination conditional on various assumptions about the policy instruments that national social planners can use. I now use the country index j but drop the time index 1. The period-1 variables will be denoted without time subscript from now on.

We consider symmetric equilibria in which all countries have the same inflation target π^* and the same discount rate,

$$\forall j, \beta_j = \beta. \tag{25}$$

The level of β determines global demand (a lower β means higher demand).

There are two global market clearing conditions. The countries' trade balances sum up to zero,

$$\int X_j dj = 0, \tag{26}$$

and global imports are the sum of imports across all countries,

$$C_F^W = \alpha_F \int \left(S_j^m\right)^{\alpha_H} C_j dj. \tag{27}$$

Integrating $X_j = \left(S_j^x\right)^{1-\gamma} C_F^W - \alpha_F (S_j^m)^{\alpha_H} C_j$ over all countries j then implies

$$\int \left(S_j^x\right)^{1-\gamma} dj = 1. \tag{28}$$

The terms of trade in export markets are relative prices that cannot all move in the same direction.

This section compares equilibria under different assumptions about the policy instruments that countries can use. Since countries enjoy monetary sovereignty the interest rate is always part of the available policy instruments but the use of the other policy instruments could be limited by international agreements such as for example the WTO. Section 4.1 considers the case of monetary wars where the only weapons are the nominal interest rate and the inflation targets. Section 4.2 adds the trade taxes to the policy mix. Section 4.3 assumes that countries can use the nominal interest rate and capital controls only, and section 4.4 considers the case where all the policy instruments can be used. In all cases we compare the Nash equilibrium between national social planners with the allocation chosen by a global social planner who maximizes the welfare of the representative country. The global social planner allocation can also be interpreted as the result of international coordination between the national social planners. Finally, section **??** compares the quantitative implications of using different sets of policy instruments.

Numerical illustration. As we go along I will illustrate the quantitative properties of the model based on the following parameter values. The elasticity of intertemporal substitution of consumption, σ, is set to 0.5, which corresponds to a risk aversion of 2, a standard value in the literature. The elasticity of substitution between foreign goods, γ, is set to 3, which is consistent with the recent estimates of Feenstra et al. (2018). Note in particular that the "microelasticity" between the differentiated imported goods is substantially larger than the "macroelasticity" between the home good and imports (which is 1 because of the Cobb-Douglas specification). Finally, we assume $\alpha_H = 0.6$, i.e., home goods amount to 60 percent of total consumption.

4.1 Monetary wars (i, π^*)

This section considers the case where countries only have the interest rate as a policy instrument. A Nash equilibrium is then composed of global economic conditions $\left(R, C_F^W\right)$, monetary policies (i_j) and allocations $(C_{Hj}, C_{Fj}, C_j, L_j, \pi_j)$ for all countries $j \in [0, 1]$ such that: (i) the monetary policy of any country j maximizes domestic welfare given the global economic conditions; (ii) country allocations satisfy the equilibrium conditions given country policies and global economic conditions; and (iii) the global markets clearing conditions (26) and (27) are satisfied.

It results from Proposition 1 that either there is full employment or the ZLB constraint is binding. There cannot be a Nash equilibrium with less than full employment and a strictly positive interest rate sincen any country would be better off deviating and lowering the interest rate.

In an equilibrium with full employment, $C_1 = 1$, $S_1^m = 1$ and equation (20) imply

$$1 + i = \frac{1 + \pi^*}{\beta}.$$

Hence the ZLB constraint can be satisfied if and only if $\beta \leq 1 + \pi^*$. If this condition is violated, all countries set their nominal interest rate to zero and by equation (20), global consumption and employment are given by

$$C_1 = L = u'^{-1}\left(\frac{\beta}{1+\pi^*}\right) < 1.$$

Our results so far are summarized in the following Proposition.

Proposition 3 *(Conventional monetary war) Assume that the only policy instrument available to national social planners is the nominal interest rate. Then there is a unique Nash equilibrium between national planners and the nominal interest rate is given by,*

$$i = \left(\frac{1+\pi^*}{\beta} - 1\right)^+. \tag{29}$$

There is full employment in period 1 if and only if $\beta \leq 1 + \pi^*$. If this condition is violated the economy falls in a global liquidity trap with the same level of unemployment in all countries. There is no gain from international coordination.

Proof. See discussion above. ■

No gain from international coordination because monetary stimulus is a positive-sum game as emphasized by Bernanke.

The model can easily be extended to the case where national social planners can choose their inflation targets (an "inflation target war"). Let us assume that each country j sets its inflation target π_j^* at the beginning of period 1. The Nash equilibrium from that point onwards is then determined conditional on the inflation targets as before. In the period-0 Nash equilibrium each country sets its inflation target so as to maximize domestic welfare taking the other countries' inflation target as given. Then we have the following result.

Proposition 4 *(Unconventional monetary war) Assume that the national social planners can choose their inflation targets before period 1. Then in a symmetric Nash equilibrium social planners set an inflation target $\pi^* \geq \beta - 1$ and $i_j = (1+\pi^*)/\beta - 1$. There is full employment in all countries and welfare is at the first-best level. There is no gain from international coordination.*

Proof. See Appendix C. ∎

This result comes from the fact that the international spillovers associated with an increase in the inflation target are the same as with a decrease in the nominal interest rate. An inflation target war, thus, is a positive-sum game.

4.2 Trade wars (i, τ^m, τ^x)

In a tariff war the available policy instruments are the nominal interest rate, the tariff on imports and the tax on exports. First I consider the case where national social planners can use only tariff on imports, and then add the export tax to the policy mix. The Nash equilibria with tariffs on imports are characterized in the following proposition. The equilibrium tariff depends on whether or not the world economy is in a global liquidity trap with a binding ZLB and unemployment in all countries.

Proposition 5 *Consider a symmetric Nash equilibrium in which all national social planners use instruments i and τ^m. The equilibrium level of tariff depends on the state of the global economy as follows:*

(i) outside of a global liquidity trap the tax on imports τ^m is set in all countries at a level satisfying

$$\frac{\tau^m}{\alpha_F} = \frac{1}{\sigma} - 1 - \gamma \left(\frac{\alpha_F}{\alpha_H} + \frac{1}{\sigma} \right) \left(1 - \frac{1}{1+\tau^m} \right), \tag{30}$$

(ii) in a global liquidity trap the tax on imports τ^m is set to

$$\tau^m = \alpha_H \left(\frac{1}{\sigma} - 1 \right), \tag{31}$$

(iii) the equilibrium tariff on imports is larger in a global liquidity trap, i.e., the level of τ^m given by (30) is larger than the level implied by (31).

Proof. See Appendix C. ∎

The equilibrium tariff on imports depends on whether the global economy is at full employment. If there is full employment national social planners use tariffs to manipulate the terms of trade. Each country can increase its terms of trade by simultaneously raising its tariff and its nominal interest rate. The equilibrium tariff, given by (30) is reached when the marginal welfare gain from increasing the terms of trade is perfectly offset by the marginal welfare cost of the intratemporal and intertemporal distortions of consumption induced by the tariff. The equilibrium tariff decreases with the export elasticity γ, which reduces the country's monopoly power in foreign markets, and with the elasticity of intertemporal substitution, which reduces the impact of tariffs on employment and welfare.

In a global liquidity trap with less than full employment, national social planners use tariffs on imports to increase home employment. For a zero tariff this increases welfare (see Proposition 1) but as the tariff rate increases the welfare gain from higher production is increasingly offset by the consumption distortion. The equilibrium tariff decreases with the elasticity of intertemporal substitution. It does not depend on the export elasticity because the tariff does not affect exports in a liquidity trap. The equilibrium tariff level is higher than under full employment because the benefit of increasing employment is larger than that of manipulating the terms of trade.

Under what condition does the Nash equilibrium between national social planners lead to a global liquidity trap? Using (20) to substitute out C in the labor demand equation (13), and noting that in a symmetric allocation $C_F^W = \alpha_F (S^m)^{\alpha_H} C$, $S^m = 1/(1+\tau^m)$ and $S^x = 1$, one can rewrite the demand for labor (13) as

$$L = \ell(1+\tau^m)\left(\beta\frac{1+i}{1+\pi^*}\right)^{-\sigma}, \tag{32}$$

where function $\ell(\cdot)$ is defined by

$$\ell(1+\tau^m) \equiv \alpha_H (1+\tau^m)^{\alpha_F(1-\sigma)} + \alpha_F (1+\tau^m)^{-\alpha_H-\alpha_F\sigma}. \tag{33}$$

Differentiating $\ell(\cdot)$ shows that it is decreasing function of τ^m as long as $\tau^m < \frac{\sigma}{\alpha_H(1-\sigma)}$. When all countries impose the same tariff, increasing the tariff decreases global demand for the consumption good and for labor when the tariff is not too high. When the tariff is very high and consumption is very distorted, it can take an increasing quantity of labor to produce a decreasing quantity of consumption good.

Let us denote by $\underline{\tau}^m$ and $\overline{\tau}^m$ the equilibrium tariffs respectively defined by (30) and (31). I assume that both tariffs are in the region where global labor demand decreases with the tariff, i.e., $\overline{\tau}^m < \frac{\sigma}{\alpha_H(1-\sigma)} = 1/\overline{\tau}^m$. This is true if and only if $\overline{\tau}^m < 100\%$, or equivalently $\sigma > \alpha_H/(1+\alpha_H)$. This condition is satisfied under our benchmark calibration, for which $\underline{\tau}^m = 10.2\%$ and $\overline{\tau}^m = 60\%$.

The Nash equilibrium in tariffs then depends on the discount factor as shown by Figure 2. The figure shows (for our benchmark calibration) the variation of labor demand with the tariff for $i = 0$. The left-hand-side panel shows the case where β is low enough that the demand for labor conditional on a zero interest rate is higher than the supply even for the higher tariff $\overline{\tau}^m$. In this case there is one unique equilibrium, corresponding to point G, with full employment and a positive nominal interest rate. The right-hand side panel shows the case where β is high enough that the demand for labor conditional on a zero interest rate is lower than the supply even for the lower tariff $\overline{\tau}^m$. In this case there is one unique equilibrium, corresponding to point B, with unemployment and a binding ZLB.

There are two equilibria if β is in an intermediate range, as shown by the middle panel of Figure 2. A good equilibrium with full employment (point G) coexists with a bad equilibrium with a global liquidity trap (point B). A tariff war can lead to a self-fulfilling global liquidity trap because high tariffs decrease the demand for global output in general equilibrium and so the natural rate of interest.

The implications of a tariff war for employment are illustrated by Figure 3 under the benchmark calibration. The figure shows the variation of global unemployment $\overline{L} - L$ with $1/\beta$, a measure of extrinsic global demand. A self-fulfilling tariff war has a large impact on employment and can arise for a wide range of β.

The equilibrium is very different if the tax on exports is added to the set of usable policy instruments. A Nash equilibrium must now feature full employment. This is because each national social planner can increase home employment and trade balance by lowering the tax (or increasing the subsidy) on exports. At the national level, the tax on exports does not affect consumption. Thus, decreasing the tax on exports unambiguously raises welfare by raising the trade balance—see equation (21). A global liquidity trap with a binding ZLB can still exist but it is no longer associated with unemployment.

The Nash equilibrium with both trade taxes is characterized in the following result.

Proposition 6 *A symmetric Nash equilibrium in which all national social planners use instruments i, τ^m and τ^x has the following properties.*

(i) There is full employment in all countries.

(ii) If the ZLB constraint is not binding the equilibrium trade taxes are given by

$$\tau^m = 0,\ \tau^x = \frac{1}{\gamma - 1}.$$

(iii) If the ZLB constraint is binding, the equilibrium taxes τ^m and τ^x are respectively increasing and decreasing with β. Exports are subsidized ($\tau^x < 0$) if $\beta \geq 1 + \pi^*$.

(iv) The ZLB constraint is binding if and only if β is larger than $(1 + \pi^*)\,\ell\left(\frac{\gamma}{\gamma-1}\right)^{1/\sigma}$.

Proof. See Appendix C. ■

Figure 4 shows the variation of the equilibrium trade taxes and the nominal interest rate with $1/\beta$. When global demand is high, countries attempt to improve their terms of trade with a high tax on exports and can keep a positive nominal interest rate. Under full employment and non-binding ZLB it is more efficient to use a tax on exports than a tax on imports to manipulate the terms of trade because the former does not distort home consumption. Thus, the terms of trade manipulation is done exclusively through the tax on exports. The equilibrium tax is the same as in the static tariff war.

As demand $1/\beta$ decreases the ZLB constraint becomes binding and national social planners lose one instrument. They reduce τ^x to maintain full employment and increase τ^m to continue manipulating the terms of trade. At some point τ^x turns negative, i.e., national social planners start to subsidize exports. If the Nash equilibrium without trade taxes is a global liquidity trap with unemployment, then the equilibrium with trade taxes involves positive subsidies on exports.

Import and export taxes have very different multilateral effects. We can generalize the expression for the demand for labor, equation (32), by noting that $S = 1/(1 + \tau^x)$ so that

$$L = \ell\left((1 + \tau^m)(1 + \tau^x)\right)\left(\beta\frac{1 + i}{1 + \pi^*}\right)^{-\sigma} \tag{34}$$

In general equilibrium the allocation depends only on the product $(1 + \tau^m)(1 + \tau^x)$, not on the trade taxes taken separately. This product measures the wedge in the relative price of the foreign good in terms of home good that matters for the intratemporal allocation of consumption between the two goods and the intertemporal allocation between the two periods. It does not matter in general equilibrium whether the tax is imposed by exporters or by importers.

A key difference between a tariff on imports and a tax on exports is in the congruence between partial and general equilibrium. In partial equilibrium, national social planners increase home employment by increasing τ^m or decreasing τ^x. These policy changes have an opposite impact on global demand. Both τ^m and τ^x act like intertemporal consumption taxes that affect global demand. An increase in τ^m reduces global demand but a decrease in τ^x stimulates global demand.

The global social planner maximizes welfare $u(C)$ subject to the constraints $L \leq \overline{L}$ and $i \geq 0$. Thus the GSP problem can be written

$$\max_{i,S^m} C = \left(\beta \frac{1+i}{1+\pi^*}\right)^{-\sigma} (S^m)^{\alpha_F \sigma},$$

subject to

$$L = \ell\left(1/S^m\right)\left(\beta \frac{1+i}{1+\pi^*}\right)^{-\sigma} \leq \overline{L}$$

and the ZLB constraint $i \geq 0$.

The solution is to set $1+i = (1+\pi^*)/\beta$ and $S^m = 1$ if this is consistent with the ZLB constraint, and set $i = 0$ and S^m at the level that ensures full employment otherwise. The GSP allocation is characterized in the following proposition.

Proposition 7 *The GSP sets the interest rate according to (29) and the trade taxes such that*

$$\ell\left((1+\tau^m)(1+\tau^x)\right) = \left(\frac{\beta(1+i)}{1+\pi^*}\right)^{\sigma}. \tag{35}$$

If the ZLB constraint is not binding the GSP allocation is achieved with zero trade taxes. If the ZLB constraint is binding the GSP allocation is achieved by subsidizing exports or imports. The allocation in this case is the same as in the Nash equilibrium where national social planners can use both τ^m and τ^x.

Proof. See discussion above. ■

The GSP cares about the total wedge $(1+\tau^m)(1+\tau^x)$, not about τ^m and τ^x separately. The allocation of the total wedge between τ^m and τ^x is indeterminate.

Whether the GSP needs to use the trade taxes depends on whether the ZLB constraint is binding. If the ZLB constraint is not binding, the GSP can achieve full employment by using the nominal interest rate and does not use the trade taxes, which are distortionary.[14] If the ZLB constraint is binding, the GSP achieves full employment by subsidizing consumption with the trade taxes. The subsidy can indifferently be on imports or exports.

There is a strictly positive welfare gain from international cooperation except in one case: if the ZLB is binding in the absence of trade taxes and both trade taxes can be used. In this case, the Nash equilibrium leads to full employment and selects a combination of trade taxes that belongs to GSP set.

Figure 5 shows the variation of the welfare gains from international coordination with $1/\beta$. Those gains crucially depend on the trade taxes used by national social planners. If they use only tariffs on imports, the welfare gains are the largest for low demand when international coordination avoid an increase in tariffs that raise unemployment. The welfare gains are modest for high demand because the tariff rate $\underline{\tau}^m$ is not very high. If national social planners can use export subsidies,[15] the welfare gains from international coordination are larger for high demand, when national social planners set a high level of tax on exports to manipulate the terms of trade. As demand falls, the level of trade taxes falls to maintain full employment in the Nash equilibrium, which reduces the gains from international coordination.

4.3 Capital wars (i, τ^b)

We now assume that national social planners can use the tax on capital inflows τ^b in a global liquidity trap. If there is unemployment national social planners can increase employment and welfare by imposing a tax on capital inflows, which depreciates the domestic currency and stimulates exports, at the cost of distorting consumption. When all national social planners impose the same tax τ^b, the only variable that is affected by the capital inflow tax is the global real interest rate. It follows from (**??**) and $S = 1$ that

$$R = \frac{1+i}{(1+\tau^b)(1+\pi^*)}.$$

[14]More precisely, the GSP ensures that a tax on imports is offset by an equivalent subsidy on exports, i.e., $(1+\tau^m)(1+\tau^x) = 1$.

[15]The figure reports only the case where they can use both τ^m and τ^x. The case where they can use only τ^x is quantitatively very close.

A symmetric increase in the tax on capital inflows reduces the global rate of interest. For any individual country, this tilts the balance against exporting and accumulating foreign assets. The equilibrium level of τ^b is achieved when the real interest rate is so low that national social planners do not find it profitable to increase their net exports above zero.

The global social planner is indifferent about the level of τ^b since it does not affect welfare. These insights are developed more precisely in the following proposition.

Proposition 8 *Assume all national social planners can use the tax on capital inflows τ^b in a global liquidity trap. There is a symmetric Nash equilibrium if and only if $\gamma < 2$. The level of the tax in this equilibrium is given by*

$$\tau^b = \frac{\gamma - \alpha_H \sigma - \alpha_F}{\sigma}. \tag{36}$$

Employment and welfare are the same as in the equilibrium without capital control. The global social planner is indifferent about the level of τ^b, and there is no benefit from international policy coordination.

Proof. See Appendix C. ■

Under our benchmark calibration equation (36) implies $\tau^b = 460\%$ and $R = 0.175$. All countries tax capital inflows at a very high rate, implying that the equilibrium return on foreign assets is very negative—a situation close to financial autarky.[16]

The symmetric Nash equilibrium described in Proposition 8 exists only if $\gamma < 2$. Higher levels of the export elasticity make the symmetric Nash equilibrium unstable. On one hand, a high level of γ implies that exports are highly sensitive to the exchange rate, so that it is possible to achieve full employment at the cost of small increase in capital controls. On the other hand, a high level of γ also implies that the global economy is close to financial autarky and that the return on foreign assets is very low. This increases the benefits of borrowing rather than lending.

When the export elasticity exceeds the threshold $\gamma = 2$, this tension makes the symmetric Nash equilibrium unsustainable. In the symmetric

[16]Financial autarky is reached in the limit where τ^b goes to infinity. In this case it is impossible to invest abroad, i.e., countries cannot accumulate their trade surpluses for future consumption ($R = 0$).

allocation where τ^b is given by (36), countries are better off deviating either by increasing τ^b to increase their employment or decreasing τ^b to finance a larger level of consumption at a low borrowing rate. This is illustrated by Figure 6, which shows how the welfare of an individual country varies with its own tax τ^b in the symmetric allocation as well as in the asymmetric Nash equilibrium.[17] Welfare is a convex function of τ^b in the symmetric allocation (point C), so that countries are better off deviating from the level of τ^b that satisfies the first-order condition in a symmetric allocation.

As a result, the global economy endogenously divides itself into two groups of countries: a group of countries with a more competitive currency, a trade surplus, and full employment (point A on Figure 6), and a group of countries with a less competitive currency, a trade deficit and some unemployment (point B). For our benchmark calibration the surplus countries impose a tax $\tau^b = 409\%$ on their capital inflows whereas the deficit countries impose a tax $\tau^b = 93\%$. This decreases the interest rate at which deficit countries borrow to a very low level ($R = 0.2027$). The low cost of borrowing comes from the fact that deficit countries have an appreciated currency when they borrow and a depreciated currency when they repay. The fraction of countries that have trade surpluses (88.3% for our benchmark calibration) is such that surplus and deficit countries have the same level of welfare level. The level of welfare is higher in points A and B than in point C. Since point C corresponds to the level of welfare without capital war,[18] the capital war actually increases the welfare of all countries. Thus, there is no case for avoiding a capital war through international coordination.

4.4 Total wars $(i, \tau^m, \tau^x, \tau^b)$

If national social planners can use all the policy instruments there is indeterminacy because of Proposition 2. Each national social planner can achieve the desired allocation with an infinity of policy combinations. However, Proposition shows that all these combinations imply the same wedge $(1 + \tau^m)(1 + \tau^x)$. One of these combinations involve a zero tax on capital inflows so that the Nash equilibrium is the same as in the case, already analyzed in section 4.2, where national social planners can use (i, τ^m, τ^x).

[17] The figure is built using the parameter values presented in the next section.

[18] This is because applying the same tax τ^b to all countries does not change welfare in a symmetric allocation.

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To conclude this section, Figure 7 shows the welfare impact of different kinds of trade and currency wars under the benchmark calibration. The discount factor was set at the level that implies an unemployment rate of 5 percent if national social planners use only monetary policy. The main lesson from the figure is that the welfare impact of these wars crucially depends on which instrument is used. The worst welfare impact comes from a trade war relying on tariffs on imports because of the resulting increase in unemployment. By contrast, a trade war increases welfare if it involves subsidies on exports. A currency war based on capital controls (or foreign exchange interventions) slightly increases welfare. Welfare is increased to the first-best level if the policy instrument the inflation target.

5 Dynamic Extensions

Assume a finite number of periods $t = 1, 2..., T$ where T is arbitrarily large. As we show in this section, the T-period model inherits the properties of the two-period model if utility is linear in the final period.

In all periods except the final one, the utility of the representative consumer is defined recursively by,

$$U_t = u\left(C_t\right) + \beta_t U_{t+1}. \tag{37}$$

The instrument of monetary policy is the nominal interest rate and we omit the utility of real money balances.

Like in the two-period model, utility in the final period is linear in the consumption of the two goods,

$$U_T = C_{HT} + C_{FT}. \tag{38}$$

On the side of production and the labor market, the model keeps the same assumptions as the two-period model. There is downward nominal wage rigidity in all periods t with employment and the wage inflation rate satisfying $L_t \leq \overline{L}$, $\pi_t \geq \underline{\pi}$, and equation (8). The national social planner sets inflation equal to π^* in any period in which there is full employment. The budget constraints are reported in Appendix D.

In each county policies are chosen by a benevolent national social planner who maximizes the welfare of the representative consumer taking the global economic conditions $(R_t)_{t=1,...,T-1}$ and $\left(C_{Ft}^W\right)_{t=1,...,T}$ as given. We assume that social planners are time-consistent and cannot commit to a policy path. Under these conditions the intertemporal quasi-linearity of the utility function implies that the social planner's problem can be decomposed into a sequence of two-period problems analogous to the social planner's problem in the two-period model.

To see this, we first observe that (as shown in Appendix D), the country's intertemporal welfare can be written

$$U_1 = \sum_{t=1}^{T-1} \left(\prod_{t'=0}^{t-1} \beta_{t'} \right) \underbrace{\left[u\left(C_t\right) + \left(\prod_{t'=t}^{T-1} \beta_{t'} R_{t'} \right) X_t \right]}_{V_t}, \tag{39}$$

with $\beta_0 = 1$. The country's welfare is equal to the discounted sum of terms V_t that are analogous to equation (21) in the two-period model. This is an implication of the model's intertemporal quasi-linearity: an increase in the trade balance in any period t is invested in foreign assets and consumed in the last period.

As shown in Appendix D, the analogs of equations (15), (19) and (**??**) in the T-period model are,

$$X_t = \left(S_t^x\right)^{1-\gamma} C_{Ft}^W - \alpha_F \left(S_t^m\right)^{\alpha_H} C_t, \tag{40}$$

$$S_t = \prod_{t'=1}^{T-1} \frac{1 + i_{t'}}{R_{t'} \left(1 + \tau_{t'}^b\right) \left(1 + \pi_{t'+1}\right)}, \tag{41}$$

$$u'\left(C_t\right) \left(S_t^m\right)^{\alpha_F} = \prod_{t'=t}^{T-1} \beta_{t'} \frac{1 + i_{t'}}{1 + \pi_{t'+1}} \tag{42}$$

In a time-consistent equilibrium, the national social planner takes the values of the future policy instruments as given, so that C_t and X_t depend on the current policy instruments in the same way as in the two-period model.

First, let us consider the subgame perfect Nash equilibrium between the national social planners using only the nominal interest rate as a policy instrument. This equilibrium is defined as time paths for (i) global economic conditions $\left(R_t, C_{Ft}^W\right)_{t=1,...,T}$; (ii) monetary policies $(i_{jt})_{t=1,...,T}$ and allocations $(C_{Hjt}, C_{Fjt}, C_{jt}, L_{jt}, \pi_{jt})_{t=1,...,T}$ for all countries $j \in [0,1]$ such that:

- the monetary policy of any country j in any period t, i_{jt}, maximizes domestic welfare given the global economic conditions and future policies

$(i_{it'})_{t'>t}$,

- country allocations satisfy the equilibrium conditions given country policies and global economic conditions;
- the global markets clearing conditions (26) and (27) are satisfied in every period t.

Then one can show that the analog of Proposition 3 is as follows.

Proposition 9 *Assume that the only policy instrument available to national social planners if the nominal interest rate. Then there is a unique subgame perfect Nash equilibrium between national planners. There is full employment in all periods if and only if the condition $\beta_t \leq 1 + \pi^*$ is satisfied for all t. The global economy is in a liquidity trap with less than full employment in any period in which this condition is violated. If the national social planners can use tariffs on imports in a period t, the Nash equilibrium tariffs are the same as in the two-period model.*

Proof. See Appendix D ■

The Nash equilibrium is constructed by iterating backwards. In the last period there is full employment and inflation is equal to the target in all countries. In the previous period, $T-1$, the equilibrium can be constructed like in the 2-period model and the global economy is either at full employment or in a liquidity trap, which determines whether wage inflation π_{T-1} is equal to $\underline{\pi}$ or π^*. The equilibrium can be derived by continuing these iterations backwards. Note that a liquidity trap in period t lowers inflation to $\underline{\pi}$ so that the condition for a liquidity trap in the previous period, $\beta_t \leq 1 + \underline{\pi}$, is weaker. That is, the expectation of a liquidity trap in the next period tends to pull the economy into a liquidity trap in the current period.[19]

Figure 8 illustrates how the dynamics of unemployment are affected by a trade war. The figure is based on the following parameter values: $\pi^* = 2\%$, $\underline{\pi} = 0$, $T = 4$, $\beta_t = \exp(-3\%)$ for $t < 4$. The equilibrium is constructed by backward induction, taking into account that unemployment reduces inflation and so increases the real interest rate. With or without tariff, unemployment is maximum at the beginning of the liquidity trap and decreases over time. A tariff war increases global unemployment by about 6% because of the depressing effect of the tariff on global demand.

[19] In some models this leads to self-fulfilling traps that last forever. This is not the case here because the model has a final period in which it is not in a liquidity trap.

6 Conclusions

We have analyzed a tractable model in which countries use transitory trade taxes and capital controls in a context of low global demand. The analysis suggests that there is one case where uncoordinated policies are mutually destructive and lead to large welfare losses: when countries use tariffs on imports. The uncoordinated use of all the other instruments we have looked at (interest rate, inflation target, export subsidy and capital controls) is Pareto optimal. Unfortunately, tariffs seem to be the instrument of choice in the real world. One interesting question is why revealed preferences favor tariffs over other instruments such as export subsidies. A possible explanation is that subsidies on exports are financed with distortionary taxation.

The paper opens several directions for further research. Making the model less symmetric would allow us to look at questions that have not been analyzed in this paper. For example, assuming that countries differ in their time preferences would make it possible to examine how a "global savings glut" in one part of the world may affect the benefits of international policy coordination. Another relevant source of asymmetry is if countries have access to different policy instruments. In the real world some countries are committed not to use certain policy instruments. For example, OECD and EU membership preclude the use of capital controls except in exceptional circumstances. WTO membership also puts restrictions on trade policies (although the limits of these restrictions are increasingly tested). One could also assume that countries have different sizes or home bias.

Another question is the robustness of trigger strategy equilibria in which free trade is supported by the threat of a trade war. It would be interesting to know, in this context, whether a trade war is made more or less likely by a fall in global demand leading to unemployment.

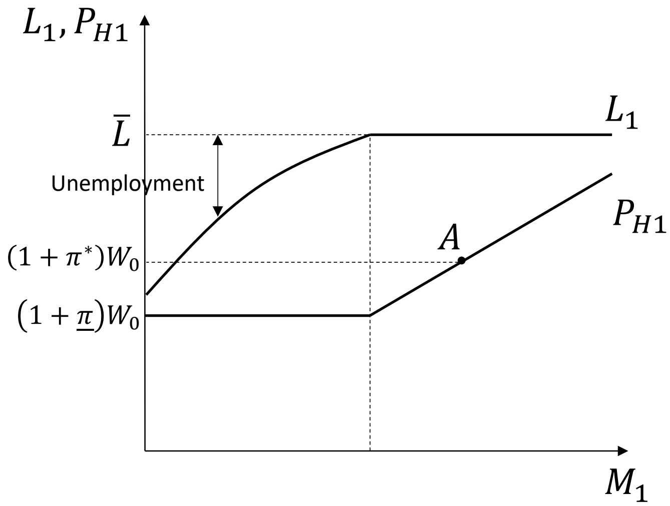

Figure 1: Money supply, price level and demand for labor

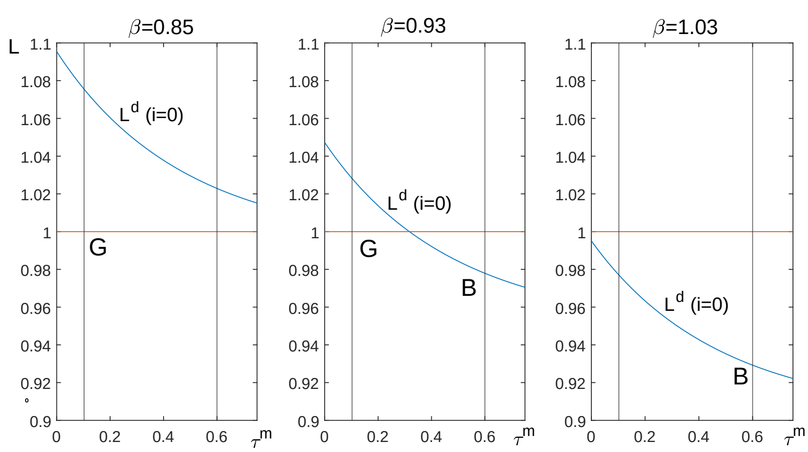

Figure 2: Equilibrium of labor market with tariff on imports

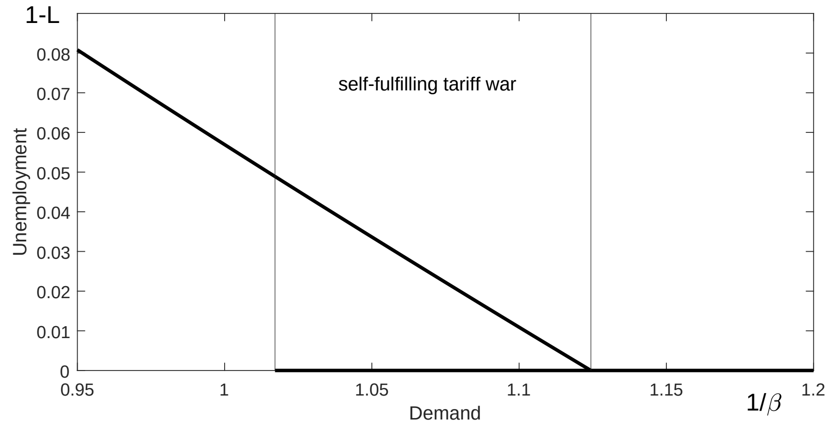

Figure 3: Variation of unemployment $\overline{L} - L$ with demand $1/\beta$

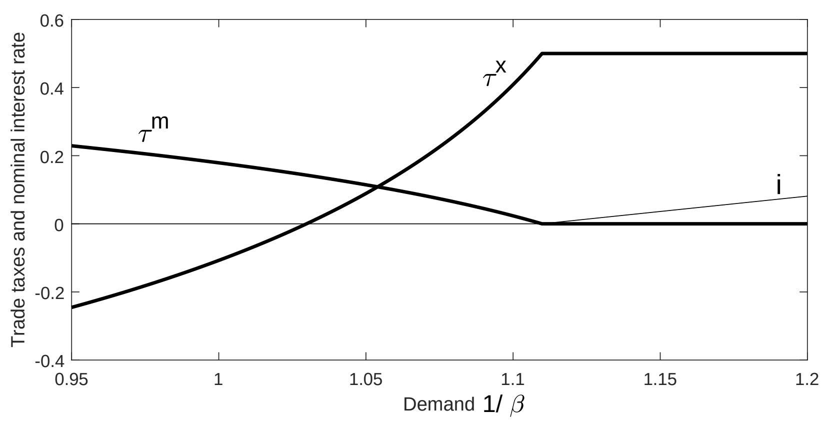

Figure 4: Variation with demand $1/\beta$ of trade taxes and nominal interest rate in Nash equilibrium

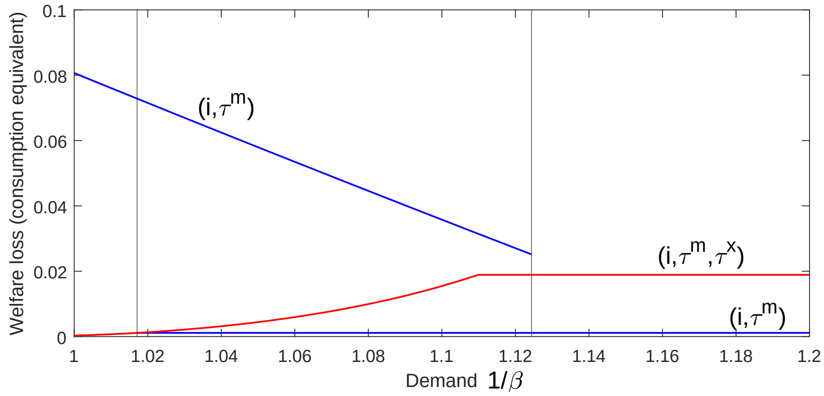

Figure 5: Variation of welfare gains from international coordination with demand $1/\beta$

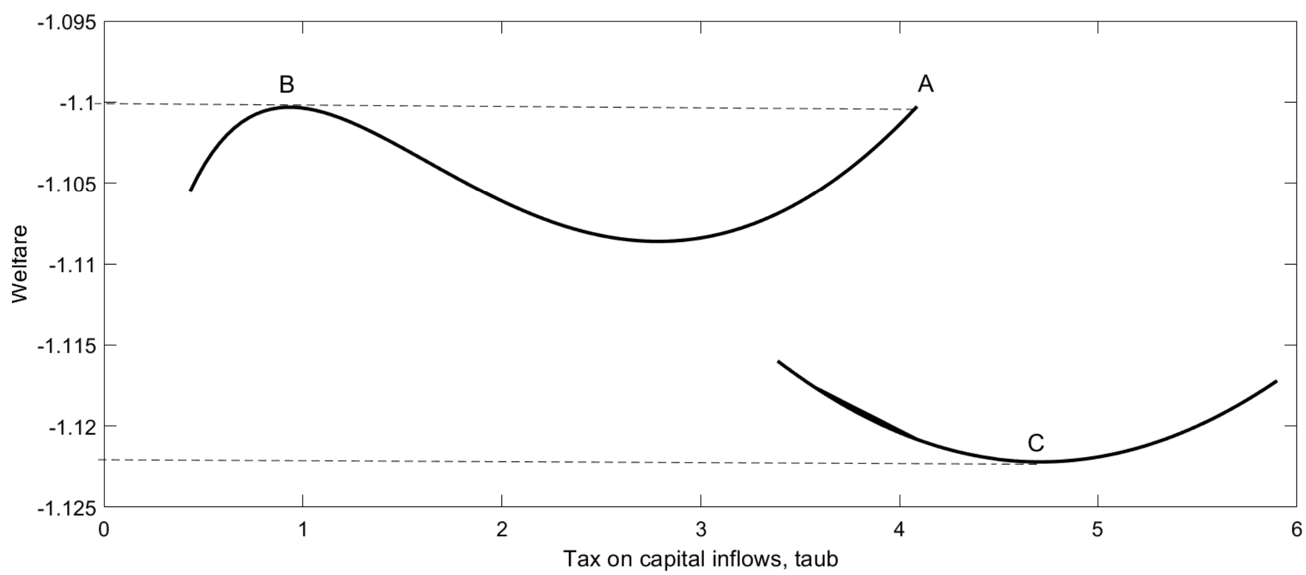

Figure 6: Variation of welfare with tax on capital inflows in symmetric allocation and Nash equilibrium

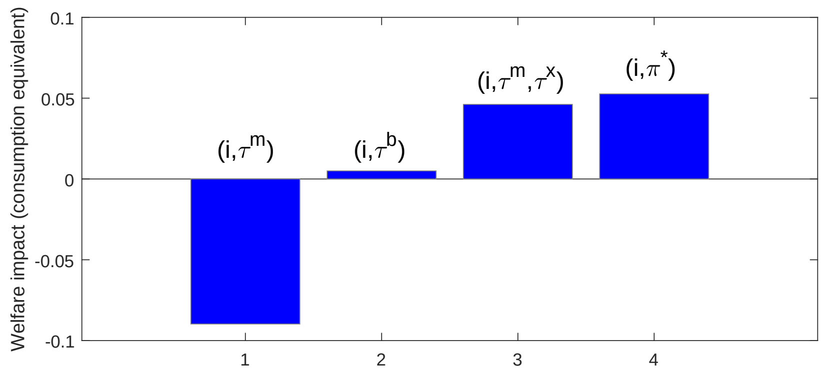

Figure 7: Impact of trade and currency wars on welfare

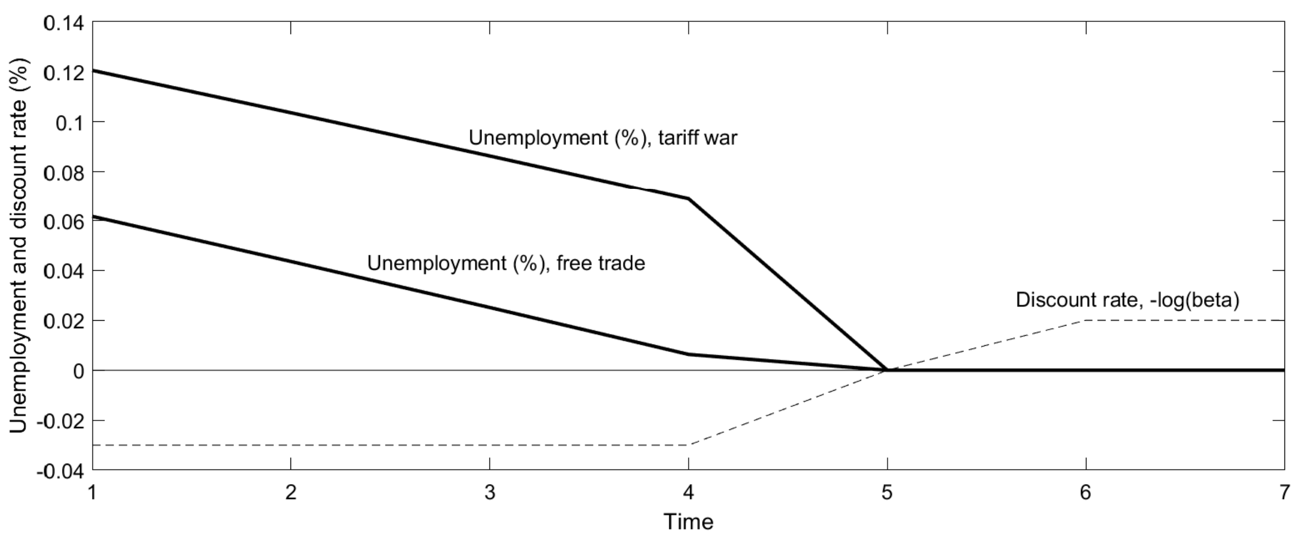

Figure 8: Unemployment rate in a dynamic trade war

APPENDICES

APPENDIX A. CONSUMER'S FIRST-ORDER CONDITIONS

This appendix derives the consumer's first-order conditions in the two-period model. Using (3) and (5) to substitute out C_2 from (21), and omitting variables that are taken as given by the consumer's problem can be written

$$\max_{C_1,B,M_1,M_2} U_1 = u\left(C_1\right) + v\left(\frac{M_1}{P_{H1}}\right) + \beta\left[B + \frac{M_1 - M_2}{P_{H2}} + v\left(\frac{M_2}{P_{H2}}\right)\right],$$

subject to

$$P_{F1}\frac{B}{R\left(1+\tau^b\right)} + M_1 + P_1^c C_1 = W_1 L_1 + Z_1,$$

where $P_1^c = \left(P_{H1}\right)^{\alpha_H}\left(\left(1+\tau^m\right)P_{F1}\right)^{\alpha_F}$ is the price index for consumption.

The first-order condition for M_2 is (16). Denoting by λ the shadow cost of the period-1 budget constraint the first-order conditions for C_1, B and M_1 are respectively

$$\begin{aligned} u'\left(C_1\right) &= \lambda P_1^c, & (43)\\ \beta &= \lambda\frac{P_{F1}}{R\left(1+\tau^b\right)}, & (44)\\ \frac{1}{P_{H1}}v'\left(\frac{M_1}{P_{H1}}\right) + \frac{\beta}{P_{H2}} &= \lambda. & (45) \end{aligned}$$

Using (44) to substitute out λ in (43) gives (20). Using (18) to substitute out $R\left(1+\tau^b\right)$ in (44) as well as $P_{F2} = P_{H2} = \left(1+\pi^*\right)P_{H1}$ gives

$$\lambda = \beta\frac{1+i}{\left(1+\pi^*\right)P_{H1}}. \tag{46}$$

Using (46) to substitute out λ in (43) gives (20). Using (46) to substitute out λ in (45) and $P_{H2}/P_{H1} = 1+\pi^*$ gives (17).

APPENDIX B. SOCIAL PLANNER'S FIRST-ORDER CONDITIONS

This appendix derives the first-order conditions for the national social planner and the elasticities of macroeconomic variables with respect to the policy instruments. Both will be used to prove our main results in Appendix C. Dropping the country index, the Lagrangian for the social planner's problem is

$$\pounds = u(C) + \beta R\left[X + \lambda\left(\overline{L} - L\right)\right] + \mu i.$$

Using (20) the first-order condition for instrument $n = i, \tau^m, \tau^x, \tau^b$ can be written

$$\left(1+\tau^b\right) e(C,n) + (1+\tau^x)\frac{\alpha_H C_t}{C_{Ht}}\left[e(X,n) - \lambda e(L,n)\right] + \mu 1_{n=i} = 0, \quad (47)$$

for $n = i, \tau^m, \tau^x, \tau^b$ where $e(\bullet, n)$ denotes the elasticity of variable $\bullet = S, C, L, X$ with respect to instrument $n = i, \tau^m, \tau^x$ and τ^b defined as follows,

$$\begin{aligned} e(S,n) &= \frac{1+n}{S}\frac{\partial S}{\partial n},\ e(C,n) = \frac{1+n}{C}\frac{\partial C}{\partial n}, \\ e(L,n) &= \frac{1+n}{C}\frac{\partial L}{\partial n},\ e(X,n) = \frac{1+n}{C}\frac{\partial X}{\partial n}. \end{aligned}$$

The elasticities are computed in a symmetric allocation assuming less than full employment. They are reported in Table B1.

Table B1. Elasticities in a symmetric allocation with unemployment

	i	τ^m	τ^x	τ^b
S	1	0	0	-1
C	$-\alpha_H\sigma$	$-\alpha_F\sigma$	0	$-\alpha_F\sigma$
L	$-(\alpha_H\sigma + \alpha_F)\frac{C_H}{C} - \gamma\frac{C_F}{C}$	$\alpha_F(1-\sigma)\frac{C_H}{C}$	$-\gamma\frac{C_F}{C}$	$\alpha_F(1-\sigma)\frac{C_H}{C} + \gamma\frac{C_F}{C}$
X	$-(\gamma - \alpha_H\sigma - \alpha_F)\frac{C_F}{C}$	$(\alpha_H + \alpha_F\sigma)\frac{C_F}{C}$	$-(\gamma-1)\frac{C_F}{C}$	$[\gamma - \alpha_F(1-\sigma)]\frac{C_F}{C}$

The values of $e(S,n)$ reported in the first row of Table B1 directly follows from (19). Using (19) to substitute out S_1^m from (20) one can write period-1 consumption as

$$C_1 = \beta^{-\sigma}\left(\frac{1+i}{1+\pi^*}\right)^{-\alpha_H\sigma}\left[R\left(1+\tau^m\right)\left(1+\tau^b\right)\right]^{-\alpha_F\sigma}. \tag{48}$$

Differentiating this expression gives the elasticities $e(C,n)$ in the second row of Table B1.

Equations (**??**) and (48) can be used to differentiate (13) and (15). Using that $C_{F1}^W = C_{F1}$ in a symmetric equilibrium we obtain

$$e(L,n) = \left[e(C,n) - \alpha_F e(S^m,n)\right]\frac{C_H}{C} - \gamma e(S^x,n)\frac{C_F}{C}, \tag{49}$$

$$e(X,n) = -\left[(\gamma-1)e(S^x,n) + e(C,n) + \alpha_H e(S^m,n)\right]\frac{C_F}{C}. \tag{50}$$

Using the elasticities for C and S given in the first two rows of Table B1 we can use (49) and (50) to derive the expressions in the bottom two rows of Table B1.

APPENDIX C. PROOFS

Proof of Proposition 1. The proof follows from the elasticities reported in Table C1. The fact that $C_H = \alpha_H C$ and $C_F = \alpha_F C$ in a symmetric equilibrium with $\tau^m = 0$, and the elasticities reported in the bottom two rows of Table B1, gives the expressions for $e(L,n)$ and $e(X,n)$ given in the top two rows of Table C1. To alleviate the expressions we introduce the notation

$$\eta = \alpha_F\left(\gamma - \alpha_H\sigma - \alpha_F\right),$$

for the elasticity of the trade deficit with respect to the nominal interest rate.

We have $U_1 = u(C_1) + \beta RX$. In a symmetric undistorted equilibrium we have $S_1^m = 1$ and so by equation (20), $u'(C_1) = \beta R$. Differentiating U_1 for any instrument $n = i, \tau^m, \tau^x, \tau^b$ gives the elasticity of welfare with respect to the policy instruments,

$$e(U,n) \equiv \frac{1+n}{Cu'(C)}\frac{\partial U}{\partial n} = \frac{1+n}{C}\frac{\partial C}{\partial n} + \frac{1+n}{C}\frac{\partial X}{\partial n}.$$

The elasticities in the bottom row of Table C1 can thus be derived by adding up the elasticities for C and X.

Table C1. Elasticities in a symmetric allocation with unemployment and $\tau^m = \tau^x = 0$.

	i	τ^m	τ^x	τ^b
L	$-\alpha_H\sigma - \alpha_F - \eta$	$\alpha_H\alpha_F(1-\sigma)$	$-\alpha_F\gamma$	$\eta + \alpha_F$
X	$-\eta$	$\alpha_F(\alpha_H + \alpha_F\sigma)$	$-\alpha_F(\gamma - 1)$	$\eta + \alpha_F\sigma$
U	$-\alpha_H\sigma - \eta$	$\alpha_H\alpha_F(1-\sigma)$	$-\alpha_F(\gamma - 1)$	η

Proof of Proposition 2. The allocation (C_1, C_{H1}, C_{F1}, X) stays the same. It then follows from (11), (12) and (15) that S_1^m and S_1^x must stay the same. Since $S_1^x/S_1^m = (1+\tau^m)(1+\tau^x)$ this implies (22). The fact that C_1 and S_1^m stay the same in equation (20) implies that i must also stay the same. Then equation (19) and the fact that S_1^m must stay the same imply (23). Conversely, (22) and (23) imply that the allocations are not changed by the alternative policy mix.

Proof of Proposition 4. There cannot be unemployment in a symmetric Nash equilibrium, otherwise any national social planner could increase domestic welfare by raising the domestic inflation target π_j^*. Hence social planners set an inflation target (it must be the same in all countries in a symmetric equilibrium) such that $\beta \leq 1 + \pi^*$ or $\pi^* \geq \beta - 1$. The inflation target is indeterminate as long as it satisfies this condition.

Proof of Proposition 5. The social planner maximizes domestic welfare over the policy instruments i and τ^m.

First, we ignore the ZLB constraint to obtain the level of tariff in equilibria where this constraint is nonbinding (point (i) of the proposition). The first-order conditions are given by (47) for $n = i, \tau^m$ with $\mu = 0$. We can use the expression for $e(C,n)$, $e(L,n)$ and $e(X,n)$ from Table B1 to substitute out the elasticities from (47), which gives

$$\begin{aligned}
(\alpha_F + \alpha_H\sigma)(\lambda C_H + C_F) &= \sigma C_H + \gamma(1-\lambda) C_F, \\
(\alpha_H + \alpha_F\sigma) C_F &= \alpha_F \left[\frac{\sigma}{\alpha_H} + \lambda(1-\sigma)\right] C_H.
\end{aligned}$$

Eliminating λ between these two expressions and $1 + \tau^m = \frac{\alpha_F}{\alpha_H}\frac{C_H}{C_F}$ give equation (30). The r.h.s. of this equation is positive for $\tau^m = 0$ and decreases with τ^m. This equation has one unique solution satisfying $\tau^m \geq -1$ and it is positive. Furthermore τ^m decreases with γ and σ because the r.h.s. of (30) does too.

Second, we set $i = 0$ and ignore the constraint $L \leq \overline{L}$ to obtain the level of tariff in equilibria with a global liquidity trap and unemployment (point (ii) of the proposition). We apply equation (47) for $n = \tau^m$ with $\tau^b = 0$, $\tau^x = 0$ and $\lambda = 0$, which gives

$$e\left(C, \tau^m\right) + \frac{\alpha_H C}{C_H} e\left(X, \tau^m\right) = 0.$$

We use the expressions for $e(C, \tau^m)$ and $e\left(X, \tau^m\right)$ from Table B1 to substitute out the elasticities from this equation, and $1 + \tau^m = \frac{\alpha_F}{\alpha_H} \frac{C_H}{C_F}$ to derive (31).

One can show that the equilibrium tariff is larger in a global liquidity trap because the r.h.s. of (30) is negative if τ^m is given by (31). One can show that global employment decreases with the tariff rate if and only if

$$\tau^m < \frac{\sigma}{\alpha_H \left(1 - \sigma\right)}. \tag{51}$$

This condition is satisfied by the equilibrium tariff whether there is unemployment or not.

Proof of Proposition 6 The national social planner's first-order conditions are given by (47) with $\tau^b = 0$ for $n = i, \tau^m, \tau^x$. The first-order condition for τ^x implies

$$\lambda = \frac{\gamma - 1}{\gamma}.$$

First, assume that the ZLB is not binding. Then the first-order conditions for i and τ^m can respectively be written (after some manipulations)

$$\begin{aligned} -\sigma - \alpha_F \left(1 - \sigma\right) \frac{1}{1 + \tau_m} + \lambda \left(\alpha_H \sigma + \alpha_F\right)\left(1 + \tau^x\right) &= 0, \qquad (52) \\ -\sigma + \left(\alpha_H + \alpha_F \sigma\right) \frac{1}{1 + \tau_m} - \lambda \alpha_H \left(1 - \sigma\right)\left(1 + \tau^x\right) &= 0. \qquad (53) \end{aligned}$$

Solving for τ^m and τ^x gives $\tau^m = 0$ and $\tau^x = \frac{1}{\lambda} - 1 = 1/\left(\gamma - 1\right)$ as reported in point (ii) of the Proposition.

Second, assume that the ZLB is binding. Then τ^m and τ^x satisfy (35) since there is full employment and (53). Let us denote by S^* the terms of trade such that

$$\ell\left(1/S^*\right) = \left(\frac{\beta}{1 + \pi^*}\right)^{\sigma}. \tag{54}$$

Using $(1+\tau^m)(1+\tau^x) = 1/S^*$ and $\lambda = (\gamma-1)/\gamma$ in equation (53) gives

$$\begin{aligned} 1+\tau^m &= \frac{1}{\sigma}\left[\alpha_H + \alpha_F\sigma - \frac{\alpha_H(1-\sigma)}{S^*}\frac{\gamma-1}{\gamma}\right], \\ 1+\tau^x &= \frac{\sigma}{(\alpha_H+\alpha_F\sigma)S^* - \alpha_H(1-\sigma)(\gamma-1)/\gamma}. \end{aligned}$$

An increase in β increases S^* by equation (54) and so lowers τ^x and increases τ^m as indicated in point (iii) of the Proposition.

Proof of Proposition 8 We assume $\pi^* = 0$, without loss of generality and to alleviate the algebra. We look at equilibria in which there is unemployment and $i=0$. It follows from (19), (20), and $S_1^m = S_1$ that

$$C_1 = \beta^{-\sigma} S_1^{\alpha_F\sigma}.$$

Using this expression, (60) and $C_{F1}^W = \alpha_F\ \beta^{-\sigma}$to substitute out C_1 and X from $U_1 = u(C_1) + \beta R X$ gives

$$U_1 = \beta^{1-\sigma}\left[\frac{S_1^{-\alpha_F(1-\sigma)}}{1-1/\sigma} + \alpha_F R\left(S_1^{1-\gamma} - S_1^{\alpha_H+\alpha_F\sigma}\right)\right].$$

Because there is a simple correspondance between the terms of trade and the tax on capital inflows, $S_1 = \frac{1}{R\left(1+\tau^b\right)}$, it is equivalent for the social planner to maximize U_1 over τ^b or over S_1. Differentiating U_1 gives

$$\frac{\partial U_1}{\partial S_1} = \alpha_F\beta^{1-\sigma}\left\{\sigma S_1^{-\alpha_F(1-\sigma)-1} - R\left[(\gamma-1)S_1^{-\gamma} + (\alpha_H+\alpha_F\sigma)S_1^{-\alpha_F(1-\sigma)}\right]\right\}.$$

In a symmetric Nash equilibrium one should have $\partial U_1/\partial S_1 = 0$ for $S_1 = 1$ and $R = 1/\left(1+\tau^b\right)$. Solving for τ^b gives equation (36).

Given the equilibrium value of R one can compute the second derivative

$$\left.\frac{\partial^2 U_1}{\partial S_1^2}\right|_{S_1=1} = \alpha_F\beta^{1-\sigma}\sigma(\gamma-2).$$

The value of τ^b given by equation (36) maximixes domestic welfare if and only if this second derivative is negative, i.e., if and only if $\gamma \leq 2$.

APPENDIX D. DYNAMIC EXTENSIONS

We omit money from the model.

Budget constraints. Consumers trade one-period bonds denominated in the global good. In any period $t < T$ the budget constraint of the representative consumer is

$$P_{Ft} \frac{B_{t+1}}{R_t \left(1 + \tau_t^b\right)} + P_{Ht} C_{Ht} + (1 + \tau_t^m) P_{Ft} C_{Ft} = W_t L_t + Z_t + P_{Ft} B_t, \quad (55)$$

with notations similar to the two-period model. In the final period the budget constraint is given by,

$$P_{HT} C_{HT} + P_T C_{FT} = W_T \overline{L} + P_{FT} B_T. \quad (56)$$

There is full employment and no trade tax in the final period. As a result the terms of trade are equal to 1 ($W_T = P_{HT} = P_{FT}$), and welfare is given by,

$$U_T = \overline{L} + B_T. \quad (57)$$

The period-t demand for home labor takes the same form as equation (13),

$$L_t = \alpha_H \left(S_t^m\right)^{-\alpha_F} C_t + \left(S_t^x\right)^{-\gamma} C_{Ft}^W. \quad (58)$$

The the balance of payments equation is

$$\frac{B_{t+1}}{R_t} = B_t + X_t, \quad (59)$$

where net exports in terms of global good are given by

$$X_t = \left(S_t^x\right)^{1-\gamma} C_{Ft}^W - \alpha_F \left(S_t^m\right)^{\alpha_H} C_t. \quad (60)$$

Welfare. By iterating on (37) and using $U_T = \overline{L} + B_T$ one can write period-1 welfare as

$$U_1 = \sum_{t=1}^{T-1} \left(\prod_{t'=0}^{t-1} \beta_{t'} \right) u\left(C_t\right) + \left(\prod_{t=0}^{T-1} \beta_t \right) \left(\overline{L} + B_T\right), \quad (61)$$

where $\beta_0 = 1$. By iterating on (59) one can write the final period foreign assets as

$$B_T = \sum_{t=1}^{T-1} \left(\prod_{t'=t}^{T-1} R_{t'} \right) X_t.$$

Using this equation to substitute out B_T from equation (61), and dropping the unimportant constant term in $\overline{L}$, gives equation (39) in the text.

Equilibrium conditions. Equation (19) is replaced by

$$S_t = \frac{1+i_t}{R_t\left(1+\tau_t^b\right)(1+\pi_{t+1})} S_{t+1}, \tag{62}$$

Iterating forward on this equation and using $S_T = 1$ gives equation (41).

The Euler equation is

$$u'(C_t)\left(S_t^m\right)^{\alpha_F} = \beta_t \frac{1+i_t}{1+\pi_{t+1}} u'(C_{t+1})\left(S_{t+1}^m\right)^{\alpha_F}, \tag{63}$$

in any period $t < T-1$, and

$$u'(C_{T-1})\left(S_{T-1}^m\right)^{\alpha_F} = \beta_{T-1}\frac{1+i_{T-1}}{1+\pi_T},$$

in period $T-1$. Iterating forward on (63) then gives equation (42).

Proof of Proposition 9. It follows from (41) and (42) that S_t and C_t can be written

$$S_t = \kappa_t^S \frac{1+i_t}{1+\tau_t^b}, \tag{64}$$

$$C_t = \kappa_t^C (1+i_t)^{-\alpha_H \sigma} (1+\tau_t^m)^{-\alpha_F \sigma}\left(1+\tau_t^b\right)^{-\alpha_F \sigma}, \tag{65}$$

where $\kappa_t^S \equiv \frac{S_{t+1}}{R_t(1+\pi_{t+1})}$ and κ_t^c are taken as given by the social planner in period t, because they depend on variables that are either global or determined after t.

In each period t the social planner maximizes

$$V_t = u(C_t) + \left(\prod_{t'=t}^{T-1} \beta_{t'} R_{t'}\right) X_t,$$

over i_t, where C_t is given by (65), $X_t = (S_t)^{1-\gamma} C_{Ft}^W - \alpha_F (S_t)^{\alpha_H} C_t$ and S_t is given by (64). If $L_t < \overline{L}$ reducing i_t raises both C_t and X_t and so V_t. Hence either there is full employment or the ZLB constraint is binding. Equation (42) then implies that if there is full employment in all periods one must have

$$\prod_{t'=t}^{T-1} \beta_{t'} \frac{1+i_{t'}}{1+\pi^*} = 1,$$

for all t. This is consistent with the ZLB if and only if $\beta_t \leq 1+\pi^*$ for all t.

The following proposition generalizes Proposition 2 to the dynamic setting.

Proposition 10 *Any allocation* $(C_{Ht}, C_{Ft}, L_t, \pi_t)_{t=1,\dots,T}$ *achieved by policy path* $(i_t, \tau_t^m, \tau_t^x, \tau_t^b)_{t=1,\dots,T-1}$ *can also be achieved without export tax by policy path* $(i_t, \widetilde{\tau}_t^m, 0, \widetilde{\tau}_{jt}^b)_{t=1,\dots,T-1}$. *with*

$$\begin{aligned} 1+\widetilde{\tau}_t^m &= (1+\tau_t^m)(1+\tau_t^x), \\ 1+\widetilde{\tau}_t^b &= \frac{1+\tau_{t+1}^x}{1+\tau_t^x}\left(1+\tau_t^b\right). \end{aligned} \tag{66}$$

Proof. Let us denote with tilde the policy and terms of trade that yield the same allocation as the original policy but with a zero tax on exports. Given the inflation targeting rule, the allocation $(C_{Ht}, C_{Ft}, L_t, \pi_t)_{t=1,\dots,T}$ is entirely determined by the terms of trade relevant for imports and exports, $S_t^m = S_t/(1+\tau_t^m)$ and $S_t^x = S_t(1+\tau_t^x)$. Thus we must have $S_t^x = \widetilde{S}_t^x$ and $S_t^m = \widetilde{S}_t^m$ for $t = 1, \dots, T-1$. Since there is no export tax in the tilde allocation one has $\widetilde{S}_t^x = \widetilde{S}_t$, which together with $\widetilde{S}_t^x = S_t^x$ imply $\widetilde{S}_t = S_t(1+\tau_t^x)$. Then $\widetilde{S}_t^m = \widetilde{S}_t/(1+\widetilde{\tau}_t^m) = S_t(1+\tau_t^x)/(1+\widetilde{\tau}_t^m)$ and $\widetilde{S}_t^m = S_t^m = S_t/(1+\tau_t^m)$ imply $1+\widetilde{\tau}_t^m = (1+\tau_t^m)(1+\tau_t^x)$ as reported in the Proposition.

The fact that all the variables except i_t in equation (63) are unchanged by the alternative policy implies that i_t is unchanged too, that is, $\widetilde{i}_t = i_t$. Then dividing (62) by the corresponding relationship in the tilde equilibrium,

$$\widetilde{S}_t = \frac{1+\widetilde{i}_t}{R_t\left(1+\widetilde{\tau}_t^b\right)(1+\widetilde{\pi}_{t+1})}\widetilde{S}_{t+1},$$

and using $\widetilde{i}_t = i_t$, $\widetilde{\pi}_{t+1} = \pi_{t+1}$, $\widetilde{S}_t = S_t(1+\tau_t^x)$ and $\widetilde{S}_{t+1} = S_{t+1}\left(1+\tau_{t+1}^x\right)$ gives the expression for $\widetilde{\tau}_t^b$ stated in the Proposition. ■

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