Optimal Macroprudential and Monetary Policy in a Currency Union*

Dmitriy Sergeyev†

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Abstract

I solve for optimal macroprudential and monetary policies for members of a currency union in an open economy model with nominal price rigidities, demand for safe assets, and collateral constraints. Monetary policy is conducted by a single central bank, which sets a common interest rate. Macroprudential policy is set at a country level through the choice of reserve requirements. I emphasize two main results. First, with asymmetric countries and sticky prices, the optimal macroprudential policy has a country-specific stabilization role beyond optimal regulation of financial sectors. This result holds even if optimal fiscal transfers are allowed among the union members. Second, there is a role for global coordination of country-specific macroprudential policies. Without coordination, members of the union face tighter financial and monetary policies. These results build the case for coordinated macroprudential policies that go beyond achieving financial stability objectives.

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†Bocconi University, Department of Economics, e-mail: dmytro.sergeyev@unibocconi.it

1 Introduction

Macroprudential regulation—policies that target financial stability by emphasizing the importance of general equilibrium effects—has become an important tool of financial regulation in recent years (Hanson *et al.*, 2011). For example, the 2010 Basel III accord, an international regulatory framework for banks, introduced a set of tools that require financial firms to hold larger liquidity and capital buffers, which could depend on the credit cycle (BSBC, 2010).

Macroprudential regulation may be in conflict with traditional monetary policy that stabilizes inflation and output (Stein, 2013, 2014). On the one hand, variation in the monetary policy rate shapes private incentives to take on risks, use leverage, and short-term debt financing. On the other hand, changes in macroprudential regulation constrain financial sector borrowing, which affects aggregate output.

In contrast, in a currency union, regional macroprudential policies may help achieve traditional monetary policy objectives. Monetary policy cannot fully mitigate asymmetric shocks in a currency union, because fixed nominal exchange rate and a single monetary policy rate are constraints that prevent full stabilization. Macroprudential regulation at a regional level can help smooth asymmetric shocks due to its ability to affect local economic activity.

The goal of this paper is to solve for optimal union-wide monetary and regional macroprudential policies in an environment where these policies interact. I address the optimal policy problem by solving a model that combines a standard New Keynesian model with a recent literature on macroprudential regulation of the financial sector, which I then extend to a currency union setting.

The first step is to define a fundamental market failure that justifies policy interventions. I set up a model, which is a variant of the model proposed in Stein (2012), with the following key features. Households value safe securities above and beyond their pecuniary returns because these securities are useful for transactions. This is formally introduced via a safe-assets-in-advance constraint. Financial firms can manufacture a certain amount of these securities by posting durable goods as collateral. The resulting endogenous collateral constraint on safe debt issuance, which features durable goods price, leads to a negative pecuniary externality (a fire-sale externality). Financial firms issue too many safe securities, which leads to social welfare losses. This provides a role for macroprudential policy to limit issuance of safe debt by financial firms.

Financial regulation can address this externality using a number of tools.[1] In this paper, I study reserve requirements (with interests paid on reserves) applied universally

[1]See Claessens (2014) for a recent review of various macroprudential tools.

to all riskless liabilities of all financial firms. I follow Kashyap and Stein (2012) and Woodford (2011), who argue that this tool can address financial stability concerns in a closed economy. The universal reserve requirements studied in this paper resemble traditional reserve requirements and the liquidity coverage ratio, introduced in the Basel III accord. Traditional reserve requirements policy orders banks to keep a minimum amount of central bank reserves relative to their deposits. The liquidity coverage ratio broadens the scope of traditional reserve requirements by obliging various types of financial firms (and not just traditional banks) to hold a minimum amount of liquid assets relative to various liabilities, and not just deposits. The macroprudential policy tool in this paper differs from the liquidity coverage ratio, in that financial firms are required to hold central bank reserves only.

In a closed economy version of the model, optimal monetary and macroprudential policies are not in conflict. Optimal monetary policy achieves the flexible price allocation, and optimal macroprudential policy only corrects the fire-sale externality. However, if any of the two policies is suboptimal, there is a scope for the other policy to address both inefficiencies.

I extend the model to a currency union setting along the lines of Obstfeld and Rogoff (1995) and Farhi and Werning (2013). Households have preferences over traded and non-traded goods. The safe-assets-in-advance constraint is applied to traded goods. Durable goods are produced by local financial firms out of non-traded goods. The last assumption allows local macroprudential policy to affect output of non-traded goods. Only safe securities are traded internationally.

I emphasize two main results in this paper. First, optimal macroprudential policy is used to stabilize business cycles. When monetary and macroprudential policies are set optimally in a coordinated way across monetary union members, optimal macroprudential policy is country-specific, and it depends on the amount of slack in a country. Optimal monetary policy sets average across countries labor wedge to zero. However, the central bank cannot replicate flexible price allocation in each country. This provides room for regional financial policy to stabilize local shocks. Optimal macroprudential policy trades off its financial stability objective, mitigation of the pecuniary externality, and stabilization of inefficient business cycle fluctuations due to presence of sticky prices.

Optimal macroprudential policy is used to stabilize business cycles even when fiscal transfers are allowed among the union members, and these transfers are set optimally. Optimal fiscal transfers equalize the social marginal value of traded goods across countries. However, in general, the fiscal transfers cannot achieve a flexible price allocation in every country. As a result, macroprudential policy are partly used to stabilize inefficient business cycle fluctuations. This result emphasizes that optimal regional macroprudential policy must be directed toward business cycle stabilization even when some regional stabilization tools are available.

The second main result underscores the benefits of global coordination of regional macroprudential policies. Tighter macroprudential policy in a given country reduces the supply of safe assets and pushes the international real interest rate on safe assets down. This decline in the interest rate affects the other economies in the union through two channels. First, it increases the marginal utility of consumption which, in turn, reduces the price of durable goods and tightens the collateral constraint in the other countries. This is a negative spillover of local macroprudential policy because it raises the cost of financing for banks in the other countries. Second, when the nominal interest rate in the union is fixed, for example, due to a binding zero-lower bound constraint on the interest rate, a lower real interest rate requires the current price of traded goods to decline. This leads to expenditure switching from non-traded to traded goods reducing production of non-traded goods. When the labor wedge is positive (a country is in a recession), this effect leads to first order welfare losses resulting in a second negative spillover of macroprudential policy. If a union-wide regulator sets local financial policies and union-wide monetary policy, it takes into account these spillovers.

If, however, the regional financial policies are set in a non-cooperative fashion, the union-wide central bank fosters coordination by generating a union-wide recession. Intuitively, any global regulator who internalizes international spillovers prefers to set looser macroprudential policies than independent local regulators. The central bank cannot directly manage local financial policies. It can, however, indirectly influence the choice of local regulators by affecting business cycles to which regulators react. To make local regulators relax their policies, the central bank engineers a union-wide recession. As a result, when macroprudential and monetary policies are not set in a coordinated way, the members of monetary union face tighter financial and monetary policies than in the case of coordinated solution.

Related Literature. The elements of the model are related to several strands of literature. The model builds on the recent paper by Stein (2012) who argues that the fire-sale externality creates a role for macroprudential interventions.[2] The idea that it is useful to use safe and liquid securities for transactions, and the financial sector can create such securities, is rationalized in Gorton and Pennacchi (1990); Dang *et al.* (2012). Woodford (2011) introduces a model similar to Stein (2012) into a standard closed-economy New

[2] Stein (2012) relies on the earlier literature which emphasizes fire-sales. See, for example, Shleifer and Vishny (1992); Gromb and Vayanos (2002); Lorenzoni (2008). A number of recent papers suggested that a system of Pigouvian taxes can be used to bring financial sector incentives closer to social interests (Bianchi, 2011; Jeanne and Korinek, 2010).

Keynesian model and shows that optimal monetary policy must partly address the fire-sale externality when macroprudential policy is suboptimal. In a related model, Caballero and Farhi (2015) show that unconventional monetary policy can be more effective than traditional monetary policy in fighting the shortage of safe assets. In this paper, I extend the model with a special role for safe assets to an international setting following the New Open Economy Macro literature.[3] I build on the models of Obstfeld and Rogoff (1995) and Farhi and Werning (2013).

The results in this paper are connected to four strands of literature. First, the optimal currency area literature deals with the inability of traditional monetary policy to fully stabilize asymmetric shocks in a currency union. This literature proposes that factors mobility (Mundell, 1961), higher level of openness (McKinnon, 1963), and fiscal integration (Kenen, 1969) are necessary for stabilization of asymmetric shocks. More recent contributions emphasize the importance of regional fiscal purchases (Beetsma and Jensen, 2005; Gali and Monacelli, 2008), distortionary fiscal taxes (Ferrero, 2009), fiscal transfers (Farhi and Werning, 2013), and capital controls (Schmitt-Grohe and Uribe, 2012). Adao *et al.* (2009); Farhi *et al.* (2014) show that with a sufficient number of fiscal tools the flexible price allocation can be achieved in a monetary union. However, it is possible that a sufficient number of policy tools is not available to policy makers. The current paper complements this literature by analyzing regional macroprudential policy as a potential macroeconomic stabilization tool.

Second, a number of papers solve for monetary and macroprudential policies in a closed economy environment with aggregate demand externality due to nominal rigidities and pecuniary externality due to collateral constraints. Farhi and Werning (forthcoming) and Korinek and Simsek (2016) present models in which constrained monetary policy and the presence of the aggregate demand externality provide an active stabilization role for macroprudential policy. The authors show that when the endogenous collateral constraints are present in these models, macroprudential policy trades off the mitigation of pecuniary externality and the stabilization of aggregate economy due to aggregate demand externality. In a model with aggregate demand and pecuniary externalities, Woodford (2016) compares optimal monetary, macroprudential, and quantitative easing policies and concludes that quantitative easing policy can be a useful policy tool even when the zero lower bound constraint does not bind. Cesa-Bianchi and Rebucci (2016) analyze monetary and macroprudential policy in a model with nominal rigidities, collateral constraints, and monopolistic competition friction in the banking sector. In contrast, in this paper, I study a model with aggregate demand and pecuniary externalities extended to a

[3] See, for example, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Benigno and Benigno (2003) for early contributions and Corsetti *et al.* (2010) for a recent overview.

monetary union setting.

Third, there is a growing literature that studies macroprudential and monetary policy in a small open economy. Benigno *et al.* (2013), Bianchi (2011), Jeanne and Korinek (2010) study macroprudential capital controls in models where foreign borrowing by a country is limited by a collateral constraint. Fornaro (2012) compares different exchange rate policies, and Ottonello (2013) solves for optimal exchange rate policy in a model with wage rigidity and occasionally binding borrowing constraints. Otrok *et al.* (2012) compare different monetary and macroprudential policies in an environment with sticky prices and collateral constraints. Farhi and Werning (forthcoming) solves for optimal capital control and monetary policy under sticky prices and collateral constraints in a small open economy. In my environment, there is an explicit financial sector that can be a source of the fire-sale externality even without international capital flows. This allows me to separate capital controls and financial sector regulation policies. In addition, I am interested in deriving optimal policy in a currency union instead of a small open economy.

Finally, there are papers that address joint conduct of monetary and macroprudential policies in a currency union. Beau *et al.* (2013) and Brzoza-Brzezina *et al.* (2015) compare effects of several specifications of monetary and macroprudential policies on macroeconomic variables, and Rubio (2014) does it for welfare. Quint and Rabanal (2014) solve for the best monetary and macroprudential policies in the class of simple policy rules that are predetermined functions of macroeconomic variables. In this paper, I solve for optimal monetary and macroprudential policies and derive implications for coordination of these policies.

The rest of the paper is organized as follows. Section 2 presents a closed economy model with sticky prices and nonpecuniary demand for safe assets. Section 3 extends the model to a currency union setting. Section 4 concludes.

2 A 2-period Closed Economy Model

I first present a closed-economy model that introduces specific modeling assumptions in the most transparent way. Section 3 extends the model to a multi-country setting.

The economy lasts for two periods, $t = 0, 1$. Uncertainty affects only preferences over durable goods in period 1, the state of the world is denoted by s_1 (all endogenous variables in period 1 can depend on s_1). There are three types of goods in the economy: durable goods, (final) consumption goods and a continuum of differentiated intermediate goods. The economy is populated by a continuum of identical multi-member households with a unit mass, a continuum of final-good producing firms with a unit mass and who only operate in the initial period 0, and the government. Any state-contingent security is traded between periods 0 and 1.

2.1 Households

Each household consists of four types of agents: a firm, a banker, a consumer and a worker.[4] Household preferences are

$$\mathbb{E}\Big\{u(c_0) - v(n_0) + \beta\Big[\mathbb{U}(c_1, \underline{c}_1) + X_1(s_1)g(h_1)\Big]\Big\} \tag{1}$$

where n_0 is labor supply in t; c_t is consumption which can be bought on credit in $t = 0, 1$; $\underline{c}_1$ is consumption in period 1 that can be bought with safe assets only, h_1 is consumption of durable goods.[5] $u(\cdot)$ is strictly increasing and concave, $v(\cdot)$ is strictly increasing and convex, $g(\cdot)$ is strictly increasing, concave, and $\sigma_g \equiv -g''(h_1)\,h_1/g'(h_1) < 1$. Random variable $X_1(s_1)$ takes on two values $X_1 \in \{1, \theta\}$ with corresponding probabilities μ and $1 - \mu$. Utility from consumption of perishable goods in period 1 is given by

$$\mathbb{U}(c_1, \underline{c}_1) = u[c_1 + (1 + \nu)\underline{c}_1],$$

where ν is the parameter that controls the demand for goods bought with safe securities. When $\nu = 0$, the utility $\mathbb{U}(\cdot)$ depends only on total consumption of the household.

A worker competitively supplies n_0 units of labor and receives income $W_0 n_0$, where W_0 is the nominal wage in period 0. A firm is a monopolist and it uses a linear technology to produce differentiated good j

$$y_0^j = A_0 n_0^j,$$

The firm hires labor on a competitive market at nominal wage W_0, but pays $(1 + \tau_0^L)W_0$, where τ_0^L is the labor tax (or subsidy if it is negative). Price P_0^j of a differentiated good in period 0 is sticky. I do not explicitly model the reason for price P_0^j stickiness. One can assume that firms set prices before period 0 anticipating certain economic conditions that potentially turn out being different in period 0. The final goods producer's demand for each variety is $y_0(P_0^j/P_0)^{-\epsilon}$, where $P_0 = (\int (P_0^j)^{1-\epsilon} dj)^{1/(1-\epsilon)}$ is the price of final goods.

[4]The multi-member household construct allows to study situations in which different agents have different trading opportunities but keeps the simplicity of the representative household. See, for example Lucas (1990).

[5]The fact that preferences are not symmetric over the two periods is without loss of generality. Assuming that household enters period 0 with an endowment of safe assets and endowment of durable goods allows to make preferences symmetric without changing the results.

The profits of the firm producing variety j is

$$\Pi_0^j = \left(P_0^j - \frac{(1+\tau_0^L)\,W_0}{A_0}\right) y_0 \left(\frac{P_0^j}{P_0}\right)^{-\epsilon}.$$

There is exogenous endowment of final goods in period 1. One could assume, instead, that the same firms produce intermediate output from labor also in period 1, but that the prices for these goods are flexible. This complication does not add new economic insights.

A banker buys k_0 units of final goods in period 0 and immediately produces $h_1 = G(k_0)$ units of durable goods that he sells to consumers in the next period at flexible nominal price $\Gamma_1(s_1)$. To finance the purchase of final goods the banker issues safe bonds with face value D_1^b, and receives $D_1^b/(1+i_0)$, where i_0 is the nominal interest rate on safe bonds. In addition to safe debt, the banker can issue any state contingent security, including equity. The banker is required that a fraction z_0 of his safe liabilities is covered by central bank reserves. Formally, the banker buys R_1^b reserves by paying $R_1^b/(1+i_0^r)$, where i_0^r is the interest rate on reserves to satisfy

$$z_0 \leq \frac{R_1^b}{D_1^b}. \tag{2}$$

For safe debt to be safe, it must repay in full even in the worst possible state of the economy in period 1. Formally,

$$D_1^b \leq \min_{s_1}\{\Gamma_1(s_1)\}G(k_0) + R_1^b,$$

where $\min_{s_1}\{\Gamma_1(s_1)\}$ is the smallest possible price of durable goods in period 1.

A consumer choose financial portfolio of the household: he buys any state-contingent security that bank issues, and acquires D_1^c safe bonds by paying $D_1^c/(1+i_0)$. A consumer also buys final goods c_t on credit in both periods, and final goods $\underline{c}_1$ in period 1 with safe assets. Formally,

$$P_1\underline{c}_1 \leq D_1^c, \tag{3}$$

where P_1 is the nominal price in $t = 1$. This inequality states that consumption $\underline{c}_1$ must be purchased using risk-free assets D_1^c.[6] There is a long tradition in macroeconomic literature to assume that part of consumption goods must be bought with nominal liabilities of a central bank (Svensson, 1985; Lucas and Stokey, 1987) because of transaction frictions. I assume that not only central bank liabilities, but also other safe assets can be used to

[6] In this simple model, there is not going to be inflation risk. Thus it is not necessary to specify if the securities must be safe in real or nominal terms.

purchase these goods. These securities include government and private safe bonds.[7] By analogy with the famous cash-in-advance constraint, I call constraint 3 a *safe-assets-in-advance* (SAIA) constraint.

Consolidated household budget constraints in periods 0 and 1 are

$$T_0 + P_0 c_0 + \frac{D_1^c}{1+i_0} + \frac{R_1^b}{1+i_0^r} + P_0 k_0 \leq \frac{D_1^b}{1+i_0} + W_0 n_0 + \Pi_0^j,$$
$$P_1(c_1 + \underline{c}_1) + T_1 + \Gamma_1 h_1 + D_1^b \leq D_1^c + R_1^b + \Gamma_1 G(k_0) + P_1 y_1,$$

where P_0 is the price of perishable goods in period 0; T_0,T_1 are lump-sum taxes.[8]

If the interest rate on reserves is strictly smaller than the interest rate on other safe securities ($i_0^r < i_0$), the bankers optimally choose not to hold more reserves than required ($R_1^b = z_0 D_1^b$). As a result, the collateral constraint and the budget constraints in both periods can be written as

$$\widetilde{D}_1^b \leq \min_{s_1}\{\Gamma_1(s_1)\} G(k_0), \tag{4}$$
$$T_0 + P_0 c_0 + \frac{D_1^c}{1+i_0} + P_0 k_0 \leq \frac{\widetilde{D}_1^b}{1+i_0}(1-\tau_0^b) + W_0 n_0 + \Pi_0^j, \tag{5}$$
$$P_1(c_1 + \underline{c}_1) + T_1 + \Gamma_1 h_1 + \widetilde{D}_1^b \leq D_1^c + W_1 n_1 + \Gamma_1 G(k_0) + P_1 y_1, \tag{6}$$

where $\widetilde{D}_1^b \equiv D_1^b - R_1^b$ is bankers safe debt liabilities net of reserves deposited at the central bank, and $\tau_0^b \equiv z_0/(1-z_0) \cdot (i_0 - i_0^r)/(1+i_0^r)$. Constraints (4)-(6) do not separately depend on i_0^r and z_0 but only through their combination expressed by τ_0^b, which can be interpreted as the Pigouvian tax on safe debt issuance. I will call τ_0^b a *macroprudential tax*. If the interest rate on reserves equals to the interest rate on other safe securities ($i_0^r = i_0$), a banker may choose to hold excess reserves in which case the reserve requirements constraint does not bind, but the constraints faced by the household are still identical to (4)-(6) with $\tau_0^b = 0$.

A household maximizes (1) subject to (3)-(6) by choosing consumption $c_0, c_1, \underline{c}_1$, h_1, safe debt position $D_1^c, \widetilde{D}_1^b$, labor supply n_0, investment in production of durable goods k_0 and price P_1^j (price P_0^j is exogenously fixed).

Household optimality conditions with respect to consumption, portfolio choice, and

[7]See Krishnamurthy and Vissing-Jorgensen (2012a,b) for evidence that the U.S. treasuries and some financial sector liabilities command both safety and liquidity premia.

[8]Note that this representation of the budget constraint does not feature state-contingent securities issued by banks and state-contingent securities bought by the consumers. This is without loss of generality because bankers and consumers are members of multi-member households. It can be thought that bankers issue state-contingent securities within its household.

labor supply are

$$
\begin{aligned}
u'(c_0) =& (1+\nu)(1+i_0)\beta\mathbb{E}_0\left[\frac{P_0}{P_1}u'[c_1+(1+\nu)\underline{c}_1]\right], && (7)\\
\frac{\Gamma_1(s_1)}{P_1} =& \frac{X_1(s_1)g'(h_1)}{u'[c_1+(1+\nu)\underline{c}_1]}, && (8)\\
\frac{W_0}{P_0} =& \frac{v'(n_0)}{u'(c_0)}, && (9)
\end{aligned}
$$

Equation (7) is the Euler equation for safe bonds. It is similar to the standard Euler equation except for the presence of the term $(1+\nu)$ in the square brackets. I will call ν a safety yield. This term introduces a wedge in the safe bonds Euler equation. This effectively adds more discounting to the model.[9]

Equation (8) is the demand for durable goods: period 1 real price of durable goods equals the ratio of durable goods marginal utility over perishable goods marginal utility. The optimality condition (9) is the labor supply.

The Lagrange multiplier on the SAIA constraint expressed in units of utility is $\eta_1 = \nu$. The bankers' optimal choice of investment in durable goods production and issuance of safe assets implies

$$
u'(c_0) = \beta G' k_0 \mathbb{E}_0 u'[c_1+(1+\nu)\underline{c}_1]\left(\frac{\Gamma_1}{P_1}+\zeta_0\frac{\min_{s_1}\{\Gamma_1\}}{P_1}\right), \tag{10}
$$

where $\zeta_0 \geq 0$ is the Lagrange multiplier on the collateral constraint (4) expressed in units of utility. Optimality condition (10) equates the cost of using one unit of consumption good, marginal utility of consumption, to the marginal benefit of investing that consists of two parts. First, a unit of investment produces $G'(k_0)$ units of durable goods which are sold to households at real price Γ_1/P_1. Second, after investing a unit into durable goods production, the banker relaxes the collateral constraint (6). The benefit of relaxing is proportional to the Lagrange multiplier ζ_0.

The bankers' optimal choice of safe debt issuance relates the Lagrange multiplier to other equilibrium objects as follows

$$
(1+\zeta_0)\mathbb{E}_0\frac{u'[c_1+(1+\nu)\underline{c}_1]}{P_1} = \frac{1-\tau_0^B}{1+i_0}\cdot\frac{u'(c_0)}{P_0}. \tag{11}
$$

[9]The recent literature, for example, Giannoni *et al.* (2015), McKay *et al.* (2015), has emphasized that the standard New Keynesian models greatly overstate the impact of announcements about future monetary policy — the forward guidance puzzle. Campbell *et al.* (2016) show that in a New Keynesian model augmented with non-pecuniary preferences for safe and liquid bonds and calibrated to the U.S. data, the strength of forward guidance policy is similar to the one found in empirical studies. These results from additional discounting in the Euler equation due to the wedge introduced by the liquidity preferences.

This expression equates the marginal cost of issuing a unit of safe debt (the left-hand side) to the benefit of raising additional funds in period 0 (the right-hand side). The marginal cost consists of two parts: the cost of repayment in period 1 and the costs of making the collateral constraint tighter. By changing the macroprudential tax τ_0^B, the financial regulator affects the marginal cost of safe debt issuance, which affects the incentives to invest in durable goods production. This, in turn, changes the aggregate demand in the economy.

2.2 Final Goods Firms

Final goods are produced by competitive firms that combine a continuum of varieties $j \in [0,1]$ using the CES technology

$$y_0 = \left(\int y_0^{j\,\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

with elasticity $\epsilon > 1$. Each firm solves in $t = 0$

$$\max_{y_0^j} P_0 y_0 - \int P_0^j y_0^j dj,$$

Optimal choice of inputs leads to differentiated goods demand $y_0^j = y_0 (P_0^j / P_0)^{-\epsilon}$ and the aggregate price index is defined as $P_0 = (\int (P_0^j)^{1-\epsilon} dj)^{1/(1-\epsilon)}$.[10]

2.3 Government

The government consists of a central bank and a treasury, and a financial regulator, who may or may not be a part of the central bank.

Financial regulator. The financial regulator chooses the level of reserve requirements z_0 and the interest on reserves i_0^r, which is equivalent to choosing the macroprudential tax τ_0^b. It rebates the proceeds to the fiscal authority.

Central bank. The central bank sets the nominal interest rate i_0 on safe assets and targets inflation rate $\Pi^* = P_1 / P_0$ which is assumed not to depend on state s_1. To motivate

[10]It must be noted that the formulation of final goods firm's problem implicitly assumes that they sell goods at a single price to those who buy goods on credit and to those who buy goods with safe assets. This assumption rules out an equilibrium in which final goods producers sell their output at different prices to those who buy with credit and to those who buy with safe assets.

the monetary authority control over the nominal price level in period 1 and the nominal interest rate between periods 0 and 1, one can assume that fraction $\kappa \in [0,1]$ of purchases has to be made with cash M_0, M_1 that does not pays zero nominal interest rate.[11] Formally, $\kappa P_0 c_0 = M_0, \kappa P_1(c_1 + \underline{c}_1) = M_1$. Because consumers want to economize on cash holdings when the safe nominal interest rate is positive, there is demand for cash which depends on nominal interest rate. By setting nominal interest rate i_0, the central bank is ready to satisfy any demand for cash in period 0. The price level in period 1 is $P_1 = M_1/[\kappa(c_1 + \underline{c}_1)]$. When announcing the price level for period 1, the central bank adjusts M_1 to keep the price level fixed at the announced level. Allowing κ and M_0, M_1 to go to zero so that ratios $M_0/\kappa, M_1/\kappa$ stay positive and finite, the government determines price P_1, but there is no need to explicitly consider equilibrium in the cash market. This limit is sometimes called a cashless economy.

Treasury. The treasury sets lump sum taxes T_0, T_1, and proportional labor tax τ_0^L.

The government issues D_1^g of safe securities. This amount consists of safe government bonds and the reserves purchased by the banks. Note that under the assumption that the financial regulator sets the interest rate on reserves i_0^r and the reserve requirement z_0, the quantity of reserves must adjust to satisfy banks reserves demand. The identity of the authority that issues public safe securities does not matter for equilibrium as long as the consolidated government budget constraint is satisfied. However, it matters whether the overall amount of public safe securities D_1^g reacts to changes in the economy. For example, if the reserve requirement constraint binds ($R_1^b = z_0 D_1^b$), the total amount of reserves demanded by the banks is an endogenous variable, which may affect the overall public safe securities supply. I assume that the government targets the overall public safe securities supply D_1^g, which implies that any equilibrium variation in the amount of outstanding reserves is offset by the mirror change in the supply of safe public bonds.

The consolidated government budget constraints in both periods are

$$0 = T_0 + \tau_0^b \frac{\widetilde{D}_1^b}{1+i_0} + \frac{D_1^g}{1+i_0} + \tau_0^L W_0 n_0, \tag{12}$$

$$D_1^g = T_1. \tag{13}$$

2.4 Auxiliary Variables

It will prove useful to introduce a number of new variables which will simplify notation. $d_1^c \equiv D_1^c/P_1$ is the real household demand for safe assets, $\widetilde{d}_1^b \equiv \widetilde{D}_1^b/P_1$ is the real private supply of safe assets by bankers, $d_1^g \equiv D_1^g/P_1$ is the real public supply of safe assets,

[11]See Mankiw and Weinzierl (2011) for similar treatment of a monetary policy in a 2-period model.

$\gamma_1 \equiv \Gamma_1/P_1$ is the price of durable goods expressed in units of period-1 consumption goods, $w_0 \equiv W_0/P_0$ is the real wage, and $r_0 \equiv (1+i_0)P_0/P_1$ is the safe real interest rate. Let us define the durable goods price elasticity as

$$\sigma_g \equiv -\frac{\partial \log \gamma_1}{\partial \log h_1} = -\frac{g''(h_1)h_1}{g'(h_1)}.$$

The elasticity is positive and, as was assumed earlier, less than one. It can depend on the durable goods consumption. The labor wedge is defined as

$$\tau_0 \equiv 1 - \frac{v'(n_0)}{A_0 u'(c_0)}.$$

The labor wedge is zero when a marginal benefit of consumption equals a marginal cost of working. The labor wedge equals zero if prices are flexible (economy is stabilized). It is positive when equilibrium labor and consumption are too small (a recession). It is negative when labor and consumption are too high (a boom).

2.5 Equilibrium

An equilibrium specifies consumption $c_0, c_1, \underline{c}_1$, labor n_0, investment in durable goods k_0, durable goods consumption h_1, real safe debt supply by bankers and the government $\widetilde{d}_1^b, d_1^g$, safe debt demand d_1^c by consumers, real wage w_t, nominal interest rate i_0, government lump-sum taxes T_0, T_1, labor taxes τ_0^L, and macroprudential tax τ_0^b such that households and firms maximize, the government budget constraints are satisfied as equalities in every period, final goods markets clear

$$k_0 + c_0 = A_0 n_0, \tag{14}$$
$$c_1 + \underline{c}_1 = y_1, \tag{15}$$

durable goods market clears

$$h_1 = G(k_0), \tag{16}$$

and safe assets market clears

$$d_1^c = \widetilde{d}_1^b + d_1^g. \tag{17}$$

The complete set of equilibrium conditions (4)-(17) can be simplified as follows. First, because the SAIA constraint binds in equilibrium, consumption bought with safe assets $\underline{c}_1$ is determined by the amount of safe assets acquired in the previous period. This implies that consumption c_1 and $\underline{c}_1$ do not depend on the realization of preferences over durable goods $X_1(s_1)$. Second, the only period-1 endogenous variable that depends on realization of s_1 is the price of durable goods. Equation (8) implies that there are only two possible realizations of the price: $\gamma(X_1 = 1) = g'(h_1)/u'(y_1^*)$ and $\gamma(X_1 = \theta) = \theta g'(h_1)/u'(y_1^*) = \theta\gamma(X_1 = 1)$.

It is intuitive that allocations in period 1 do not depend on the realization of state s_1. Realization of s_1 directly affects durable goods price γ_1 by changing the marginal utility of durable goods. However, the realization of this price only redistributes resources between bankers and consumers. Because bankers and consumers belong to a large household, they effectively pool their resources together in the end of period 1. Thus, the allocation in period 1 is not affected.

The SAIA constraint binds in equilibrium ($\underline{c}_1 = \widetilde{d}_1^b + d_1^g$) because marginal utility of consumption bought with safe assets is always positive. Taking this into account, Euler equation (7) links three endogenous variables: consumption c_0, real interest rate on safe debt $(1 + i_0)/\Pi^*$ and safe debt supply $\widetilde{d}_1^b$:

$$u'(c_0) = \beta(1+\nu)\frac{1+i_0}{\Pi^*}u'[y_1 + \nu(\widetilde{d}_1^b + d_1^g)], \tag{18}$$

Household demand for durable goods (8), bankers' choice of investment in durable goods (10), and durable goods market clearing condition (16) lead to

$$u'(c_0) = \beta g'[G(k_0)]G'(k_0)[\mu + (1-\mu)\theta + \zeta_0\theta], \tag{19}$$

where multiplier ζ_0, given by (11), can be rewritten taking into account safe assets Euler equation (18) as follows

$$\zeta_0 = (1 - \tau_0^b)(1+\nu) - 1.$$

The last expression implies that whenever $\tau_0^b > \nu/(1+\nu)$, the collateral constraint becomes slack. Next, the collateral constraint (4) can be expressed in real terms, taking into account equilibrium durable goods price (8), as follows

$$\widetilde{d}_1^b \leq \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1 + \nu(\widetilde{d}_1^b + d_1^g)]}. \tag{20}$$

Note that the minimal real durable good price $\gamma_1 = \theta g'[G(k_0)]/u'[y_1 + \nu(\widetilde{d}_1^b + d_1^g)]$ depends on the level of investment in durable goods production. This price in the collateral constraint is a source of pecuniary externality that affects welfare. I will call it a fire-sale externality.

Equations (18)-(20), together with complementarity slackness conditions on the last inequality, describe equilibrium and determine there unknowns $c_0, k_0, \widetilde{d}_1^b$.

2.6 Ramsey Planning Problem

The financial regulator and the central bank face all of the equilibrium equations (4)-(17) as constraints when choosing their optimal policies. The full system of equilibrium conditions was reduced to system (18)-(20).[12] Note that the full set of equilibrium conditions can be unambiguously recovered from (18)-(20).

Following the public finance literature (Lucas and Stokey, 1983), I further drop certain variables and constraints from the optimal policy problem. First, given quantities $c_0, k_0, \widetilde{d}_1^b$, optimal choice of investment in durable goods (19) can be dropped because it can be used to express optimal macroprudential tax τ_0^b when the collateral constraint binds. We will see that whenever the planner's optimal choice of investment in durable goods does not lead to binding collateral constraints, the planner's optimum will coincide with the private one. Finally, (18) can be dropped because it can be used to back out the nominal interest rate.

Proposition 1. *An allocation $c_0, k_0, \widetilde{d}_1^b$ form part of an equilibrium if and only if condition* (20) *holds.*

I now solve the Ramsey problem by choosing the competitive equilibrium that maximizes the social welfare. Formally, the planner solves

$$\max_{\{c_0,k_0,\widetilde{d}_1^b\}} u(c_0) - v\left(\frac{c_0+k_0}{A_0}\right) + \beta\left\{u[y_1+\nu(\widetilde{d}_1^b+d_1^g)] + [\mu+(1-\mu)\theta]g[G(k_0)]\right\}$$
$$s.t.: \quad \widetilde{d}_1^b \leq \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}.$$

The form of the planner's objective takes into account that only preferences over durable goods depend on realization of s_1. Specifically, $\mathbb{E}X_1(s_1) = \mu + (1-\mu)\theta$. The planner's optimal behavior leads to

$$\begin{aligned}
\tau_0 &= 0, \\
\frac{\widetilde{\zeta}_0}{u'(y_1+\nu\underline{c}_1)} &= \frac{\nu}{1+\frac{\nu\widetilde{d}_1^b u''(y_1+\nu\underline{c}_1)}{u'(y_1+\nu\underline{c}_1)}}, \\
u'(c_0) &= \beta g'[G(k_0)]G'(k_0)\left[\mu+(1-\mu)\theta+\frac{\widetilde{\zeta}_0\theta(1-\sigma_g)}{u'(y_1+\nu\underline{c}_1)}\right],
\end{aligned}$$

[12] In addition, the collateral and safe-assets-in-advance constraints are accompanied by the complementarity slackness conditions.

where $\widetilde{\zeta}_0$ is the Lagrange multiplier on the collateral constraint. The first equation states that the labor wedge equals zero (the economy is stabilized). The second line states that in planner's optimum the collateral constraints binds. Moreover, the multiplier on the collateral constraint is proportional to the safety wedge ν. The third equation is the regulator's choice of investment in durable goods production. Compared to private optimal choice of investment in durable goods (19), the planner's optimal condition reveals that she internalizes the impact of durable goods investment on future durable goods price. This is formally represented by term $1-\sigma_g$ on the right-hand side of the equation, as well as the term $\nu \widetilde{d}_1^b u''(y_1+\nu \underline{c}_1)/u'(y_1+\nu \underline{c}_1)$ in the expression for the Lagrange multiplier $\widetilde{\zeta}_0$. The first of these two terms reflects the externality that stems from the fact that a banker does not internalize all costs that he imposes on the other bankers when he issues more safe debt. Additional resources from issuing safe debt are invested in production of durable goods. Higher durables production reduces the minimal durable goods price that enters the collateral constraints of all the bankers. As a result, the collateral constraints are tightened for all bankers in the economy, which is the cost that the banker does not internalize. The second term is due to the fact that more safe debt reduces marginal utility of consumption $u'(y_1+\nu \underline{c}_1)$, and, hence, increases the price of durable goods relaxing the collateral constraint. This results in a positive externality.

I next characterize the implementation of the constrained efficient allocation. Comparison of planner's optimality with private optimality condition (19) leads to following result.

Proposition 2. *Constrained efficient allocation can be implemented by setting the macroprudential tax and the nominal interest rate so that*

$$\tau_0^b = \frac{\sigma_g \nu}{1+\nu} - \frac{(1-\sigma_g)\nu}{1+\nu} \cdot \frac{\nu \sigma \widetilde{d}_1^b}{y_1 + \nu[(1-\sigma)\,\widetilde{d}_1^b + d_1^g]} \text{ and } \tau_0 = 0. \tag{21}$$

When monetary and macroprudential policies are chosen optimally, the economy is stabilized (the labor wedge equals zero) and the financial regulation reduces welfare losses due to the pecuniary externalities. The first term of the macroprudential tax τ_0^b is proportional to durable goods price elasticity and the safety yield. When durable goods price elasticity σ_g is zero, the first term of the macroprudential tax is also zero because private investment decisions do not affect future price of durable goods. In addition, in the absence of safety wedge, $\nu = 0$, the planner also sets the macroprudential tax to zero. This is because the collateral constraint does not bind when the safety yield is zero. If the yield is not zero, its higher value leads to higher firs term of the macroprudential tax because safety yield is proportional to the social marginal cost of safe debt issuance. The second term of the macroprudential tax is proportional to the safety yield squared, making it potentially less important for realistic value of ν.

2.7 Macroprudential and Monetary Policy Interaction

The results in Proposition 2 do not depend on the fact that the two policies are set cooperatively. This is because both policies are chosen to maximize the same objective subject to the same constraints.[13] If the macroprudential policy is chosen optimally, it is optimal for the monetary policy to set flexible price allocation (the labor wedge is zero).[14] However, if one of the two policies is suboptimal, the other policy will have an additional role.

It is sometimes proposed that monetary policy should be directed towards financial stability objectives because macroprudential policy may not be chosen optimally. For example, Stein (2013, 2014) argues that some market participants may evade macroprudential regulation leading to inability of the financial regulator to set optimal policy. However, monetary policy has a universal effect on all market participants. Symmetrically, one can argue that sometimes monetary policy may not be set optimally, for example, due to the zero lower bound or because a country belongs to a monetary union, which precludes control over the nominal interest rate. In this case, the macroprudential policy should be directed toward the stabilization of inefficient business cycle fluctuations due to sticky prices. The model of this section can be used to analyze these two situations. The following proposition describes the optimal monetary policy when macroprudential policy is not set optimally and the optimal macroprudential policy when monetary policy is not set optimally.

Proposition 3. *(i) When the macroprudential tax is set at the fixed leveτ_0^b, optimal monetary policy is such that*

$$\tau_0 = -\frac{1}{Z_1}\left[\tau_0^b - \left(\frac{\sigma_g \nu}{1+\nu} - \frac{\nu(1-\sigma_g)}{1+\nu} \cdot \frac{\nu\sigma \widetilde{d}_1^b}{y_1 + \nu[(1-\sigma)\,\widetilde{d}_1^b + d_1^g]}\right)\right], \tag{22}$$

[13] De Paoli and Paustian (2013) show that there is a scope for coordination between the two policy choices when the objectives of monetary and macroprudential authorities differ.

[14] It must be noted that the separation between optimal monetary and macroprudential policies, i.e., the fact that monetary policy sets the labor wedge to zero and macroprudential policy corrects pecuniary externality, relies on the specific price setting assumption: all firms costlessly set prices one period ahead. In models that allow for costs of price setting (Rotemberg, 1982) or non-trivial price dispersion across firms (Calvo, 1983), these nominal rigidies lead to additional welfare losses. As a result, there can be a non-trivial policy trade-off even when both monetary and macroprudential policies are set optimally.

In addition, in a multi-period model there can be an interaction between current macroprudential and future monetary policy. Specifically, in a multi-period extenssion of the model, inequality (20) will have the following form $\widetilde{d}_{t+1}^b \leq \theta G(k_t) g'[G(k_t)]/u'(\widetilde{c}_{t+1})$. The future monetary policy will have a direct effect on this collateral constraint through its effect on consumption $\widetilde{c}_{t+1}$. If a policymaker can commit to ease future monetary policy, she can relax the current collateral constraint. As a result, the optimal monetary policy will depart from setting the labor wedge to zero.

where $Z_1 > 0$ is a variable that depends on the optimal allocation;
(ii) when monetary policy is such that $\tau_0 \neq 0$, optimal macroprudential tax is

$$\tau_0^b = \frac{\nu\sigma_g}{1+\nu} - \frac{\nu(1-\sigma_g)}{1+\nu} \cdot \frac{\nu\sigma\widetilde{d}_1^b}{y_1 + \nu[(1-\sigma)\,\widetilde{d}_1^b + d_1^g]} - \tau_0 Z_2, \tag{23}$$

where $Z_2 > 0$ is a variable that depends on the optimal allocation.

Proof and the formal expressions for Z_1, Z_2 are in Appendix A.1.2. The first part of Proposition 3 states that the optimal monetary policy takes into account the deviation of macroprudential tax from its optimum. Formally, the planner solves a problem in which she has an additional constraint—banker's optimality condition with respect to investment in durable goods. The optimal monetary policy generates a recession ($\tau_0 > 0$) if the financial regulator sets the macroprudential tax below the optimal level that prevails under the zero labor wedge. Intuitively, if the tax τ_0^b is not high enough, monetary authority generates a recession in the whole economy to reduce banks incentives to issue safe debt and invest in durables production. If the macroprudential tax is above its optimum, the monetary authority generates an inefficient boom to undo overly strict financial regulation.

The second part of proposition 3 shows that the optimal macroprudential tax not only mitigates the losses of the pecuniary externalities (the first two terms in equation (23)), but also corrects the so-called "aggregate demand externality" due to sticky prices. The third term formally represents this externality. Intuitively, when prices are sticky, additional purchases of perishable goods either by bankers or by consumers increase aggregate demand and, hence, contemporaneous output because prices are sticky. Higher output makes agents richer and induce them to spend more, which increases output further. An individual agent does not internalize this. Proposition 2 shows that monetary policy can correct this externality and macroprudential policy only addresses the fire-sale externality. Proposition 3, however, states that macroprudential policy optimally addresses both types of externalities when monetary policy is not set optimally. For example, when a country is in an inefficient recession, $\tau_0 > 0$, the optimal macroprudential tax is reduced to induce the bankers to invest more in durables production.

3 A 2-period Model of Currency Union

This section extends the model presented in the previous section to a multi-country setting, and presents the main results of the paper.

The international extension of the model features traded and non-traded goods as in

Obstfeld and Rogoff (1995) and Farhi and Werning (2013). Non-traded goods are produced with labor in period 0 and they come in the form of endowment in period 1.[15] There is inelastic supply of traded goods in both periods. Durable goods are produced with non-traded goods and are consumed only locally. Labor is immobile across countries. Agents can trade only safe bonds across borders. Only non-traded goods prices in period 0 are sticky, all of the other prices are flexible. There is a continuum of countries of measure one.

The following household preferences extend the closed-economy preferences (1) by adding traded and non-traded goods

$$\mathbb{E}\left\{U_0(c^i_{NT,0}, c^i_{T,0}) - v(n^i_0) + \beta U_1[c^i_{NT,1}, c^i_{T,1} + (1+\nu^i_T)\underline{c}^i_{T,1}] + \beta X_1(s_1) g(h^i_1)\right\} \tag{24}$$

where superscript i is the country index, $c^i_{NT,t}, c^i_{T,t}$ is country i household consumption of non-traded (NT) and traded (T) goods in period t, and $\underline{c}^i_{T,1}$ is non-traded and traded goods consumption in period 1 that must be purchased with safe assets. $U_0(\cdot,\cdot)$ and $U_1(\cdot,\cdot)$ are period 0 and 1 utility functions and they are strictly increasing and concave. I assume that the non-pecuniary demand for safe assets is due to traded goods only. This assumption makes the exposition more transparent without altering the main conclusions.

Household's consolidated budget constraint in period 0 is

$$\begin{aligned} T^i_0 + P^i_{NT,0} c^i_{NT,0} + P_{T,0} c^i_{T,0} + \frac{D^{c,i}_1}{1+i_0} + P^i_{NT,0} k^i_0 \\ \leq P_{T,0} e^i_{T,0} + \frac{\widetilde{D}^{b,i}_1}{1+i_0}(1-\tau^{b,i}_0) + W^i_0 n^i_0 + \Pi^i_0(j), \end{aligned} \tag{25}$$

where $P^i_{NT,0}$ is the sticky price index of non-traded goods in country i in period 0, $P_{T,0}$ is the flexible price of traded goods in period 0, $e^i_{T,0}$ is the household endowment of traded goods in period 0, k^i_0 is the input in production of durable goods, $D^{c,i}_1$ is country i consumer nominal purchases of safe debt, $\widetilde{D}^{b,i}_1$ is country i banker nominal issuance of safe debt net of reserves held at the central bank, i_0 is safe debt nominal interest rate, $\Pi^i_0(j)$ are the profits of non-traded goods firm that produces differentiated good j

$$\Pi^i_0(j) = \left(P^i_{NT,0}(j) - \frac{1+\tau^{L,i}_0}{A^i_0}\right) y^i_0 \left(\frac{P^i_{NT,0}(j)}{P^i_{NT,0}}\right)^{-\epsilon}.$$

[15]If non-traded goods are produced using labor also in period 1, the derivations are identical because non-traded goods prices are flexible in period 1.

Budget constraint (25) is an international extension of the closed-economy budget constraint (5).

Household budget constraint in period 1 is

$$
\begin{aligned}
P^i_{NT,1}c^i_{NT,1} + P_{T,1}(c^i_{T,1} + \underline{c}^i_{T,1}) + T^i_1 + \Gamma^i_1 h^i_1 + \widetilde{D}^{b,i}_1 \\
\leq P_{T,1}e^i_{T,1} + D^{c,i}_1 + \Gamma^i_1 G\left(k^i_0\right) + P^i_{NT,1}e^i_{NT,1}.
\end{aligned} \tag{26}
$$

where $P^i_{NT,1}, P_{T,1}, \Gamma^i_1$ are non-traded, traded and durable goods nominal prices in period 1, $e^i_{T,1}, e^i_{NT,1}$ are the endowments of traded and non-traded goods.

Traded goods nominal prices $P_{T,0}, P_{T,1}$ and nominal interest rate i_0 have no country superscripts reflecting that countries belong to a monetary union.

Country i bankers' constraint on the issuance of safe debt is

$$
\widetilde{D}^{b,i}_1 \leq \min_{s_1}\{\Gamma^i_1\}G(k^i_0), \tag{27}
$$

Part of traded consumption in period 1 must be purchased with safe assets

$$
P_{T,1}\underline{c}^i_{T,1} \leq D^{c,i}_1, \tag{28}
$$

A typical household in country i maximizes (24) subject to (25)-(28) by choosing consumption of traded and non-traded goods $c^i_{NT,0}, c^i_{T,0}, c^i_{NT,1}, c^i_{T,1}, \underline{c}^i_{T,1}$, consumption of durable goods h^i_1, safe assets portfolio $D^{c,i}_1, \widetilde{D}^{b,i}_1$, labor supply n^i_0, investment in production of durable goods k^i_0.

The household's optimality conditions with respect to consumption are[16]

$$
\frac{U^i_{NT,0}}{P^i_{NT,0}} = \frac{U^i_{T,0}}{P_{T,0}}, \tag{29}
$$

$$
\frac{U^i_{NT,1}}{P^i_{NT,1}} = \frac{U^i_{T,1}}{P_{T,1}}. \tag{30}
$$

The first two equations characterize optimal intraperiod consumption choices in both periods. Household optimal labor supply satisfies

$$
\frac{v'\left(n^i_0\right)}{U^i_{NT,0}} = \frac{W^i_0(s_0)}{P^i_{NT,0}}, \tag{31}
$$

[16] $U^i_{NT,t}, U^i_{T,t}$ are partial derivatives of household preferences with respect to $c^i_{NT,t}, c^i_{T,t}$.

Durable goods demand is described by

$$\frac{X_1(s_1)g'\left(h_1^i\right)}{U_{T,1}^i} = \frac{\Gamma_1^i}{P_{T,1}}. \tag{32}$$

Household optimal choice of safe bonds is summarized by the following Euler equation

$$U_{T,0}^i = \beta \mathbb{E}_0 \frac{1+i_0}{P_{T,1}/P_{T,0}} U_{T,1}\left(1+\nu^i\right), \tag{33}$$

Optimal choice of investment in durable goods leads to

$$U_{NT,0}^i = \beta \mathbb{E}_0 U_{T,1}^i G_{NT,0}^i(k_0^i)\left(\frac{\Gamma_1^i}{P_{T,1}} + \zeta_0^i \min_{s_1} \frac{\Gamma_1^i}{P_{T,1}}\right), \tag{34}$$

where the Lagrange multiplier on the collateral constraint is pinned down using the optimality with respect to safe bonds issuance by bankers

$$\zeta_0^i = (1-\tau_0^{b,i})(1+\nu^i) - 1 \geq 0. \tag{35}$$

Note that optimality conditions (31)-(35) are analogues to the closed-economy case, and conditions (29) and (30) result from the international dimension of the model.

3.1 Government

The government consists of a union-wide central bank, national treasuries and financial regulators. The central bank sets the nominal interest rate on safe bonds i_0 and period-1 price of traded goods $P_{T,1}$, so that the price level does not depend on state s_1.[17] A financial regulator in country i sets the level of reserve requirements z_0^i and country-specific interest rate on reserves $i_0^{r,i}$, which is equivalent to setting macroprudential tax $\tau_0^{b,i}$ on local issuance of safe debt. It rebates the proceeds to the local treasury. In turn, a local treasury sets lump sum taxes T_0^i, T_1^i, labor tax $\tau_0^{L,i}$ and issues safe bonds $D_1^{g,i}$. The

[17] Similarly to the closed-economy case, I motivate the monetary authority control over the nominal price level of traded goods in period 1 and the nominal interest rate between periods 0 and 1 by assuming that fraction $\kappa \in [0,1]$ of traded goods purchases has to be bought with monetary authority nominal liabilities M_0, M_1 that pay zero nominal interest rate (cash): $\kappa P_{T,0}\int c_{T,0}^i di = M_0, \kappa P_{T,1}\int (c_{T,1}^i + \underline{c}_{T,1}^i)di = M_1$. Making κ, M_0, M_1 tend to zero such that $M_0/\kappa, M_1/\kappa$ are finite and bounded from zero allows the monetary authority to have a control over nominal variables $P_{T,1}, i_0$ but does not require explicit treatment of cash.

consolidated government budget constraints in both periods in country i are

$$T_0^i + \tau_0^{L,i} W_0^i n_0^i + \tau_0^{b,i} \frac{\widetilde{D}_1^{b,i}}{1+i_0} + \frac{D_1^{g,i}}{1+i_0} = 0, \tag{36}$$

$$T_1^i = D_1^{g,i}. \tag{37}$$

The government budget constraint in period 0 states that the revenue from lump-sum taxes T_0^i (transfers if negative), revenue from labor taxes $\tau_0^{L,i} W_0^i n_0^i$, revenue from reserve requirement policy $\tau_0^{b,i} \widetilde{D}_1^{b,i} / (1+i_0)$, and revenue from issuing government safe debt $D_1^{g,i}/(1+i_0)$ must add up to zero. The budget constraint in period 1 requires the treasury to repay its safe debt $D_1^{g,i}$ by collecting lump-sum taxes T_1^i.

3.2 Auxiliary Variables

Similarly to the closed-economy model, I introduce real variables and several wedges. First, I express period-1 nominal non-traded goods and durable goods prices in units of traded goods as follows: $p_1^i \equiv P_{NT,1}^i / P_{T,1}, \gamma_t^i \equiv \Gamma_1^i / P_{T,1}$; workers nominal wages in units of traded goods as $w_0^i \equiv W_0^i / P_{T,0}$, and the interest rate on safe debt deflated by traded goods inflation $r_0 \equiv (1+i_0) P_{T,0} / P_{T,1} - 1$. Second, I express nominal quantities in units of traded goods: $\widetilde{d}_1^{b,i} \equiv \widetilde{D}_1^{b,i} / P_{T,1}$, $d_1^{g,i} \equiv D_1^{g,i} / P_{T,1}$, $d_1^{c,i} \equiv D_1^{c,i} / P_{T,1}$. Third, I denote total consumption of traded goods in period 1 as $\widetilde{c}_{T,1}^i \equiv c_{T,1}^i + \underline{c}_{T,1}^i$. Finally, the labor wedge is

$$\tau_0^i \equiv 1 - \frac{v'(n_0^i)}{A_0^i U_{NT,0}^i}.$$

3.3 Equilibrium

An equilibrium specifies consumption $\{c_{NT,t}^i, c_{T,t}^i\}, \underline{c}_{T,1}^i$, labor $\{n_t^i\}$, investment in durable goods k_0^i, durable goods production h_1^i, real (in terms of traded goods) safe debt supply by bankers and the government $\widetilde{d}_1^{b,i}, d_1^{g,i}$, real safe debt demand $d_1^{c,i}$, real wages $\{w_t^i\}$, traded and non-traded goods prices $\{P_{NT,t}^i, P_{T,t}\}$, real interest rate r_0, government lump-sum taxes T_0^i, T_1^i, and labor taxes $\tau_0^{L,i}$ in every country $i \in [0,1]$ such that households and firms optimize, the government budget constraints are satisfied, final non-traded goods markets in both periods clear in every country

$$k_0^i + c_{NT,0}^i = A_0^i n_0^i, \tag{38}$$

$$c_{NT,1}^i = e_{NT,1}^i, \tag{39}$$

traded goods market clears in both periods

$$\int c^i_{T,0} di = \int e^i_{T,0} di, \tag{40}$$

$$\int \widetilde{c}^i_{T,1} di = \int e^i_{T,1} di, \tag{41}$$

durable goods markets clear in every country

$$h^i_1 = G(k^i_0), \tag{42}$$

and international safe assets market clears

$$\int d^{c,i}_1 di \quad = \quad \int d^{g,i}_1 di + \int \widetilde{d}^{b,i}_1 di. \tag{43}$$

3.4 Functional Forms

The utility functions $U_0(c_{NT}, c_T)$ and $U_1(c_{NT}, c_T)$ have the following forms

$$U_0(c_{NT,0}, c_{T,0}) = \frac{a^\sigma c^{1-\sigma}_{NT,0}}{1-\sigma} + c_{T,0},$$

$$U_1(c_{NT,1}, \widetilde{c}_{T,1} + \nu \underline{c}_{T,1}) = \frac{a^\sigma c^{1-\sigma}_{NT,1} + (1-a)^\sigma [c_{T,1} + (1+\nu)\underline{c}_{T,1}]^{1-\sigma}}{1-\sigma}.$$

These functional forms embed three assumptions. First, in period 1, the intratemporal elasticity of substitution between traded and non-traded goods is equal to $1/\sigma$ and equals one over the relative risk aversion. This special case is routinely utilized in the international macro literature (see, for example, Cole and Obstfeld, 1991). Second, utility is linear in traded goods consumption in period 0. This simplifies the exposition without qualitatively altering the main conclusions of the paper.

The durable goods utility function take the form $g(h_1) = \psi_h h_1^{1-\sigma_g}/(1-\sigma_g)$, where $\psi_h > 0$ and $\sigma_g < 1$. The durable goods production function is $G(k_0) = k_0^{\alpha_G}$.

3.5 Equilibrium Characterization

This section simplifies the complete set of equilibrium conditions (25)-(43) before turning to the optimal policy characterization. The intratemporal optimality conditions (29)-(30) lead to: $c^i_{NT,0} = a(P^i_{NT,0}/P_{T,0})^{-1/\sigma}$, $c^i_{NT,1} = (p^i_1)^{-1/\sigma} c^i_{T,1} a/(1-a)$. The demand for traded goods in period 0 does not depend on the demand for non-traded goods because traded goods marginal utility equals one.

Household budget constraint (25), government budget constraint (36), and non-traded goods market clearing condition can be combined to express country i consolidated budget constraint in period 0

$$c^i_{T,0} - e^i_{T,0} = \frac{\widetilde{d}^{b,i}_1 + d^{g,i}_1 - d^{c,i}_1}{1 + r_0}. \tag{44}$$

It states that country i excess consumption of traded goods (the left-hand side) must be financed by issuing safe bonds on the international market. Similarly, (26), (37), and (39) can be combined to express country-wide budget constraint in period 1

$$\widetilde{c}^i_{T,1} - e^i_{T,1} = d^{c,i}_1 - \widetilde{d}^{b,i}_1 - d^{g,i}_1, \tag{45}$$

It shows that excess consumption of traded goods in period 1 results from the accumulation of safe claims on other countries. The household Euler equation can be rewritten as

$$1 = [\beta(1 + r_0)(1 + \nu^i)]^{-\frac{1}{\sigma}} \frac{\widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}}{1 - a}, \tag{46}$$

Durable goods demand (32) and supply (34) lead to

$$\frac{P_{T,0}}{P^i_{NT,0}} = \beta g'[G(k^i_0)]G'(k^i_0)[\mu + (1 - \mu)\theta^i + \zeta^i_0 \theta^i], \tag{47}$$

where $\zeta^i_0 = (1 - \tau^{b,i}_0)(1 + \nu^i) - 1 \geq 0$ is the Lagrange multiplier on the collateral constraint in country i. The real interest rate on safe debt expressed in units of traded goods is related to price level of traded goods as follows

$$P_{T,0} = \frac{1 + r_0}{1 + i_0} P_{T,1}. \tag{48}$$

Recall that the central bank has a control over i_0 and $P_{T,1}$. The last expression states that price $P_{T,0}$ is related to real interest rate r_0 and monetary policy choices $i_0, P_{T,1}$.

Finally, collateral constraint (27) and safe-assets-in-advance constraint (28) can be simplified as follows

$$\widetilde{d}^{b,i}_1 \leq \frac{\theta^i g'[G(k^i_0)]G(k^i_0)}{(1 - a)^\sigma (\widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1})^{-\sigma}}, \tag{49}$$

$$\underline{c}^i_{T,1} \leq d^{c,i}_1. \tag{50}$$

Collateral constraint (49) is analogues to the closed-economy case.

Equations (40), (41), (44)-(50), and the complementarity slackness conditions on the last two inequalities, describe the equilibrium. This system determines the remaining unknown endogenous variables $\{c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \widetilde{d}^{b,i}_1, d^{c,i}_1\}$, r_0, $P_{T,0}$. There are two cross-country equations to determine interest rate r_0 and price level $P_{T,0}$ that are common across countries. There are six conditions for every country i to determine six country-level endogenous variables $c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \widetilde{d}^{b,i}_1, d^{c,i}_1$.

The simplifying assumptions introduced so far allow me to explicitly express the country-specific endogenous variables through policies $P_{T,1}, i_0, \{\tau^{b,i}_0\}$ and the international real interest rate on safe debt r_0. Specifically, combining the intratemporal optimality with no-arbitrage equations 29 and 48, I can write

$$c^i_{NT,0} = a \left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1 + r_0}{1 + i_0} \right)^{\frac{1}{\sigma}} .$$

Intuitively, when a union-wide central bank increases its interest rate or reduces targeted price level $P_{T,1}$, then the consumption of non-traded goods drops because this policy change increases the relative price of non-traded to traded goods in period 0 leading to expenditure switching towards traded goods. Banks' optimal choice of investment in traded goods (47) and a no-arbitrage condition (48) lead to

$$k^i_0 = (\Omega^i)^{\frac{1}{\alpha_G(1-\sigma_g)}} \left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1 + r_0}{1 + i_0} \right)^{\frac{1}{1-\alpha_G(1-\sigma_g)}} ,$$

where new variable $\Omega^i \equiv (\alpha_G \beta \{\mu(1-\theta^i) + (1-\tau^{b,i}_0)(1+\nu^i)\theta^i\})^{\alpha_G(1-\sigma_g)/[1-\alpha_G(1-\sigma_g)]}$ depends on model parameters and the stance of macroprudential policy. The optimal demand for capital depends on monetary policy and the international real interest rate in the same manner as consumption for non-traded goods.

When the collateral constraint (49) binds, the Euler equation (46) and the explicit expression for the investment in durable goods demand lead to

$$\widetilde{d}^{b,i}_1 = \theta^i \beta \left(1 + \nu^i\right) \Omega^i \left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1 + r_0}{1 + i_0} \right)^{\frac{(1-\sigma_g)\alpha_G}{1-\alpha_G(1-\sigma_g)}} (1 + r_0). \tag{51}$$

An important implication of this formula is that a decline in world interest rate r_0 reduce the equilibrium issuance of safe debt keeping everything else equal. This will be an important source of international spillovers of macroprudential policy choices that I study below.

The supply of safe debt, expressed in the last formula, together with the flow budget constraint of country i in period 1, shown in equation (45), result in the expression for demand for safe assets by consumers in country i

$$d_1^{c,i} = \beta \left[(1+\nu^i)\beta\right]^{\frac{1}{\sigma}-1} (1+r_0)^{\frac{1}{\sigma}} + \theta^i \beta \Omega^i \left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0} \right)^{\frac{(1-\sigma_g)\alpha_G}{1-\alpha_G(1-\sigma_g)}} (1+r_0) + \frac{d_1^{g,i} - e^i_{T,1}}{1+\nu^i}.$$

This formula shows among other things that higher interest rate r_0 increase the demand for safe assets. When $1/\sigma$ is larger than $1/[1-\alpha_G(1-\sigma_g)]$, then the demand $d_1^{c,i}$ is steeper than the supply $\widetilde{d}_1^{b,i}$.

Finally, the equilibrium on the safe debt market (43) leads to the following implicit formula for the work real interest rate

$$1+r_0 = \frac{1}{\beta} \left[\frac{\int \left(e^i_{T,1} + \nu^i d_1^{c,i} \right) di}{\int \left[(1+\nu^i)\right]^{\frac{1}{\sigma}} di} \right]^{\sigma}.$$

3.6 Ramsey Planning Problem

The central bank and financial regulator face all equilibrium conditions (25)-(43) as constraints when choosing their optimal policies. The full system of equilibrium conditions was reduced to system (40), (41), (44)-(50). Note that the full set of equilibrium conditions can be unambiguously recovered from (40), (41), (44)-(50).

I further drop certain variables and constraints from the optimal policy problem. First, given quantities $\{c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \widetilde{d}_1^{b,i}, d_1^{c,i}\}$ and prices $r_0, P_{T,0}, \{P^i_{NT,0}\}$, the optimal choice of investment in durable goods (47) can be dropped because it can be used to express optimal macroprudential tax $\tau_0^{b,i}$ when the collateral constraint binds. (48) can be dropped because it can be used to express the ratio of the nominal interest rate and price of traded goods in period 1.

Proposition 4. *An allocation $\{c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \widetilde{d}_1^{b,i}, d_1^{c,i}\}$ and prices $r_0, P_{T,0}, \{P^i_{NT,0}\}$ form part of an equilibrium if and only if conditions (40), (41), (44), (45), (49) and (50) hold.*

After taking into account intratemporal consumption choices by the household, the objective in country i can be simplified as in the following lemma.

Lemma 1. *Country i indirect household utility is*

$$V^i(c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0; r_0, i_0, P_{T,1}) = \frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} - v(n^i_0)$$
$$+ \beta\frac{(1-a)^\sigma(\widetilde{c}^i_{T,1} + \nu^i d^{c,i}_1)^{1-\sigma}}{1-\sigma} + \beta[\mu + (1-\mu)\theta]g[G(k^i_0)] + O^i, \tag{52}$$

where $n^i_0 = \{a[P_{T,1}/P^i_{NT,0} \cdot (1+r_0)/(1+i_0)]^{\frac{1}{\sigma}} + k^i_0\}/A^i_0$, O^i is the term which depends only on exogenous variables and model parameters.[18]

3.6.1 Local Planner

I start by solving a local planner problem. In this case, the planner maximizes local welfare taking international prices as given. I will later compare this solution to a union wide planner's solution. The two solutions will turn out to be different.

Formally, the local planner maximizes (52) subject to country budget constraints (44) and (45), banker's collateral constraint (49), safe-assets-in-advance constraint (50), and Euler equation (46) by choosing allocation $c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \widetilde{d}^{b,i}_1, d^{c,i}_1$ conditional on prices international prices $r_0, i_0, P_{T,1}$. The solution to the planner's problem is derived in Appendix A.2.3. The following Lemma presents the implementation of the planner's solution.

Proposition 5. *Constrained Pareto efficient allocation in country i, given international prices, can be implemented by setting the macroprudential tax to*

$$\tau^{b,i}_0 = \frac{\nu^i\sigma_g}{1+\nu^i} - \frac{\tau^i_0}{1-\tau^i_0} \cdot \frac{\mu\left(1-\theta^i\right) + \theta^i[1+\nu^i(1-\sigma_g)]}{\theta^i(1+\nu^i)}. \tag{53}$$

Proof is in Appendix A.2.3. The interpretation of this formula highlights the externalities that the planner takes into account when choosing the optimal macroprudential tax. There are two terms that correspond to two externalities. The first term reflects to a pecuniary externality that arises from the fact that bankers do not internalize the durable effect of their choices (Stein, 2012). The second term represents the financial regulator desire to mitigate the aggregate demand externality when the labor wedge is not zero (Farhi and Werning, forthcoming; Korinek and Simsek, 2016). Note that optimal tax (53) somewhat resembles the optimal tax in the closed economy when the monetary policy is

[18] Because indirect utility function depends on country-specific parameters that do not enter this function as arguments, I added index i to $V^i(\cdot)$.

not set optimally, equation (23). The key difference is that the second term in the closed-economy formula (23) is not present in the international context. The reason for that is straightforward: the local regulator of small-open economy i takes the international real interest rate r_0 as given, while the regulator in a closed economy internalizes his effect on the real interest rate.

3.6.2 Global Planner

In this section, I solve the global planner's problem and show that global planner chooses a different allocation compared to the independent local planners. The global planner's problem corresponds to coordinate choice of union-wide monetary policy and country-specific financial regulation. Formally, the global planner maximizes an average of country-specific welfare functions (52) subject to constraints (40)-(50) by choosing allocation $\{c^i_{T,0}, k^i_0, \tilde{c}^i_{T,1}, \underline{c}^i_{T,1}, \tilde{d}^{b,i}_1, d^{c,i}_1\}$ and prices r_0, i_0.

The full characterization of the global planner problem is in Appendix A.2.4. The following proposition summarizes the optimal monetary and macroprudential policy implementation.

Proposition 6. *At a constrained Pareto efficient equilibrium*
(i) average (across countries) labor wedge is zero

$$\int \tau^i_0 c^i_{NT,0} U^i_{NT,0} di = (1+i_0)\sigma\tilde{\xi},$$

where $\tilde{\xi} \geq 0$ is the planner's problem Lagrange multiplier on the nominal interest rate i_0 zero-lower bound (ZLB) constraint;
(ii) optimal choice of $\{c^i_{T,0}, k^i_0, \tilde{c}^i_{T,1}, \underline{c}^i_{T,1}, \tilde{d}^{b,i}_1, d^{c,i}_1\}$ and r_0, i_0 is implemented by setting macroprudential tax

$$\tau^{b,i}_0 = \tau^{b,i}_0\Big|_{local} + \frac{\tilde{\varphi}_0}{1-\tau^i_0} \cdot \frac{\nu^i(1-\sigma_g)}{1+\nu^i}, \tag{54}$$

where $\tau^{b,i}_0\Big|_{local}$ is the expression identical to local macroprudential tax (53), and

$$\tilde{\psi}_0 = -\sigma \frac{\int \nu^i \tilde{d}^{b,i}_1 U^i_{T,1} di + \beta^{-1}(1+i_0)\tilde{\xi}}{\int (\tilde{c}^i_{T,1} + \nu^i d^{c,i}_1 - \sigma\nu^i \tilde{d}^{b,i}_1) U^i_{T,1} di} \leq 0. \tag{55}$$

The proof is in Appendix A.2.4. The first part of proposition 6 show that the central bank sets the average labor wedge across countries to zero when it is not constrained by the ZLB. This result is similar to the one derived in Farhi and Werning (2012). The linearized version of this condition would equalize the average output gap to zero (Benigno, 2004; Gali and Monacelli, 2008). If all of the countries are symmetric, the monetary authority stabilizes all economies with just one policy tool. When, instead, the ZLB constraint binds, the average labor wedge is positive across countries of the union.

The second part of the proposition characterizes the implementation of macroprudential policy by highlighting that the globally optimal macroprudential tax deviates from a local one. Specifically, the deviation is proportional to $\widetilde{\psi}_0$, which captures the sum of the two spillovers of local prudential policies on the other countries. Both of these spillovers arise from the fact that tighter macroprudential regulation reduces the supply of safe assets and, hence, reduces the international real interest rate r_0.

The first channel through which a lower international interest rate affects other members of the union has to do with the collateral constraints of the bankers. As the equation for the supply of safe debt (51) demonstrates, a lower interest rate r_0 tightens the collateral constraint hurting bankers in all of the countries of the monetary union. This happens because lower interest rate increases the marginal utility of traded goods in period 1, which reduces the price of durable goods. The global regulator captures this negative *international pecuniary externality* and sets a lower macroprudential tax in all of the countries in monetary union.

A lower interest rate r_0 works not only through the financial channel however. When the monetary union central bank does not set its monetary policy optimally, because it reached its zero lower bound on the nominal interest rate i_0, the international aggregate demand externality comes into play. Intuitively, a lower interest rate r_0, holding fixed nominal interest rate $i_0 = 0$ due to the ZLB constraint and the price $P_{T,1}$ due to the unchanged price-level target, results in a lower period-0 price of traded goods $P_{T,0}$. This is because of the no-arbitrage condition (48). Since the price of non-traded goods is fixed, the relative price non-traded to traded goods goes up and consumers and bankers demand for non-traded goods collapses reducing the output of non-traded goods. With non-zero labor wedge, the international aggregate demand externality has first order negative welfare effects.

When the zero lower bound constraint does not bind, the international aggregate demand externality surprisingly has no effect on the choice of macroprudential taxes by the global regulator. After all, the international aggregate demand externality is still present even when the ZLB constraint does not bind. Moreover, because the union-wide interest rate i_0 does not meaningfully respond to conditions in each individual member of the union, the labor wedge in a given country may not be zero resulting in first order welfare losses (or gains) from macroprudential policies in the other countries. To square this logic with seemingly paradoxical result that macroprudential taxes do not take into account the international aggregate demand externality when $\widetilde{\xi} = 0$, note that $\tau_0^{b,i}$ reflects the international externalities through the variable $\widetilde{\psi}_0$, which, in turn, is common across countries and it represents the average losses from the spillovers. By reducing output everywhere in the union, a tighter macroprudential regulation in country i reduces welfare in countries with recessions ($\tau_0^i > 0$) and increases welfare in countries with booms ($\tau_0^i < 0$). But because a union-wide monetary policy sets the average across countries labor wedge to zero, the net effect of these gains and losses is zero. This explains why international aggregate demand externality only affect macroprudential taxes in the ZLB.

3.6.3 Regulation Games

What will be the outcome if the union-wide central bank and local financial regulators fail to coordinate? In such a situation, optimal monetary policy will not coincide with the global planner's solution I derived in previous section. In this section, I characterize policy choices in the absence of coordination. Specifically, I assume that local financial regulators set their optimal policies by maximizing local welfare. This is formally identical to the local regulator's solution in Section 3.6.1. The union-wide central bank internalizes non-coordinated behavior of financial regulators that results in negative international spillovers. When the bank is not constrained by the ZLB, it will try to correct for the lack of coordination by tilting its monetary policy.

Formally, as the global regulator from Section 3.6.2, the central bank maximizes the average across countries welfare of households conditional on all equilibrium condition. The only difference is that private optimal choice of investment in durable goods cannot be dropped anymore. Instead, it is preserved as a constraint and the macroprudential tax in this optimality condition is given by local regulator's optimal solution (53).

Proposition 7. *When financial regulators maximizes local welfare only, optimal monetary policy is such that*

$$\int \tau_0^i U_{NT,0}^i c_{NT,0}^i di = \sigma(1+i_0)\widetilde{\xi} - \widetilde{\psi}_0 Z,$$

where $\widetilde{\psi}_0 \leq 0$ is give by equation (55), $Z > 0$ is a variable that depends on the optimal allocation and is presented in the Appendix, and $\widetilde{\xi} \geq 0$ is the Lagrange multiplier on the ZLB constraint.

Proof is in Appendix A.2.5. This proposition states that the choice of monetary policy deviates from the choice under complete coordination expressed in Proposition 6. Specifically, the second term reflects the desire of central bank to accomodate spillover effects. This term is proportional to $\widetilde{\psi}_0$ that, as I discussed in the previous section, summarizes the international spillovers in the model. When, for example, the ZLB constraint does not bind, i.e., $\widetilde{\xi} = 0$, then the average labor wedge across countries is positive. Knowing that local financial regulators react to the state of the business cycles, summarized by the labor wedge τ_0^i, the central bank sets the average labor wedge to be positive, i.e., chooses tighter monetary policy than under complete coordination. This, in turn, prompts the local regulators to relax their financial regulation which brings their choices closer to the optimal choices under coordination. All this comes at a cost of generating a recession however.

Corollary 1. *When countries are symmetric and $1-\sigma = \alpha_G(1-\sigma_g)$, in non-coordinated solution the labor wedge is positive and macroprudential taxes are larger than in the global regulator solution.*

Proof is in Appendix A.2.6.

3.7 Countries Outside of Monetary Union

The focus of the analysis so far was on policy for members of the currency union. In this section, I solve for optimal monetary and macroprudential policy in counties outside of the monetary union.

Countries outside of the monetary union have control over their monetary policy. In each of these countries, for example, country i, the central bank sets local price of traded goods in period 1, i.e., $P_{T,1}^i$, and the level of safe nominal interest rate i_0^i between periods 0 and 1. The nominal exchange rate is then given by $E_t^i = P_{T,t}^i / P_{T,t}$, where $P_{T,t}$ is the traded goods price in period t in the countries that belong to the currency union.

Due to the potential presence of the nominal exchange rate risk in period 1, safe assets issued in the currency union may not be safe in countries outside of the currency union. In the analysis below, I assume that period 1 monetary policy in the currency union and outside of it is perfectly predictable, which removes exchange rate risk completely. As a result, safe debt in the currency union is also safe outside of it.[19]

I next solve for optimal monetary and macroprudential policy in countries that do not belong to a currency union. I first study the local policy maker's problem. Then, I show that there are gains from coordination of macroprudential policies even for the countries outside of the union.

Optimal monetary and macroprudential policy can be written as a Ramsey planning problem. Repeating steps similar to those in section 3.6, I can reduce the whole set of equilibrium conditions to a smaller set that uniquely defines some of the equilibrium variables as in the following lemma.

Lemma 2. *An allocation $\{c_{T,0}^i, \tilde{c}_{T,0}^i, \underline{c}_{T,1}^i, k_0^i, \tilde{d}_1^{b,i}, d_1^{c,i}\}$ and prices i_0^i (given safe real interest rate r_0 expressed in units of traded goods) form part of an equilibrium if and only if conditions* (44), (45), (46), (49), (50) *(where $P_{T,t}$ is replaced with $P_{T,t}^i$) hold.*

[19] The assumption may be a reasonable approximation of the countries with relatively transparent and predictable monetary policies.

The proof is in Appendix A.2.7. The local planner in country i, which is outside of the currency union, maximizes the expectations of indirect household utility function $V^i(c^i_{T,0}, \tilde{c}^i_{T,0}, \underline{c}^i_{T,1}, k^i_0, p^i_0)$, expressed in equation (52), conditional on country i budget constraints (44) and (45), banker's collateral constraint (49), safe-assets-in-advance constraint (50), and Euler equation (46) by choosing allocation $c^i_{T,0}, \tilde{c}^i_{T,0}, \underline{c}^i_{T,1}, k^i_0, \tilde{d}^{b,i}_1, d^{c,i}_1$, and prices p^i_0, i^i_0 taking r_0 as given.

Proposition 8. *Constrained Pareto efficient allocation in country i with independent monetary policy and flexible exchange rate can be implemented by setting the macroprudential tax and nominal interest rate so that*

$$\tau^{b,i}_0 = \frac{\tau^i_A \sigma_g}{1+\tau^i_A} \text{ and } \tau^i_0 = 0. \tag{56}$$

Proof is in Appendix A.2.8. The proposition states that when monetary and macroprudential policies are optimal, the monetary policy fully stabilizes the economy in the sense that the labor wedge equals zero. Macroprudential policy addresses the negative pecuniary externalities associated with over-investment in production of durable goods. Because the labor wedge equals zero, the financial regulation policy is not used to stabilize the local business cycle.

There are gains from coordinating macroprudential policies even for the countries outside of monetary union. To formally show this, I solve the global regulator problem who maximizes the following objective

$$\int_{i\in\mathcal{I}} \omega^i \mathbb{E}_0 V^i(c^i_{T,0}, \tilde{c}^i_{T,1}, \underline{c}^i_{NT,1}, k^i_0, \frac{P^i_{NT,0}}{P_{T,0}})di + \int_{i\notin\mathcal{I}} \omega^i \mathbb{E}_0 V^i(c^i_{T,0}, \tilde{c}^i_{T,1}, \underline{c}^i_{NT,1}, k^i_0, p^i_0)di,$$

where $\mathcal{I} \subseteq [0,1]$ is a set of countries that belong to the currency union. The planner chooses $\{c^i_{T,0}, \tilde{c}^i_{T,0}, \underline{c}^i_{T,1}, k^i_0, \tilde{d}^{b,i}_1, d^{c,i}_1\}_{i\in[0,1]}$, $\{p^i_0\}_{i\notin\mathcal{I}}$, and $P_{T,0}, r_0$ subject to the equilibrium conditions listed in Proposition 4 and Lemma 2.

Proposition 9. *At a constrained Pareto efficient equilibrium when the ZLB constraints do not bind*

(i) average (across countries in the currency union) labor wedge and labor wedge in every country outside of the currency union are zero

$$\begin{aligned} \int_{i\in\mathcal{I}} \tau^i_0 c^i_{NT,0} U^i_{NT,0} di &= 0, \\ \tau^i_0 &= 0, \quad i \notin \mathcal{I}, \end{aligned}$$

(ii) optimal choice of $\{c^i_{T,0}, \tilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \tilde{d}^{b,i}_1, d^{c,i}_1\}_{i\in[0,1]}, \{p^i_0\}_{i\notin\mathcal{I}}, P_{T,0}, r_0$ *is implemented by set-*

ting macroprudential tax

$$\tau_0^{b,i} = \frac{\nu^i \sigma_g}{1+\nu^i} - \frac{\tau_0^i}{1-\tau_0^i} \cdot \frac{\mu\left(1-\theta^i\right) + \theta^i[1+\nu^i(1-\sigma_g)]}{\theta^i(1+\nu^i)} + \frac{\widetilde{\varphi}_0}{1-\tau_0^i} \cdot \frac{\nu^i(1-\sigma_g)}{1+\nu^i},\ i \in \mathcal{I},$$

$$\tau_0^{b,i} = \frac{\nu^i \sigma_g}{1+\nu^i} + \frac{\widetilde{\varphi}_0}{1-\tau_0^i} \cdot \frac{\nu^i(1-\sigma_g)}{1+\nu^i},\ i \notin \mathcal{I},$$

where

$$\widetilde{\varphi}_0 = -\sigma \frac{\int \nu^i \widetilde{d}_1^{b,i} U_{T,1}^i di}{\int (\widetilde{c}_{T,1}^i + \nu^i d_1^{c,i} - \sigma \nu^i \widetilde{d}_1^{b,i}) U_{T,1}^i di} < 0.$$

The proof is in Appendix A.2.8. The first part of the proposition states that in a global optimum, monetary policies are still chosen to set the average labor wedge across the countries of the currency union and labor wedge in each individual country outside of monetary union to zero. As a result, even after taking international spillovers into account, economies outside of currency union are stabilized in the sense of closing the labor wedge.

The second part of the proposition states that there are gains from coordination of local macroprudential policies both inside and outside of the currency union. This is formally represented by terms featuring $\widetilde{\varphi}_0$. Note that $\widetilde{\varphi}_0$ is the same for all of the countries.

4 Conclusion

When monetary and macroprudential policies are set optimally in a currency union, local macroprudential policy has a regional macroeconomic stabilization role beyond the correction of the fire-sale externality in the financial sector. There are gains from setting macroprudential policy in a coordinated manner.

The proposed model considered only macroprudential regulation. One direction for future research is to consider unconventional monetary policy tools. For example, directed purchases of regional risky assets by the central bank in exchange of newly created reserves can also be used to stabilize local business cycles.

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A Appendix

A.1 A 2-period Closed Economy Model

This section presents closed economy derivations and proofs omitted from the main text.

A.1.1 Representative Household Problem Solution

The Lagrangian for the household problem is

$$\begin{aligned}\mathcal{L}_0 =&\mathbb{E}\Big\{ u\left(c_0\right) - v\left(n_0\right) + \beta\left[u\left(c_1 + (1+\nu)\underline{c}_1\right) + X_1 g\left(h_1\right)\right] \\ &- \frac{\lambda_0}{P_0}\left[T_0 + P_0 c_0 + \frac{D_1^c}{1+i_0} + P_0 k_0 - \frac{\widetilde{D}_1^b}{1+i_1}\left(1-\tau_0^B\right) - W_0 n_0 - \Pi_0^j\right] \\ &- \beta\frac{\lambda_1}{P_1}\left[P_1\left(c_1 + \underline{c}_1\right) + T_1 + \Gamma_1 h_1 + \widetilde{D}_1^b - D_1^c - \Gamma_1 G(k_0) - P_1 y_1\right] \\ &- \beta\frac{\lambda_1}{P_1}\zeta_0\left[\widetilde{D}_1^b - \min_{s_1}\{\Gamma_1\}G\left(k_0\right)\right] \\ &-\beta\frac{\lambda_1}{P_1}\eta_1\left[P_1\underline{c}_1 - D_1^c\right]\Big\},\end{aligned}$$

where

$$\Pi_0^j = \left(P_0^j - \frac{\left(1+\tau_0^L\right)W_0}{A_0}\right)y_0\left(\frac{P_0^j}{P_0}\right).$$

The first order conditions are

$$\begin{aligned}\partial c_0 :&u'\left(c_0\right) = \lambda_0, \\ \partial c_1 :&u'\left(c_1 + (1+\nu)\underline{c}_1\right) = \lambda_1, \\ \partial \underline{c}_1 :&(1+\nu)u'\left(c_1 + (1+\nu)\underline{c}_1\right) = \lambda_1\left(1+\eta_1\right), \\ \partial D_1^c :&\frac{\lambda_0}{P_0(1+i_0)} = \beta\mathbb{E}_0\frac{\lambda_1}{P_1}(1+\eta_1), \\ \partial \widetilde{D}_1^b :&\frac{\lambda_0}{P_0(1+i_0)}(1-\tau_0^B) = \beta\mathbb{E}_0\frac{\lambda_1}{P_1}(1+\zeta_0), \\ \partial n_0 :&v'(n_0) = \lambda_0\frac{W_0}{P_0}, \\ \partial h_1 :&X_1 g'(h_1) = \lambda_1\frac{\Gamma_1}{P_1}, \\ \partial k_0 :&\lambda_0 = \beta G'\left(k_0\right)\mathbb{E}_0\left\{\lambda_1\frac{\Gamma_1}{P_1} + \lambda_1\zeta_0\frac{\min_{s_1}\{\Gamma_1\}}{P_1}\right\}.\end{aligned}$$

The complementarity slackness conditions are

$$\begin{aligned}CSC_1 :&\widetilde{D}_1^b \leq \min_{s_1}\{\Gamma_1\}G(k_0),\ \zeta_0 \geq 0,\ [\widetilde{D}_1^b - \min_{s_1}\{\Gamma_1\}G(k_0)]\zeta_0 = 0, \\ CSC_2 :&P_1\underline{c}_1 \leq D_1^c,\ \eta_1 \geq 0,\ [P_1\underline{c}_1 - D_1^c]\eta_1 = 0.\end{aligned}$$

The first order condition can be simplified as follows

$$
\begin{aligned}
\zeta_0 &= (1-\tau_0^B)\frac{1}{1+i_0}\cdot\frac{u'(c_0)/P_0}{\mathbb{E}_0\left[u'(c_1+(1+\nu)\underline{c}_1)/P_1\right]}-1\geq 0,\\
&\qquad\qquad \widetilde{D}_1^b \leq \min_{s_1}\{\Gamma_1\}G(k_0),\ [\widetilde{D}_1^b-\min_{s_1}\{\Gamma_1\}G(k_0)]\zeta_0=0,\\
\eta_1 &= \nu\geq 0,\ P_1\underline{c}_1\leq D_1^c, [P_1\underline{c}_1-D_1^c]\eta_1=0,\\
u'(c_0) &= (1+i_0)\beta\mathbb{E}_0\left\{\frac{P_0}{P_1}u'(c_1+(1+\nu)\underline{c}_1)(1+\nu)\right\},\\
\frac{v'(n_0)}{u'(c_0)} &= \frac{W_0}{P_0},\\
u'(c_0) &= \beta G'(k_0)\mathbb{E}_0\left\{X_1g'(h_1)+\zeta_0\min_{s_1}\{X_1g'(h_1)\}\right\}.
\end{aligned}
$$

The optimality conditions, the market clearing conditions, and the fact that only durable goods price is affected by s_1 lead to the following full set of equilibrium equations

$$
\begin{aligned}
\zeta_0 &= (1-\tau_0^B)(1+\nu)-1\geq 0,\\
&\qquad \widetilde{d}_1^b \leq \theta\frac{g'[G(k_0)]}{u'(y_1+\nu\underline{c}_1)}G(k_0),\\
&\qquad \left[\widetilde{d}_1^b-\theta\frac{g'[G(k_0)]}{u'(y_1+\nu\underline{c}_1)}G(k_0)\right]\zeta_0=0,\\
\eta_1 &= \nu\geq 0,\ \underline{c}_1\leq d_1^g+\widetilde{d}_1^b, \left[\underline{c}_1-d_1^g-\widetilde{d}_1^b\right]\eta_0=0,\\
u'(c_0) &= \frac{(1+i_0)\beta}{\Pi^*}u'(y_1+\nu\underline{c}_1)(1+\nu),\\
u'(c_0) &= \beta g'[G(k_0)]G'(k_0)[\mu+(1-\mu)\theta+\zeta_0\theta],\\
y_0 &= A_0n_0,\\
y_0 &= c_0+k_0.
\end{aligned}
$$

A.1.2 Proof of Proposition 3

(i) Let's first consider optimal monetary policy conditional on macroprudential policy being set at τ_0^b. The planner solves

$$
\begin{aligned}
\max_{c_0,k_0,d_1^b}\quad & u(c_0)-v\left(\frac{c_0+k_0}{A_0}\right)+\beta\left\{u[y_1+\nu(\widetilde{d}_1^b+d_1^g)]+[\mu+(1-\mu)\theta]g[G(k_0)]\right\}\\
s.t.:&\widetilde{d}_1^b\leq\theta\frac{g'[G(k_0)]}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}G(k_0),\\
&u'(c_0)=\beta g'[G(k_0)]G'(k_0)\{\mu+[(1-\tau_0^B)(1+\nu)-\mu]\theta\}.
\end{aligned}
$$

This problem differs from the problem of optimally choosing both monetary and macroprudential policy. The banker's choice of optimal investment is now taken as a constraint because τ_0^B is not chosen optimally. The Lagrangian of this problem is

$$\begin{aligned}\widetilde{\mathcal{L}}_0 =& u(c_0) - v\left(\frac{c_0+k_0}{A_0}\right) + \beta\left\{u[y_1+\nu(\widetilde{d}_1^b+d_1^g)] + [\mu+(1-\mu)\theta]g[G(k_0)]\right\} \\ & - \beta\widetilde{\zeta}_0\left[\widetilde{d}_1^b - \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right] \\ & - \widetilde{\chi}_0\left\{u'(c_0) - \beta g'[G(k_0)]G'(k_0)(\mu+[(1-\tau_0^B)(1+\nu)-\mu]\theta)\right\}.\end{aligned}$$

The first order optimality conditions for this problem are

$$\begin{aligned}&\partial c_0 : u'(c_0) = \frac{v'(n_0)}{A_0} + \widetilde{\chi}_0 u''(c_0), \\ &\partial \widetilde{d}_1^b : \nu u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)] = \widetilde{\zeta}_0\left[1 + \widetilde{d}_1^b\frac{\nu u''[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right], \\ &\partial k_0 : \beta g'[G(k_0)]G'(k_0)\left\{\mu+(1-\mu)\theta + \frac{\widetilde{\zeta}_0\theta(1-\sigma_g)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right. \\ &\qquad\left. + \widetilde{\chi}_0[(1-\theta)\mu + (1-\tau_0^B)(1+\nu)\theta]\left[\frac{G''(k_0)}{G'(k_0)} - \sigma_g\frac{G'(k_0)}{G(k_0)}\right]\right\} = \frac{v'(n_0)}{A_0}.\end{aligned}$$

and the complementarity slackness conditions are

$$CSC_1 : \widetilde{d}_1^b \le \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]},\ \widetilde{\zeta}_0 \ge 0,\ \left\{\widetilde{d}_1^b - \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right\}\widetilde{\zeta}_0 = 0.$$

These optimality conditions imply

$$\begin{aligned}\widetilde{\chi}_0 =& \tau_0\frac{u'(c_0)}{u''(c_0)}, \\ \widetilde{\zeta}_0 =& \frac{\nu u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}{1+\widetilde{d}_1^b\frac{\nu u''[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}}, \\ u'(c_0) =& \beta\frac{g'[G(k_0)]G'(k_0)}{1-\tau_0}\left\{\mu+(1-\mu)\theta + \frac{\widetilde{\zeta}_0\theta(1-\sigma_g)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right. \\ &\left. + \tau_0\frac{u'(c_0)}{u''(c_0)}\{\mu+(1-\mu)\theta+\theta[(1-\tau_0^B)(1+\nu)-1]\}\left[\frac{G''(k_0)}{G'(k_0)} - \sigma_g\frac{G'(k_0)}{G(k_0)}\right]\right\}.\end{aligned}$$

Combining the last two equations with private durable investment optimality conditions leads to

$$\tau_0 = -\frac{1}{Z_1}\left[\tau_0^b - \left(\frac{\sigma_g\nu}{1+\nu} - \frac{\nu(1-\sigma_g)}{1+\nu}\cdot\frac{\frac{\nu\widetilde{d}_1^b\sigma}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}{1-\frac{\nu\widetilde{d}_1^b\sigma}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}\right)\right],$$

where

$$Z_1 = \left\{\frac{\mu+(1-\mu)\theta}{\theta(1+\nu)} + \left(\frac{\nu}{1+\nu} - \tau_0^B\right)\right\}\left\{1 + \frac{u'(c_0)}{u''(c_0)}\left[\frac{G''(k_0)}{G'(k_0)} - \sigma_g\frac{G'(k_0)}{G(k_0)}\right]\right\} > 0.$$

Part (ii) Let's consider optimal macroprudential policy conditional on monetary policy being set at i_0. The planner solves

$$\begin{aligned}
\max_{c_0,k_0,d_1^b} \quad & u(c_0) - v\left(\frac{c_0+k_0}{A_0}\right) + \beta\left\{u[y_1+\nu(\widetilde{d}_1^b+d_1^g)] + [\mu+(1-\mu)\theta]g[G(k_0)]\right\} \\
s.t. :& \widetilde{d}_1^b \leq \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}, \\
& \beta\frac{1+i_0}{\Pi^*}u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)](1+\nu) = u'(c_0).
\end{aligned}$$

The regulator's problem is characterized by the following Lagrangian

$$\begin{aligned}
\widetilde{\mathcal{L}}_0 =& u(c_0) - v\left(\frac{c_0+k_0}{A_0}\right) + \beta\left\{u[y_1+\nu(\widetilde{d}_1^b+d_1^g)] + [\mu+(1-\mu)\theta]g[G(k_0)]\right\} \\
& - \beta\widetilde{\zeta}_0\left[\widetilde{d}_1^b - \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right] - \widetilde{\phi}_0\left\{\beta\frac{1+i_0}{\Pi^*}u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)](1+\nu) - u'(c_0)\right\}.
\end{aligned}$$

The first order conditions are

$$\begin{aligned}
&\partial c_0 : u'(c_0) = \frac{v'(n_0)}{A_0} - \widetilde{\phi}_0 u''(c_0), \\
&\partial \widetilde{d}_1^b : \nu u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)] = \widetilde{\zeta}_0\left[1+\widetilde{d}_1^b\frac{\nu u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}{u''[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right] + \widetilde{\phi}_0\frac{1+i_0}{\Pi^*}(1+\nu)u''[y_1+\nu(\widetilde{d}_1^b+d_1^g)], \\
&\partial k_0 : \beta g'[G(k_0)]G'(k_0)\left[\mu+(1-\mu)\theta+\frac{\widetilde{\zeta}_0\theta(1-\sigma_g)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right] = \frac{v'(n_0)}{A_0}.
\end{aligned}$$

and the complementarity slackness conditions are

$$CSC_1 : \widetilde{d}_1^b \leq \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]},\ \widetilde{\zeta}_0 \geq 0,\ \left\{\widetilde{d}_1^b - \frac{\theta g'[G(k_0)]G(k_0)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right\}\widetilde{\zeta}_0 = 0.$$

The first order conditions can be rewritten as follows

$$\begin{aligned}
\tau_0 &= -\widetilde{\phi}_0\frac{u''(c_0)}{u'(c_0)}, \\
\frac{\widetilde{\zeta}_0}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]} &= \nu\frac{1+\widetilde{\phi}_0\frac{\frac{1+i_0}{\Pi^*}\cdot\frac{1+\nu}{\nu}}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}\sigma}{1-\frac{\sigma\nu\widetilde{d}_1^b}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}, \\
(1-\tau_0)u'(c_0) &= \beta g'[G(k_0)]G'(k_0)\left[\mu+(1-\mu)\theta+\frac{\widetilde{\zeta}_0\theta(1-\sigma_g)}{u'[y_1+\nu(\widetilde{d}_1^b+d_1^g)]}\right].
\end{aligned}$$

Comparing planner's optimal choice of investment in durable goods to private optimum I get

$$\tau_0^b = \frac{\nu\sigma_g}{1+\nu} - \frac{\nu(1-\sigma_g)}{1+\nu}\cdot\frac{\frac{\sigma\nu\widetilde{d}_1^b}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}{1-\frac{\sigma\nu\widetilde{d}_1^b}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}} - \tau_0 Z_2,$$

where

$$Z_2 = \frac{\tau_0}{1+\nu}\left[-\frac{u'(c_0)}{u''(c_0)}\cdot\frac{\frac{\frac{1+i_0}{\Pi^*}(1+\nu)}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}\sigma}{1-\frac{\sigma\nu\widetilde{d}_1^b}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}(1-\sigma_g)+\frac{1}{1-\tau_0}\left(\frac{\mu}{\theta}+(1-\mu)+\nu\frac{1-\frac{u'(c_0)}{u''(c_0)}\tau_0\cdot\frac{\frac{1+i_0}{\Pi^*}\cdot\frac{1+\nu}{\nu}}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}\sigma}{1-\frac{\sigma\nu\widetilde{d}_1^b}{y_1+\nu(\widetilde{d}_1^b+d_1^g)}}(1-\sigma_g)\right)\right]>0, \tag{A.1}$$

The two terms in Z_2 reflects two effects that bankers do not internalize when they decide to issue safe debt. First, higher level of safe debt allows a banker increase its investment in durable goods production. This increases "aggregate demand" in period 0. This has a positive welfare effect if a country is in recession, i.e., $\tau_0 > 0$. Second, higher level of safe debt increases consumers safe debt holdings, which allows them to buy more goods with safe debt in period 1. When the nominal (and real) interest rate does not adjust, higher consumption of goods bought with safe debt in period 1 lead to higher consumption of goods in period 0. As a result, "aggregate demand" increases. When the country is in recession, this has a positive welfare effects.

A.2 A 2-period Model of Currency Union

This section presents monetary union derivations and proofs omitted from the main text.

A.2.1 Household Problem Solution

A typical household in country i solves the following problem

$$\begin{aligned}
\mathcal{L}_0 =& \mathbb{E}\Bigg\{U(c^i_{NT,0}, c^i_{T,0}) - v(n^i_0) + \beta\Big[U[c^i_{NT,1}, \widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}] + X_1(s_1)g\left(h^i_1\right) \\
& - \Lambda^i_0\Big[T^i_0 + P^i_{NT,0}c^i_{NT,0} + P_{T,0}c^i_{T,0} + \frac{D^{c,i}_{NT,1}+D^{c,i}_{T,1}}{1+i_0} + P^i_{NT,0}k^i_0 \\
& \qquad - P_{T,0}e^i_{T,0} - \frac{\widetilde{D}^{b,i}_1}{1+i_0}\left(1-\tau^{b,i}_0\right) - W^i_0 n^i_0 - \Pi^i_0\Big] \\
& - \beta\Lambda^i_1\Big[P^i_{NT,1}c^i_{NT,1} + P_{T,1}\widetilde{c}^i_{T,1} + T^i_1 + \Gamma^i_1 h^i_1 + \widetilde{D}^{b,i}_1 \\
& \qquad - P_{T,1}e^i_{T,1} - D^{c,i}_{NT,1} - D^{c,i}_{T,1} - W^i_1 n^i_1 - \Gamma^i_1 G(k^i_0) - P^i_{NT,1}e^i_{NT,1}.\Big] \\
& - \beta\Lambda^i_1\zeta^i_0\left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\}G(k^i_0)\right] \\
& - \beta\Lambda^i_1\eta^i_1\left[P_{T,1}\underline{c}^i_{T,1} - D^c_{T,1}\right]
\end{aligned}$$

Let's introduce the following notation

$$
\begin{aligned}
U^i_{NT,0} &\equiv \frac{\partial U\left(c^i_{NT,0}, c^i_{T,0}\right)}{\partial c^i_{NT,0}},\ U^i_{T,0} \equiv \frac{\partial U\left(c^i_{NT,0}, c^i_{T,0}\right)}{\partial c^i_{T,0}},\\
U^i_{NT,1} &\equiv \frac{\partial U[c^i_{NT,1}, \widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}]}{\partial c^i_{NT,1}}, \quad U^i_{T,1} \equiv \frac{\partial U[c^i_{NT,1}, \widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}]}{\partial c^i_{T,1}},\\
\underline{U}^i_{T,1} &\equiv \frac{\partial U[c^i_{NT,1}, \widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}]}{\partial \underline{c}^i_{T,1}} = \nu^i U^i_{T,1},\\
G^i_{NT,0} &\equiv G'\left(k^i_0\right).
\end{aligned}
$$

The first order conditions can be written as follows

$$
\begin{aligned}
\partial c^i_{NT,0} &: U^i_{NT,0} = \Lambda^i_0 P^i_{NT,0},\\
\partial c^i_{T,0} &: U^i_{T,0} = \Lambda^i_0 P_{T,0},\\
\partial c^i_{NT,1} &: U^i_{NT,1} = \Lambda^i_1 P^i_{NT,1},\\
\partial c^i_{T,1} &: U^i_{T,1} = \Lambda^i_1 P_{T,1},\\
\partial \underline{c}^i_{T,1} &: U^i_{T,1} + \nu^i \underline{U}^i_{T,1} = P_{T,1}\Lambda^i_1\left(1+\eta^i_1\right),\\
\partial D^{c,i}_1 &: \frac{\Lambda^i_0}{1+i_0} = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\eta^i_1\right),\\
\partial \widetilde{D}^{b,i}_1 &: \frac{\Lambda^i_0}{1+i_0}\left(1-\tau^{b,i}_0\right) = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\zeta^i_0\right),
\end{aligned}
$$

$$
\begin{aligned}
\partial n^i_0 &: v'\left(n^i_0\right) = \Lambda^i_0(s_0) W^i_0,\\
\partial h^i_1 &: X_1(s_1) g'\left(h^i_1\right) = \Lambda^i_1 \Gamma^i_1,\\
\partial k^i_0 &: \Lambda^i_0 P^i_{NT,0} = \beta \mathbb{E}_0 G^i_{NT,0} \Lambda^i_1 \left(\Gamma^i_1 + \zeta^i_0 \min_{s_1}\{\Gamma^i_1\}\right),
\end{aligned}
$$

as well as complementarity slackness conditions

$$
\begin{aligned}
CSC_1 &: \widetilde{D}^{b,i}_1 \le \min_{s_1|s_0}\{\Gamma^i_1(s_1)\} G\left(k^i_0\right), \zeta^i_0 \ge 0, \left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\} G\left(k^i_0\right)\right]\zeta^i_0 = 0,\\
CSC_1 &: P_{T,1}\underline{c}^i_{T,1} \le D^{c,i}_1, \eta^i_1 \ge 0, \left(P_{T,1}\underline{c}^i_{T,1} - D^{c,i}_1\right)\eta^i_1 = 0.
\end{aligned}
$$

A.2.2 Proof of Lemma 1

The indirect utility function is

$$
\begin{aligned}
V^i =& U\left(c^i_{NT,0}, c^i_{T,0}\right) - v\left(n^i_0\right) + \beta \mathbb{E}_0\left[U\left(c^i_{NT,1}, c^i_{T,1} + \left(1+\nu^i\right)\underline{c}^i_{T,1}\right) + X_1(s_1) g\left(h^i_1\right)\right] \\
=& \frac{a^\sigma\left(c^i_{NT,0}\right)^{1-\sigma}}{1-\sigma} + c^i_{T,0} - v\left(n^i_0\right) + \beta\mathbb{E}_0\left[\frac{a^\sigma\left(c^i_{NT,1}\right)^{1-\sigma} + (1-a)^\sigma\left(c^i_{T,1} + \nu^i \underline{c}^i_{T,1}\right)^{1-\sigma}}{1-\sigma} + X_1(s_1) g\left(h^i_1\right)\right] \\
=& \frac{a\left(\frac{P_{T,0}}{P^i_{NT,0}}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} - v\left(\frac{a\left(\frac{P_{T,0}}{P^i_{NT,0}}\right)^{\frac{1}{\sigma}} + k^i_{NT,0}}{A^i_0}\right) \\
& + \beta\left\{\frac{a^\sigma\left(e^i_{NT,1}\right)^{1-\sigma} + (1-a)^\sigma[c^i_{T,1} + (1+\nu^i) d^{c,i}_1]^{1-\sigma}}{1-\sigma} + [\mu + (1-\mu)\theta] g[G(k^i_{NT,0})]\right\} \\
=& \frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} + \beta[\mu + (1-\mu)\theta] g\left(G(k^i_{NT,0})\right) - v\left(\frac{a\left(\frac{P_{T,0}}{P^i_{NT,0}}\right)^{\frac{1}{\sigma}} + k^i_{NT,0}}{A^i_0}\right) \\
& + \beta\frac{(1-a)^\sigma\left(c^i_{T,1} + \left(1+\nu^i\right) d^{c,i}_1\right)^{1-\sigma}}{1-\sigma} + \beta\frac{a^\sigma (e^i_{NT,1})^{1-\sigma}}{1-\sigma}
\end{aligned}
$$

$$
\begin{aligned}
=& \frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} - v\left(\frac{a\left(\frac{P_{T,0}}{P^i_{NT,0}}\right)^{\frac{1}{\sigma}} + k^i_{NT,0}}{A^i_0}\right) \\
& + \beta\frac{(1-a)^\sigma[c^i_{T,1} + \left(1+\nu^i\right) d^{c,i}_1]^{1-\sigma}}{1-\sigma} + \beta[\mu + (1-\mu)\theta] g[G(k^i_{NT,0})] + \beta\frac{a^\sigma (e^i_{NT,1})^{1-\sigma}}{1-\sigma}.
\end{aligned}
$$

The only endogenous variables in the last expression are $c^i_{T,0}, k^i_{NT,0}, c^i_{T,1} + \left(1+\nu^i\right) d^{c,i}_1$ which are functions of r_0.

A.2.3 Proof of Proposition 5

The regulator's problem can be summarized as follows

$$\max_{\substack{c^i_{T,0},\tilde{c}^i_{T,1},\underline{c}^i_{T,1}\\ k^i_0,\tilde{d}^{b,i}_1,d^{c,i}_1}} \frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} - v\left[\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}} + k^i_0}{A^i_0}\right]$$

$$+\beta\left\{\frac{(1-a)^\sigma\left(\tilde{c}^i_{T,1}+\nu^i d^{c,i}_1\right)^{1-\sigma}}{1-\sigma} + [\mu+(1-\mu)\theta]g\left(G(k^i_0)\right)\right\}$$

$$\text{s.t.: } 1 = \left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}(\tilde{c}^i_{T,1}+\nu^i\underline{c}^i_{T,1}),$$

$$\tilde{d}^{b,i}_1 \le \theta^i\beta\left(1+r_0\right)\left(1+\nu^i\right)g'[G(k^i_0)]G(k^i_0), \tag{A.2}$$

$$\underline{c}^i_{T,1} \le d^{c,i}_1, \tag{A.3}$$

$$c^i_{T,0} - e^i_{T,0} = \frac{\tilde{d}^{b,i}_1 + d^{g,i}_1 - d^{c,i}_1}{1+r_0}, \tag{A.4}$$

$$\tilde{c}^i_{T,1} - e^i_{T,1} = d^{c,i}_1 - \tilde{d}^{b,i}_1 - d^{g,i}_1. \tag{A.5}$$

Denote the Lagrange multipliers on the above constraints as $\tilde{\varphi}^i, \beta\tilde{\lambda}^i_1\tilde{\zeta}^i_0, \beta\tilde{\lambda}^i_1\tilde{\eta}^i_0, \tilde{\lambda}^i_0, \beta\tilde{\lambda}^i_1$ respectively. The first order conditions are

$$\partial c^i_{T,0}: 1 = \tilde{\lambda}^i_0,$$

$$\partial k_{NT,0}: \frac{v'(n^i_0)}{A^i_0} = \beta G'(k^i_0)g'[G(k^i_0)]\left[\mu+(1-\mu)\theta^i+\theta^i\tilde{\zeta}^i_0\beta\left(1+r_0\right)\left(1+\nu^i\right)\tilde{\lambda}^i_1\left(1-\sigma_g\right)\right], \tag{A.6}$$

$$\partial \underline{c}^i_{T,1}: \beta(1-a)^\sigma\left(c^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma}\nu^i + \tilde{\varphi}^i(1-a)^{-1}\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}\nu^i - \beta\tilde{\lambda}^i_1\tilde{\eta}^i_0 = 0,$$

$$\partial \tilde{c}^i_{T,1}: \beta(1-a)^\sigma\left(c^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma} + \tilde{\varphi}^i(1-a)^{-1}\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}} - \beta\tilde{\lambda}^i_1 = 0,$$

$$\partial \tilde{d}^{b,i}_1: -\beta\tilde{\lambda}^i_1\tilde{\zeta}^i_0 + \frac{\tilde{\lambda}^i_0}{1+r_0} - \beta\tilde{\lambda}^i_1 = 0,$$

$$\partial d^c_1: -\beta\tilde{\lambda}^i_1\tilde{\eta}^i_0 - \frac{\tilde{\lambda}^i_0}{1+r_0} + \beta\tilde{\lambda}^i_1 = 0.$$

Next we express the Lagrange multiplier $\tilde{\lambda}^i_1$

$$\tilde{\lambda}^i_1 = U^i_{T,1}\left\{1+\tilde{\varphi}^i\beta^{-1}\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{1-\frac{1}{\sigma}}\right\}.$$

Next, I use the first order condition with respect to $\tilde{d}^{b,i}_1$ to solve for $\tilde{\varphi}^i$

$$\tilde{\varphi}^i = 0.$$

Intuitively, because the household Euler equation pins down $\tilde{c}^i_{T,1} + \nu^i\underline{c}^i_{T,1}$, the planner does not alter this constraint when choosing his optimum. As a result, the Lagrange multiplier is zero. Combine formulas for

$\widetilde{\lambda}_1^i$ and $\widetilde{\phi}^i$ to obtain

$$\widetilde{\lambda}_1^i = U_{T,1}^i.$$

Finally, comparing private durable goods investment optimality condition (47) to the regulator's condition (A.6), I can express optimal prudential tax as follows

$$\tau_0^{b,i} = \frac{\nu^i \sigma_g}{1+\nu^i} - \frac{\tau_0^i}{1-\tau_0^i} \cdot \frac{\mu\left(1-\theta^i\right) + \theta^i[1+\nu^i(1-\sigma_g)]}{\theta^i(1+\nu^i)}. \tag{A.7}$$

A.2.4 Proof of Proposition 6

This section solves the global Ramsey planner problem that corresponds coordinated choice of optimal union-wide monetary and regional macroprudential policies. I start by assuming that the global regulator maximizes a weighted average of utilities across the countries to derive more general first order conditions. The weights are $\{\omega^i\}$ such that $\int \omega^i di = 1$. In the main text of the paper, I present the expression for the case with $\omega^i = 1$.

$$\begin{aligned}
\max_{\substack{\{k_{NT,0}^i, c_{T,0}^i, \underline{c}_{T,1}^i, c_{T,1}^i,\\ \widetilde{d}_1^{b,i}, d_1^{c,i}\}, P_{T,0}, r_0, i_0}} \mathbb{E} \int \omega^i \Bigg\{ & \frac{a\left(\frac{P_{T,1}}{P_{NT,0}^i}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c_{T,0}^i - v\left(\frac{a\left(\frac{P_{T,1}}{P_{NT,0}^i}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}} + k_{NT,0}^i}{A_0^i}\right) \\
& + \beta\left[\frac{(1-a)^\sigma\left(\widetilde{c}_{T,1}^i + \nu^i d_1^{c,i}\right)^{1-\sigma}}{1-\sigma} + [\mu + (1-\mu)\theta] g\left(G(k_{NT,0}^i)\right)\right]\Bigg\} di \\
\text{s.t.: } & \widetilde{d}_1^{b,i} \le \theta^i \beta\left(1+r_0\right)\left(1+\nu^i\right) g'[G(k_{NT,0}^i)]G(k_{NT,0}^i), \left[\beta\widetilde{\lambda}_1^i\widetilde{\zeta}_0^i\omega^i\right] \\
& \underline{c}_{T,1}^i \le d_1^{c,i}, \left[\beta\widetilde{\lambda}_1^i\widetilde{\eta}_0^i\omega^i\right] \\
& c_{T,0}^i - e_{T,0}^i - \frac{\widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i}}{1+r_0} = 0, \left[\widetilde{\lambda}_0^i\omega^i\right] \\
& \widetilde{c}_{T,1}^i - e_{T,1}^i - d_1^{c,i} + \widetilde{d}_1^{b,i} + d_1^{g,i} = 0, \left[\beta\widetilde{\lambda}_1^i\omega^i\right] \\
& 1 = \left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}(\widetilde{c}_{T,1}^i + \nu^i \underline{c}_{T,1}^i), [\widetilde{\phi}_i\omega_i] \\
& \int c_{T,0}^i di = \int e_{T,0}^i di, [\widetilde{\varphi}_0] \\
& \int \widetilde{c}_{T,1}^i di = \int e_{T,1}^i di, [\widetilde{\varphi}_1] \\
& -i_0 \le 0. [\widetilde{\xi}]
\end{aligned}$$

Note that the traded goods market clearing condition in one of the two periods is redundant because it can be obtained by summing the country-wide budget constraints across countries in both periods, and then using the traded goods market clearing condition in the other period. Thus, I drop the global market clearing condition for traded goods in period 1.

The first order conditions are

$$\partial c^i_{T,0}: 1-\widetilde{\lambda}^i_0-\frac{\widetilde{\varphi}_0}{\omega^i}=0,$$

$$\partial k_{NT,0}: \frac{v'(n^i_0)}{A^i_0}=\beta G'(k^i_{NT,0})g'[G(k^i_{NT,0})]\left[\mu+(1-\mu)\theta^i+\theta^i\widetilde{\zeta}^i_0\beta\left(1+r_0\right)\left(1+\nu^i\right)\widetilde{\lambda}^i_1\left(1-\sigma_g\right)\right],$$

$$\partial \underline{c}^i_{T,1}: \beta(1-a)^\sigma\left(\widetilde{c}^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma}\nu^i+\widetilde{\phi}^i\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}\nu^i-\beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0=0,$$

$$\partial \widetilde{c}^i_{T,1}: \beta(1-a)^\sigma\left(\widetilde{c}^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma}+\widetilde{\phi}^i\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}-\beta\widetilde{\lambda}^i_1=0,$$

$$\partial \widetilde{d}^{b,i}_1: -\beta\widetilde{\lambda}^i_1\widetilde{\zeta}^i_0+\frac{\widetilde{\lambda}^i_0}{1+r_0}-\beta\widetilde{\lambda}^i_1=0,$$

$$\partial d^c_1: -\beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0-\frac{\widetilde{\lambda}^i_0}{1+r_0}+\beta\widetilde{\lambda}^i_1=0,$$

$$\partial r_0: \int\omega^i\left\{\frac{1-\sigma}{\sigma}\cdot\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}(1+r_0)^{-1}-v'\left(\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}}+k^i_{NT,0}}{A^i_0}\right)\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}}(1+r_0)^{-1}}{A^i_0}\right.$$

$$\left.-\widetilde{\phi}^i\frac{1}{\sigma}\left[\beta(1+r_0)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1+r_0)^{-1}(1-a)^{-1}(\widetilde{c}^i_{T,1}+\nu^i\underline{c}^i_{T,1})+\beta\widetilde{\lambda}^i_1\widetilde{\zeta}^i_0\frac{\widetilde{d}^{b,i}_1}{1+r_0}-\widetilde{\lambda}^i_0\frac{\widetilde{d}^{b,i}_1+d^{g,i}_1-d^{c,i}_1}{(1+r_0)^2}\right\}di=0.$$

$$\partial i_0: \int\omega^i\left\{-\frac{1-\sigma}{\sigma}\cdot\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}(1+i_0)^{-1}\right.$$

$$\left.+v'\left(\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}}+k^i_{NT,0}}{A^i_0}\right)\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}}(1+i_0)^{-1}}{A^i_0}\right\}di+\widetilde{\xi}=0.$$

Next, I solve for $\widetilde{\phi}^i$ by substituting out $\widetilde{\lambda}^i_0$ and $\widetilde{\lambda}^i_1$ in the optimality condition for d^c_1:

$$\widetilde{\phi}^i=-\frac{\widetilde{\varphi}_0}{\omega^i}\beta\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{\frac{1-\sigma}{\sigma}}(1-a).$$

As a result, the Lagrange multiplier on the period-1 budget constraint is

$$\widetilde{\lambda}^i_1=U^i_{T,1}\left(1-\frac{\widetilde{\varphi}_0}{\omega^i}\right).$$

Using this expression, I can compute $\widetilde{\varphi}_0$ by substituting out $\widetilde{\lambda}^i_0$,$\widetilde{\lambda}^i_1$ and $\widetilde{\phi}^i$ in the FOC wrt r_0:

$$\widetilde{\varphi}_0=\sigma\frac{\int\omega^i(c^i_{T,0}-e^i_{T,0}-\beta\nu^i\widetilde{d}^{b,i}_1U^i_{T,1})di-(1+i_0)\widetilde{\xi}}{\beta\int\{[\beta(1+r_0)(1+\nu^i)]^{\frac{1}{\sigma}}(1-a)-\sigma\nu^i\widetilde{d}^{b,i}_1\}U^i_{T,1}di}.$$

If $\omega^i = 1$, then

$$\widetilde{\varphi}_0 = -\sigma \frac{\int \nu^i \widetilde{d}_1^{b,i} U^i_{T,1} di + \beta^{-1}(1+i_0)\widetilde{\xi}}{\int \left(\widetilde{c}^i_{T,1} + \nu^i d_1^{c,i} - \sigma \nu^i \widetilde{d}_1^{b,i}\right) U^i_{T,1} di},$$

where

$$(1+i_0)\sigma\widetilde{\xi} = \int \omega^i [\widetilde{\varphi}^i + \sigma\widetilde{\lambda}^i_0 (c^i_{T,0} - e^i_{T,0}) - \sigma\beta\widetilde{\lambda}^i_1 \nu^i \widetilde{d}_1^{b,i}] di.$$

The comparison of private and planner's optimal choices of $\{k_0^i\}$ leads to the following expression for optimal macroprudential tax in country i

$$\tau_0^{b,i} = \frac{\nu^i \sigma_g}{1+\nu^i} - \frac{\tau_0^i}{1-\tau_0^i} \cdot \frac{\theta^i(1+\nu^i - \nu^i\sigma_g) + \mu\left(1-\theta^i\right)}{1+\nu^i} + \frac{1}{1-\tau_0^i} \cdot \frac{\nu^i\left(1-\sigma_g\right)}{1+\nu^i} \cdot \frac{U^i_{T,1} - \widetilde{\lambda}^i_1}{U^i_{T,1}},$$

and

$$\frac{U^i_{T,1} - \widetilde{\lambda}^i_1}{U^i_{T,1}} = \frac{\widetilde{\varphi}_0}{\omega^i}.$$

This optimal macroprudential tax can now be expressed as follows

$$\tau_0^{b,i} = \frac{\nu^i \sigma_g}{1+\nu^i} - \frac{\tau_0^i}{1-\tau_0^i} \cdot \frac{\theta^i(1+\nu^i - \nu^i\sigma_g) + \mu\left(1-\theta^i\right)}{1+\nu^i} + \frac{1}{1-\tau_0^i} \cdot \frac{\nu^i\left(1-\sigma_g\right)}{1+\nu^i} \cdot \frac{\widetilde{\varphi}_0}{\omega^i} \tag{A.8}$$

A.2.5 Proof of Proposition 7

The union wide central bank solves

$$\max_{\substack{\{c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1}, k^i_0, \\ \widetilde{d}_1^{b,i}, d_1^{c,i}\}, r_0, i_0}} \int \Bigg\{ \frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma} + c^i_{T,0} - v\left(\frac{a\left(\frac{P_{T,1}}{P^i_{NT,0}} \cdot \frac{1+r_0}{1+i_0}\right)^{\frac{1}{\sigma}} + k^i_0}{A^i_0}\right)$$
$$+ \beta\left[\frac{(1-a)^\sigma \left(\widetilde{c}^i_{T,1} + \nu^i d_1^{c,i}\right)^{1-\sigma}}{1-\sigma} + [\mu + (1-\mu)\theta] g\left(G(k^i_0)\right)\right]\Bigg\} \omega^i di$$

$$\text{s.t.: } 1 = \left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}} (1-a)^{-1} (\widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}), \ [\widetilde{\phi}_i \omega_i]$$
$$\widetilde{d}_1^{b,i} \le \theta^i \beta\left(1+r_0\right)\left(1+\nu^i\right) g'[G(k^i_0)] G(k^i_0), \ \left[\beta\widetilde{\lambda}^i_1 \widetilde{\zeta}^i_0 \omega^i\right]$$
$$\underline{c}^i_{T,1} \le d_1^{c,i}, \ \left[\beta\widetilde{\lambda}^i_1 \widetilde{\eta}^i_0 \omega^i\right]$$
$$c^i_{T,0} - e^i_{T,0} = \frac{\widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i}}{1+r_0}, \ \left[\widetilde{\lambda}^i_0 \omega^i\right]$$

$$\widetilde{c}_{T,1}^{i}-e_{T,1}^{i}=d_{1}^{c,i}-\widetilde{d}_{1}^{b,i}-d_{1}^{g,i},\left[\beta\widetilde{\lambda}_{1}^{i}\omega^{i}\right]$$

$$\frac{v'(n_{0}^{i})}{A_{0}^{i}}=\beta G'(k_{0}^{i})g'[G(k_{0}^{i})]\left[\mu+(1-\mu)\theta^{i}+\theta^{i}\nu^{i}\left(1-\sigma_{g}\right)\right],\left[\widetilde{\psi}_{i}\omega_{i}\right]$$

$$\int c_{T,0}^{i}di=\int e_{T,0}^{i}di,\left[\widetilde{\varphi}_{0}\right],$$

$$\int\widetilde{c}_{T,1}^{i}di=\int e_{T,1}^{i}di,\left[\widetilde{\varphi}_{1}\right]$$

$$-i_{0}\leq0.[\widetilde{\xi}]$$

Note that the traded goods market clearing condition in one of the periods is redundant. On of the two conditions can be obtained by summing country-wide budget constraints across countries in both periods, and then using the traded goods market clearing condition in the other period. Thus, I drop global market clearing condition for traded goods in period 1.

The first order conditions are

$$\partial c_{T,0}^{i}:1-\widetilde{\lambda}_{0}^{i}-\frac{\widetilde{\varphi}_{0}}{\omega^{i}}=0,$$

$$\partial k_{NT,0}:-\frac{v'(n_{0}^{i})}{A_{0}^{i}}+\beta G'(k_{0}^{i})g'[G(k_{0}^{i})]\left[\mu+(1-\mu)\theta^{i}+\theta^{i}\widetilde{\zeta}_{0}^{i}\beta\left(1+r_{0}\right)\left(1+\nu^{i}\right)\widetilde{\lambda}_{1}^{i}\left(1-\sigma_{g}\right)\right]$$
$$-\widetilde{\psi}_{i}\left(\frac{v''\left(n_{0}^{i}\right)}{\left(A_{0}^{i}\right)^{2}}-\left\{\frac{G''(k_{0}^{i})}{G'(k_{0}^{i})}+G'(k_{0}^{i})\frac{g''[G(k_{0}^{i})]}{g'[G(k_{0}^{i})]}\right\}\beta G'(k_{0}^{i})g'[G(k_{0}^{i})]\left[\mu+(1-\mu)\theta^{i}+\theta^{i}\nu^{i}\left(1-\sigma_{g}\right)\right]\right)$$

$$\partial\underline{c}_{T,1}^{i}:\beta(1-a)^{\sigma}\left(\widetilde{c}_{T,1}^{i}+\nu^{i}\underline{c}_{T,1}^{i}\right)^{-\sigma}\nu^{i}+\widetilde{\phi}^{i}\left[\beta\left(1+r_{0}\right)\left(1+\nu^{i}\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}\nu^{i}-\beta\widetilde{\lambda}_{1}^{i}\widetilde{\eta}_{0}^{i}=0,$$

$$\partial\widetilde{c}_{T,1}^{i}:\beta(1-a)^{\sigma}\left(\widetilde{c}_{T,1}^{i}+\nu^{i}\underline{c}_{T,1}^{i}\right)^{-\sigma}+\widetilde{\phi}^{i}\left[\beta\left(1+r_{0}\right)\left(1+\nu^{i}\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}-\beta\widetilde{\lambda}_{1}^{i}=0,$$

$$\partial\widetilde{d}_{1}^{b,i}:-\beta\widetilde{\lambda}_{1}^{i}\widetilde{\zeta}_{0}^{i}+\frac{\widetilde{\lambda}_{0}^{i}}{1+r_{0}}-\beta\widetilde{\lambda}_{1}^{i}=0,$$

$$\partial d_{1}^{c}:-\beta\widetilde{\lambda}_{1}^{i}\widetilde{\eta}_{0}^{i}+\frac{\widetilde{\lambda}_{0}^{i}}{1+r_{0}}-\beta\widetilde{\lambda}_{1}^{i}=0,$$

$$\partial r_{0}:\int\omega^{i}\left\{\frac{1-\sigma}{\sigma}\cdot\frac{a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}(1+r_{0})^{-1}-v'\left(n_{0}^{i}\right)\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1}{\sigma}}(1+r_{0})^{-1}}{A_{0}^{i}}\right.$$
$$-\widetilde{\phi}^{i}\frac{1}{\sigma}\left[\beta(1+r_{0})\left(1+\nu^{i}\right)\right]^{-\frac{1}{\sigma}}(1+r_{0})^{-1}(1-a)^{-1}(\widetilde{c}_{T,1}^{i}+\nu^{i}\underline{c}_{T,1}^{i})+\beta\widetilde{\lambda}_{1}^{i}\widetilde{\zeta}_{0}^{i}\frac{\widetilde{d}_{1}^{b,i}}{1+r_{0}}-\widetilde{\lambda}_{0}^{i}\frac{\widetilde{d}_{1}^{b,i}+d_{1}^{g,i}-d_{1}^{c,i}}{(1+r_{0})^{2}}$$
$$\left.-\widetilde{\psi}_{i}\frac{v''\left(n_{0}^{i}\right)}{A_{0}^{i}}\cdot\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1}{\sigma}}(1+r_{0})^{-1}}{A_{0}^{i}}\right\}di=0.$$

$$\partial i_{0}:\int\omega^{i}\left\{-\frac{1-\sigma}{\sigma}\cdot\frac{a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma}(1+i_{0})^{-1}+v'\left(n_{0}^{i}\right)\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1}{\sigma}}(1+i_{0})^{-1}}{A_{0}^{i}}\right.$$
$$\left.+\widetilde{\psi}_{i}\frac{v''\left(n_{0}^{i}\right)}{A_{0}^{i}}\cdot\frac{\frac{1}{\sigma}\cdot a\left(\frac{P_{T,1}}{P_{NT,0}^{i}}\cdot\frac{1+r_{0}}{1+i_{0}}\right)^{\frac{1}{\sigma}}(1+i_{0})^{-1}}{A_{0}^{i}}\right\}di+\widetilde{\xi}=0.$$

Express $\widetilde{\psi}_i$

$$\widetilde{\psi}_i = -\frac{\theta^i \nu^i \left(1-\sigma_g\right)}{\omega^i[\mu + (1-\mu)\theta^i + \theta^i \nu^i (1-\sigma_g)]} \cdot \frac{k_0^i}{\frac{k_0^i}{k_0^i + c_{NT,0}^i}\rho_n + \sigma} \widetilde{\varphi}_0$$

Express $\widetilde{\varphi}_0$ by plugging the Lagrange multipliers in the FOC for r_0

$$\widetilde{\varphi}_0 = \sigma \frac{\int \left[(c_{T,0}^i - e_{T,0}^i) - \beta \nu^i \widetilde{d}_1^{b,i} U_{T,1}^i \right] \omega^i di - (1+i_0)\widetilde{\xi}}{\beta \int \left\{ \widetilde{c}_{T,1}^i + \nu^i \underline{c}_{T,1}^i - \sigma \nu^i \widetilde{d}_1^{b,i} \right\} U_{T,1}^i di}.$$

If $\omega^i = 1$, then

$$\widetilde{\varphi}_0 = -\sigma \frac{\int \nu^i \widetilde{d}_1^{b,i} U_{T,1}^i di + \beta^{-1}(1+i_0)\widetilde{\xi}}{\int \left[\widetilde{c}_{T,1}^i + \nu^i \left(d_1^{c,i} - \sigma \widetilde{d}_1^{b,i} \right) \right] U_{T,1}^i di} < 0.$$

As a result, $\widetilde{\psi}_i > 0$, specifically,

$$\widetilde{\psi}_i = \frac{\sigma \theta^i \nu^i \left(1-\sigma_g\right)}{\omega^i[\mu + (1-\mu)\theta^i + \theta^i \nu^i (1-\sigma_g)]} \cdot \frac{k_0^i}{\frac{k_0^i}{k_0^i + c_{NT,0}^i}\rho_n + \sigma} \cdot \frac{\int \nu^i \widetilde{d}_1^{b,i} U_{T,1}^i di + \beta^{-1}(1+i_0)\widetilde{\xi}}{\int \left[\widetilde{c}_{T,1}^i + \nu^i \left(d_1^{c,i} - \sigma \widetilde{d}_1^{b,i} \right) \right] U_{T,1}^i di}.$$

Finally, plug this expression in the FOC wrt i_0

$$\int \omega^i \left\{ \left[1 - \frac{v'\left(n_0^i\right)}{A_0^i U_{NT,0}^i} \right] U_{NT,0}^i \left(P_{NT,0}^i \right)^{-\frac{1}{\sigma}} \right\} di - \frac{1}{a} \left(P_{T,1} \frac{1+r_0}{1+i_0} \right)^{-\frac{1}{\sigma}} \sigma (1+i_0) \widetilde{\xi}$$
$$= \frac{\int \nu^i \widetilde{d}_1^{b,i} U_{T,1}^i di + \beta^{-1}(1+i_0)\widetilde{\xi}}{\int [\widetilde{c}_{T,1}^i + \nu^i (d_1^{c,i} - \sigma \widetilde{d}_1^{b,i})] U_{T,1}^i di} \cdot \int \frac{\sigma \theta^i \nu^i \left(1-\sigma_g\right)}{[\mu + (1-\mu)\theta^i + \theta^i \nu^i (1-\sigma_g)]} \cdot \frac{\rho_n k_0^i}{k_0^i \rho_n + \sigma(k_0^i + c_{NT,0}^i)} \cdot \frac{v'\left(n_0^i\right)}{A_0^i} \left(P_{NT,0}^i \right)^{-\frac{1}{\sigma}} di.$$

The last expression further simplifies to

$$\int \omega^i \tau_0^i U_{NT,0}^i c_{NT,0}^i di = \sigma (1+i_0)\widetilde{\xi} - \widetilde{\varphi}_0 Z,$$

where

$$Z \equiv \int \frac{\theta^i \nu^i (1-\sigma_g)}{\mu + (1-\mu)\theta^i + \theta^i \nu^i (1-\sigma_g)} \cdot \frac{\rho_n k_0^i c_{NT,0}^i}{k_0^i \rho_n + \sigma(k_0^i + c_{NT,0}^i)} \cdot \frac{v'\left(n_0^i\right)}{A_0^i} di.$$

A.2.6 Proof of Corollary 1

If countries are symmetric except for $\{A^i\}$, I simplify the last expression to get

$$U_{NT,0} \int \left[1 - \frac{v'(n_0^i)}{A_0^i U_{NT,0}^i} \right] di - \frac{1}{a} \left(\frac{P_{T,1}}{P_{NT,0}} \cdot \frac{1+r_0}{1+i_0} \right)^{-\frac{1}{\sigma}} \sigma (1+i_0)\widetilde{\xi}$$
$$= \frac{\sigma \theta \nu (1-\sigma_g)}{\mu + (1-\mu)\theta + \theta \nu (1-\sigma_g)} \cdot \frac{\nu U_{T,1} \int \widetilde{d}_1^{b,i} di + \beta^{-1}(1+i_0)\widetilde{\xi}}{U_{T,1} \int \{ e_{T,1}^i + \nu [d_1^{g,i} + (1-\sigma)\, \widetilde{d}_1^{b,i}] \} di} \int \frac{\rho_n k_0^i}{k_0^i \rho_n + \sigma(k_0^i + c_{NT,0}^i)} \cdot \frac{v'\left(n_0^i\right)}{A_0^i} di.$$

Assume that the union is not at the ZLB. I get

$$
\begin{aligned}
&U_{NT,0}\int\left[1-\frac{v'\left(n_0^i\right)}{A_0^i U_{NT,0}^i}\right]di\\
=&\frac{\sigma\theta\nu(1-\sigma_g)}{\mu+(1-\mu)\theta+\theta\nu(1-\sigma_g)}\cdot\frac{\nu\int\tilde{d}_1^{b,i}di}{\int\{e_{T,1}^i+\nu[d_1^{g,i}+(1-\sigma)\tilde{d}_1^{b,i}]\}di}\int\frac{\rho_n k_0^i}{k_0^i\rho_n+\sigma(k_0^i+c_{NT,0}^i)}\cdot\frac{v'\left(n_0^i\right)}{A_0^i}di,
\end{aligned}
$$

Assume that countries are symmetric then

$$
\frac{\tau_0}{1-\tau_0}=\frac{\rho_n\sigma\theta\nu(1-\sigma_g)}{\mu+(1-\mu)\theta+\theta\nu(1-\sigma_g)}\cdot\frac{\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}\cdot\frac{\Omega^{\frac{1}{1-\sigma}}}{(\rho_n+\sigma)\Omega^{\frac{1}{1-\sigma}}+\sigma a}. \quad (A.9)
$$

Using equation (**??**), I can write

$$
\begin{aligned}
\frac{\tau_0}{1-\tau_0}=&A_0^{1+\rho_n}[\beta(1+\nu)]^{\frac{\rho_n+\sigma}{\sigma}}\psi_n^{-1}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{-\frac{\rho_n+\sigma}{\sigma}}\\
&\cdot\left(e_{T,1}+\nu d_1^g\right)^{-(\rho_n+\sigma)}\left(\frac{1-\theta\nu\left[\beta(1+\nu)\right]^{\frac{\sigma-1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}^i}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{\left(a+\Omega^{\frac{1}{1-\sigma}}\right)^{\frac{\rho_n}{\rho_n+\sigma}}}\right)^{\rho_n+\sigma}-1. \quad (A.10)
\end{aligned}
$$

Combine the last two expressions

$$
\begin{aligned}
\frac{\tau_0}{1-\tau_0}=&A_0^{1+\rho_n}[\beta(1+\nu)]^{\frac{\rho_n+\sigma}{\sigma}}\psi_n^{-1}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{-\frac{\rho_n+\sigma}{\sigma}}(e_{T,1}+\nu d_1^g)^{-(\rho_n+\sigma)}\\
&\cdot\left(a+\Omega^{\frac{1}{1-\sigma}}\right)^{-\rho_n}\left\{1-\theta\nu\left[\beta(1+\nu)\right]^{\frac{\sigma-1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}^i}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}\right\}^{\rho_n+\sigma}-1\\
=&\frac{\theta\nu(1-\sigma_g)}{\mu+(1-\mu)\theta+\theta\nu(1-\sigma_g)}\cdot\frac{\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}\cdot\frac{\rho_n\Omega^{\frac{1}{1-\sigma}}}{(\rho_n+\sigma)\Omega^{\frac{1}{1-\sigma}}+\sigma a}.
\end{aligned}
$$

The last equation determines i_0 as a function of τ_0^b (recall that Ω is a function of i_0 and τ_0^b).

Finally, after expressing τ_0^b through Ω, I get

$$
\frac{\theta(1+\nu)+(1-\theta)\mu-(\alpha_G\beta)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta(1+\nu)}=\frac{\nu\sigma_g}{1+\nu}-\frac{\tau_0}{1-\tau_0}\cdot\frac{\theta\left[1+\nu\left(1-\sigma_g\right)\right]+\mu(1-\theta)}{\theta(1+\nu)}. \quad (A.11)
$$

To compare the regulation, we simply need to compare the following two expressions (LHS is part of Nash taxes and the RHS is part of coordinated taxes)

$$-\frac{\tau_0}{1-\tau_0}\cdot\frac{\theta\left[1+\nu\left(1-\sigma_g\right)\right]+\mu\left(1-\theta\right)}{\theta\left(1+\nu\right)} > -\frac{\nu\left(1-\sigma_g\right)}{1+\nu}\cdot\frac{\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega^{Coop}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0^{Coop}}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega^{Coop}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0^{Coop}}\right)^{\frac{1-\sigma}{\sigma}}},$$

$$\frac{\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega^{Coop}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0^{Coop}}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega^{Coop}\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0^{Coop}}\right)^{\frac{1-\sigma}{\sigma}}} > \frac{\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}{1-\sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}}\cdot\frac{\rho_n\Omega^{\frac{1}{1-\sigma}}}{(\rho_n+\sigma)\Omega^{\frac{1}{1-\sigma}}+\sigma a}.$$

Note that because of equation (A.11), it is clear that

$$\Omega^{Coop}(\rho_n=0) > \Omega^{Nash}(\rho_n=0)$$

Combine equations into two

$$\begin{aligned}
\frac{\theta\left(1+\nu\right)+\left(1-\theta\right)\mu-\left(\alpha_G\beta\right)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta\left(1+\nu\right)} &= \frac{\nu\sigma_g}{1+\nu}-\frac{\nu(1-\sigma_g)}{(1+\nu)}\cdot\frac{x}{1-x}\cdot\frac{\rho_n\Omega^{\frac{1}{1-\sigma}}}{(\rho_n+\sigma)\Omega^{\frac{1}{1-\sigma}}+\sigma a}, \\
\Upsilon\frac{[1-x]^{\rho_n+\sigma}}{x^{\frac{\rho_n+\sigma}{1-\sigma}}\Omega^{-\frac{\rho_n+\sigma}{1-\sigma}}\left(a+\Omega^{\frac{1}{1-\sigma}}\right)^{\rho_n}} & \\
=1-\frac{\theta\left(1+\nu\right)}{\theta[1+\nu(1-\sigma_g)]+\mu\left(1-\theta\right)}&\left(\frac{\theta\left(1+\nu\right)+\left(1-\theta\right)\mu-\left(\alpha_G\beta\right)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta\left(1+\nu\right)}-\frac{\nu\sigma_g}{1+\nu}\right),
\end{aligned}$$

where

$$\begin{aligned}
x &= \sigma\nu\theta[\beta(1+\nu)]^{1-\frac{1}{\sigma}}\Omega\left(\frac{P_{T,1}}{P^i_{NT,0}}\cdot\frac{1}{1+i_0}\right)^{\frac{1-\sigma}{\sigma}}, \\
\Upsilon &\equiv A_0^{1+\rho_n}[\beta(1+\nu)]^{\frac{\rho_n+\sigma}{\sigma}}\psi_n^{-1}\left(e_{T,1}+\nu d_1^g\right)^{-(\rho_n+\sigma)}\left\{\theta\nu\left[\beta(1+\nu)\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\rho_n+\sigma}{1-\sigma}}.
\end{aligned}$$

Note that $\Omega^{-\frac{\rho_n+\sigma}{1-\sigma}}\left(a+\Omega^{\frac{1}{1-\sigma}}\right)^{\rho_n}$ decreases with Ω for all values of Ω. Rewrite the two equations

$$\begin{aligned}
\frac{1-\sigma_g}{\sigma_g}\cdot\frac{x}{1-x} &= \left(1+\frac{\sigma}{\rho_n}+\frac{\sigma a}{\rho_n}\Omega^{-\frac{1}{1-\sigma}}\right)\left[1-\frac{\theta\left(1+\nu\right)+\left(1-\theta\right)\mu-\left(\alpha_G\beta\right)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta\nu\sigma_g}\right], \\
\Upsilon\frac{(1-x)^{\rho_n+\sigma}}{x^{\frac{\rho_n+\sigma}{1-\sigma}}} &= \Omega^{-\frac{\rho_n+\sigma}{1-\sigma}}\left(a+\Omega^{\frac{1}{1-\sigma}}\right)^{\rho_n}\cdot \\
&\quad\left[1+\frac{\theta\nu\sigma_g}{\theta[1+\nu(1-\sigma_g)]+\mu\left(1-\theta\right)}\left(1-\frac{\theta\left(1+\nu\right)+\left(1-\theta\right)\mu-\left(\alpha_G\beta\right)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta\nu\sigma_g}\right)\right].
\end{aligned}$$

I can simplify these equations further as follows

$$x = 1 - \frac{1}{1 - \left(1 + \frac{\sigma}{\rho_n} + \frac{\sigma a}{\rho_n}\Omega^{-\frac{1}{1-\sigma}}\right)\left(1 + \frac{\theta + (1-\theta)\mu - (\alpha_G \beta)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta \nu (1-\sigma_g)}\right)},$$

$$Y \frac{(1-x)^{\rho_n + \sigma}}{x^{\frac{\rho_n+\sigma}{1-\sigma}}} = \Omega^{-\frac{\rho_n+\sigma}{1-\sigma}} \left(a + \Omega^{\frac{1}{1-\sigma}}\right)^{\rho_n} \frac{(\alpha_G \beta)^{-1}\Omega^{\frac{\sigma}{1-\sigma}}}{\theta[1 + \nu(1-\sigma_g)] + \mu(1-\theta)}.$$

[Add here a figure that shows the intersection which completes the proof]

A.2.7 Proof of Lemma 2

The proof of this Lemma proceeds in several steps. First, I derive the household optimality conditions. Then I summarize all equilibrium conditions. Finally, I reduce the equilibrium conditions to a smaller set of equations that will become the constraints in the planners problem.

Household problem. The problem of a household in country i outside of monetary union

$$\begin{aligned}
\mathcal{L}_0 = & \mathbb{E}\Bigg\{ U(c^i_{NT,0}, c^i_{T,0}) - v(n^i_0) + \beta\left[U(c^i_{NT,1}, \widetilde{c}^i_{T,1} + \nu^i \underline{c}^i_{T,1}) + X_1(s_1) g\left(h^i_1\right)\right] \\
& - \Lambda^i_0 \Big[T^i_0 + P^i_{NT,0} c^i_{NT,0} + P^i_{T,0} c^i_{T,0} + \frac{D^{c,i,i}_1}{1+i^i_0} + \frac{D^{c,u}_1}{1+i_0} E^i_0 + P^i_{NT,0} k^i_0 \\
& \qquad - P^i_{T,0} e^i_{T,0} - \frac{\widetilde{D}^{b,i}_1}{1+i_0}(1 - \tau^{b,i}_0) - W^i_0 n^i_0 - \Pi^i_0\Big] \\
& - \beta \Lambda^i_1 \Big[P^i_{NT,1} c^i_{NT,1} + P^i_{T,1} \widetilde{c}^i_{T,1} + T^i_1 + \Gamma^i_1 h^i_1 + \widetilde{D}^{b,i}_1 \\
& \qquad - P^i_{T,1} e^i_{T,1} - P^i_{NT,1} e^i_{NT,1} - D^{c,i}_1 - D^{c,u}_1 E^i_1 - \Gamma^i_1 G(k^i_0). \Big] \\
& - \beta \Lambda^i_1 \zeta^i_0 \left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\} G(k^i_0)\right] \\
& - \beta \Lambda^i_1 \eta^i_1 \left[P^i_{T,1} \underline{c}^i_{NT,1} - D^{c,i,i}_1 - D^{c,i,u}_1 E^i_1\right],
\end{aligned}$$

where $D^{c,i,i}_1$ and $D^{c,i,u}_1$ are the amounts of safe debt denominated in home and monetary union currency respectively, purchased by the household in country i. Observe that the above formulation of the problem assumes that bankers in country i issue safe debt only denominated in local currency. This is without loss of generality because in equilibrium they are indifferent between issuing safe debt in local or foreign currency.

The first order conditions can be written as follows

$$\begin{aligned}
\partial c^i_{NT,0} &: U^i_{NT,0} = \Lambda^i_0 P^i_{NT,0}, \\
\partial c^i_{T,0} &: U^i_{T,0} = \Lambda^i_0 P^i_{T,0}, \\
\partial c^i_{NT,1} &: U^i_{NT,1} = \Lambda^i_1 P^i_{NT,1}, \\
\partial \widetilde{c}^i_{T,1} &: U^i_{T,1} = \Lambda^i_1 P^i_{T,1}, \\
\partial \underline{c}^i_{T,1} &: \nu^i U^i_{T,1} = P^i_{T,1} \Lambda^i_1 \left(1+\eta^i_1\right), \\
\partial D^{c,i,i}_1 &: \frac{\Lambda^i_0}{1+i^i_0} = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\eta^i_1\right), \\
\partial D^{c,i,u}_1 &: \frac{\Lambda^i_0}{1+i_0} E^i_0 = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\eta^i_1\right) E^i_1, \\
\partial \widetilde{D}^{b,i}_1 &: \frac{\Lambda^i_0}{1+i_0}(1-\tau^{b,i}_0) = \beta \mathbb{E}_0 \Lambda^i_1 \left(1+\zeta^i_0\right), \\
\partial n^i_0 &: v'\left(n^i_0\right) = \Lambda^i_0(s_0) W^i_0, \\
\partial h^i_1 &: X_1(s_1) g'\left(h^i_1\right) = \Lambda^i_1 \Gamma^i_1, \\
\partial k^i_0 &: \Lambda^i_0 P^i_{NT,0} = \beta \mathbb{E}_0 G^i_{NT,0} \Lambda^i_1 \left(\Gamma^i_1 + \zeta^i_0 \min_{s_1}\{\Gamma^i_1\}\right).
\end{aligned}$$

as well as complementarity slackness conditions

$$\begin{aligned}
CSC_1 &: \widetilde{D}^{b,i}_1 \le \min_{s_1|s_0}\{\Gamma^i_1(s_1)\} G\left(k^i_0\right), \zeta^i_0 \ge 0, \left[\widetilde{D}^{b,i}_1 - \min_{s_1}\{\Gamma^i_1\} G\left(k^i_0\right)\right] \zeta^i_0 = 0, \\
CSC_2 &: P^i_{T,1} \underline{c}^i_{T,1} \le D^{c,i,i}_1 + D^{c,i,u}_1 E^i_1, \eta^i_1 \ge 0, \left[P^i_{T,1} \underline{c}^i_{N,1} - D^{c,i,i}_1 - D^{c,i,u}_1 E^i_1\right] \eta^i_1 = 0.
\end{aligned}$$

Equilibrium conditions. Define home and monetary union safe real interest rate (in units of traded goods) as before

$$\begin{aligned}
r_0 &\equiv \frac{1+i_0}{P_{T,1}/P_{T,0}}, \\
r^i_0 &\equiv \frac{1+i^i_0}{P^i_{T,1}/P^i_{T,0}},
\end{aligned}$$

The first order conditions with respect to $\partial D^{c,i}_1$ and $\partial D^{c,i,u}_1$ together with the assumption that there is no uncertainty in nominal exchange rate can be combined to yield the interest rate parity

$$\frac{1+i^i_0}{1+i_0} = \frac{E^i_1}{E^i_0}.$$

This condition can be rewritten taking into account the definitions of safe real interest rates and exchange rate as follows

$$r_0 = r^i_0.$$

The full set of equilibrium conditions is

$$
\begin{aligned}
\zeta_0^i &= (1-\tau_0^{b,i})(1+\nu^i) - 1 \geq 0, \tilde{d}_1^{b,i} \leq G(k_0^i) \min_{s_1} \frac{X_1(s_1) g'(h_1^i)}{U_{T,1}^i}, \\
&\left[\tilde{d}_1^{b,i} - G(k_0^i) \min_{s_1|s_0} \frac{X_1(s_1) g'(h_1^i)}{U_{T,1}^i}\right] \zeta_0^i = 0, \\
\eta_1^i &= \nu^i \geq 0,\ \underline{c}_{T,1}^i \leq d_1^{c,i} + d_1^{c,i,u}, [\underline{c}_{T,1}^i - d_1^{c,i} - d_1^{c,i,u}]\eta_1 = 0, \\
U_{T,0}^i &= (1+r_0)\beta \left(1+\nu^i\right) \mathbb{E}_0 U_{T,1}^i, \\
\frac{v'(n_0^i)}{U_{NT,0}^i} &= \frac{W_0^i}{P_{NT,0}^i}, \\
U_{NT,0}^i &= \beta G_{NT,0}^i \mathbb{E}_0 \left[X_1(s_1) g'(h_1^i) + \Lambda_1^i \zeta_0^i \min_{s_1} \left\{\frac{X_1(s_1) g'(h_1^i)}{\Lambda_1^i}\right\}\right], \\
\frac{U_{NT,0}^i}{U_{T,0}^i} &= p_0^i, \\
\frac{U_{NT,1}^i}{U_{T,1}^i} &= p_1^i, \\
\frac{\underline{U}_{NT,1}^i}{\underline{U}_{T,1}^i} &= \frac{U_{NT,1}^i}{U_{T,1}^i}, \\
c_{T,0}^i - e_{T,0}^i &+ \frac{d_1^{c,i} - d_1^{b,i} - d_1^{g,i}}{1+r_0} = 0, \\
\tilde{c}_{T,1}^i - e_{T,1}^i &+ d_1^{b,i} + d_1^{g,i} - d_1^{c,i} = 0,
\end{aligned}
$$

$$
\begin{aligned}
A_0^i n_0^i &= k_0^i + c_{NT,0}^i, \\
e_{NT,1}^i &= c_{NT,1}^i, \\
h_1^i &= G(k_0^i),
\end{aligned}
$$

where $d_1^{c,i} \equiv \left(D^{c,i,i} + D^{c,i,u} E_1^i\right) / P_{T,1}^i$. The full set of equilibrium conditions can be written as follows

$$\zeta_0^i = (1-\tau_0^{b,i})(1+\nu^i) - 1 \geq 0, \widetilde{d}_1^{b,i} \leq \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i), \left[\widetilde{d}_1^{b,i} - \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i)\right] \zeta_0^i = 0$$

$$\eta_1^i = \nu^i \geq 0,\ \underline{c}_{T,1}^i \leq d_1^{c,i}, (\underline{c}_{T,1}^i - d_1^{c,i})\eta_1^i = 0,$$

$$1 = [\beta(1+r_0)(1+\nu^i)]^{-\frac{1}{\sigma}} \frac{\widetilde{c}_{T,1}^i + \nu^i \underline{c}_{T,1}^i}{1-a},$$

$$\frac{v'(n_0^i)}{U_{NT,0}^i} = \frac{W_0^i}{P_{NT,0}^i},$$

$$\left(\frac{a}{c_{NT,0}^i}\right)^{\sigma} = p_0^i,$$

$$\frac{a}{c_{NT,0}^i} = \beta g'[G(k_0^i)]G'(k_0^i)[\mu + (1-\mu)\theta^i + \zeta_0^i \theta^i],$$

$$c_{T,0}^i - e_{T,0}^i + \frac{d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}}{1+r_0} = 0,$$

$$\widetilde{c}_{T,1}^i - e_{T,1}^i + \widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i} = 0.$$

Dropping some equilibrium equations. I drop variables and equilibrium conditions that involve variables that do not affect the household utility function. The remaining conditions are

$$\zeta_0^i = (1-\tau_0^{b,i})(1+\nu^i) - 1 \geq 0, \widetilde{d}_1^{b,i} \leq \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i), \left[\widetilde{d}_1^{b,i} - \theta^i \frac{g'[G(k_0^i)]}{U_{T,1}^i} G(k_0^i)\right] \zeta_0^i = 0$$

$$\eta_1^i = \nu^i \geq 0,\ \underline{c}_{T,1}^i \leq d_1^{c,i}, (\underline{c}_{T,1}^i - d_1^{c,i})\eta_1^i = 0,$$

$$1 = [\beta(1+r_0)(1+\nu^i)]^{-\frac{1}{\sigma}} \frac{\widetilde{c}_{T,1}^i + \nu^i \underline{c}_{T,1}^i}{1-a},$$

$$c_{T,0}^i - e_{T,0}^i + \frac{d_1^{c,i} - \widetilde{d}_1^{b,i} - d_1^{g,i}}{1+r_0} = 0,$$

$$\widetilde{c}_{T,1}^i - e_{T,1}^i + \widetilde{d}_1^{b,i} + d_1^{g,i} - d_1^{c,i} = 0.$$

A.2.8 Proof of Proposition 8

Step 1. The local planner outside of monetary union solves the following problem

$$\max_{\substack{c^i_{NT,0}, c^i_{T,0}, \widetilde{c}^i_{T,1}, \underline{c}^i_{T,1} \\ k^i_0, \widetilde{d}^{b,i}_1, d^{c,i}_1}} \frac{a\left(c^i_{NT,0}\right)^{1-\sigma}}{1-\sigma} + c^i_{T,0} - v\left(\frac{c^i_{NT,0}+k^i_0}{A^i_0}\right)$$

$$+\beta\left\{\frac{(1-a)^\sigma\left(\widetilde{c}^i_{T,1}+\nu^i d^{c,i}_1\right)^{1-\sigma}}{1-\sigma} + [\mu+(1-\mu)\theta]g\left(G(k^i_0)\right)\right\}$$

$$\begin{aligned}
\text{s.t.: } 1 &= \left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}(1-a)^{-1}(\widetilde{c}^i_{T,1}+\nu^i\underline{c}^i_{T,1}),\\
\widetilde{d}^{b,i}_1 &\le \theta^i\beta\left(1+r_0\right)\left(1+\nu^i\right)g'[G(k^i_0)]G(k^i_0),\\
\underline{c}^i_{T,1} &\le d^{c,i}_1,\\
c^i_{T,0}-e^i_{T,0} &= \frac{\widetilde{d}^{b,i}_1+d^{g,i}_1-d^{c,i}_1}{1+r_0},\\
\widetilde{c}^i_{T,1}-e^i_{T,1} &= d^{c,i}_1-\widetilde{d}^{b,i}_1-d^{g,i}_1.
\end{aligned}$$

Denote the Lagrange multipliers on the above constraints as $\widetilde{\phi}^i, \beta\widetilde{\lambda}^i_1\widetilde{\zeta}^i_0, \beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0, \widetilde{\lambda}^i_0, \beta\widetilde{\lambda}^i_1$ respectively. The first order conditions are

$$\begin{aligned}
\partial c^i_{NT,0} :& \tau^i_0 = 0,\\
\partial c^i_{T,0} :& 1 = \widetilde{\lambda}^i_0,\\
\partial k_{NT,0} :& \frac{v'(n^i_0)}{A^i_0} = \beta G'(k^i_0)g'[G(k^i_0)]\left[\mu+(1-\mu)\theta^i+\theta^i\widetilde{\zeta}^i_0\beta\left(1+r_0\right)\left(1+\nu^i\right)\widetilde{\lambda}^i_1\left(1-\sigma_g\right)\right],\\
\partial\underline{c}^i_{T,1} :& \beta(1-a)^\sigma\left(c^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma}\nu^i+\widetilde{\phi}^i(1-a)^{-1}\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}\nu^i-\beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0 = 0,\\
\partial\widetilde{c}^i_{T,1} :& \beta(1-a)^\sigma\left(c^i_{T,1}+\nu^i\underline{c}^i_{T,1}\right)^{-\sigma}+\widetilde{\phi}^i(1-a)^{-1}\left[\beta\left(1+r_0\right)\left(1+\nu^i\right)\right]^{-\frac{1}{\sigma}}-\beta\widetilde{\lambda}^i_1 = 0,\\
\partial\widetilde{d}^{b,i}_1 :& -\beta\widetilde{\lambda}^i_1\widetilde{\zeta}^i_0+\frac{\widetilde{\lambda}^i_0}{1+r_0}-\beta\widetilde{\lambda}^i_1 = 0,\\
\partial d^c_1 :& -\beta\widetilde{\lambda}^i_1\widetilde{\eta}^i_0-\frac{\widetilde{\lambda}^i_0}{1+r_0}+\beta\widetilde{\lambda}^i_1 = 0.
\end{aligned}$$

Step 2. I now express the Lagrange multiplier on the Euler equation. Because $\widetilde{\phi}^i = 0$, I get that

$$\widetilde{\lambda}^i_1 = U^i_{T,1}.$$

Finally, comparing private durable goods investment optimality condition to the regulator's condition, I can express optimal prudential tax as

$$\tau^{b,i}_0 = \frac{\nu^i\sigma_g}{1+\nu^i}.$$