

# The International Medium of Exchange\*

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## Abstract

We propose a model of endogenous, persistent coordination on the international medium of exchange. An asset becomes the dominant international medium because it is widely held, and remains widely held because it is dominant. The country issuing the dominant asset is a net debtor, but earns an “exorbitant privilege” on its position. In a calibrated model, only steady states with one dominant asset are stable. The dominant country experiences a significant welfare gain, most of which is accrued during its rise to dominance. A mild trade war reduces privilege slightly, while a protracted or deep trade war eliminates it altogether.

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# 1 Introduction

The US dollar plays a central role in international trade in both goods and financial assets.<sup>1</sup> At the same time, the US has a unique external position: it is the world’s largest debtor country, but earns positive net income from its positions vis-a-vis the rest of the world.<sup>2</sup> These two phenomena, trade dollarization and exorbitant privilege, are puzzling from the viewpoint of standard models, and could potentially have large welfare implications. For these reasons, each has received significant attention in recent research, but the two have rarely been analyzed jointly.

In this paper, we propose a new theory of these facts, based on endogenous coordination on an international medium of exchange. The essential insight of our theory is that asset availability matters: when more than one asset might serve as a medium of exchange, traders tend to coordinate on using the asset that is most readily available to all counterparties. In the context of international trade, when safe dollar assets are more widely held worldwide, more of trade will be facilitated by dollars. On the other hand, if dollars are heavily used in trade, then foreigners will optimally choose to hold substantial stocks of dollar-denominated assets in order to facilitate trade. Thus, the feedback between the choice of trading currency and portfolio holdings mutually reinforce coordination on a dominant medium of exchange.

Our analysis begins with an analytical model that includes the essential ingredients of our asset availability mechanism. In the model, trading firms from a continuum of countries seek to engage in an international transaction with firms from other countries. Contracting frictions require firms to collateralize their cross-country transactions with safe assets, which they seek in frictional domestic funding markets.<sup>3</sup> On the other side of these credit markets are domestic households, who offer intra-period loans of their holdings of the two safe assets, US and EU bonds, for a fee. Lastly, if two firms trade together but use different types of collateral, the transaction surplus is reduced by a “currency mismatch” cost.<sup>4</sup>

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<sup>1</sup>Gopinath (2015) shows that dollar-invoiced trade is five times larger than trade directly involving US counterparties, and the dollar is far and away the largest third party currency used for trade invoice. Goldberg (2011) and Maggiori et al. (2017) show that 85% of foreign exchange rate transactions involve the USD, 60% international debt securities are issued in dollars, and dollar debt is virtually the only debt asset that foreign investors buy denominated in its own currency.

<sup>2</sup>Estimates of this return differential range from below 1% to 3% (e.g. Gourinchas and Rey, 2007a).

<sup>3</sup>The importance of financing frictions in trade is well-established, and many different financing strategies have emerged to serve the need for collateralization. See the overview in BIS (2014), or Amiti and Weinstein (2011), Antràs (2013), Ahn (2015), Antras and Foley (2015), and Niepmann and Schmidt-Eisenlohr (2017) for detailed evidence.

<sup>4</sup>Amiti et al. (2018) present evidence of coordination incentives using firm-level data from Belgium.

Because of search frictions, firms prefer *ceteris paribus* to search for an asset that is relatively plentiful in local funding markets. Hence, if an asset is broadly held by households worldwide, its availability to potential trading partners encourages coordination on that asset as the dominant international currency. Conversely, households are aware that an asset that is actively used in trade is more likely to be loaned in collateral markets and earn the associated fee. Thus, the incentives of households and firms are mutually reinforcing: wide holdings of an asset drive its adoption as the medium of exchange, while higher adoption encourages households around the world to maintain large positions in that asset.

We characterize the equilibrium in the analytical model in a series of propositions. In particular, we show that the feedback between households and firms described above leads the model to have three steady states: a dollar-dominant equilibrium in which all trade is denominated in dollars, a mirror-image euro-dominant equilibrium, and a symmetric multipolar equilibrium in which each currency intermediates one half of transactions. Importantly, all three steady states obtain regardless of the size of the currency mismatch costs.

We demonstrate several key features of the analytical economy that correspond well with historical experience. First, we show that when the mismatch costs are moderate, only the dominant-currency steady states are locally stable. This is true because complementarity in currency adoption means that even a small initial deviation from symmetry both pushes firms to adopt the more dominant currency and encourages households to hold more of it. This implication is consistent with the historical experience of highly persistent dominant-currency regimes, including the recent reign of the US dollar and the earlier dominance of the British pound.

Second, we find that dominant-currency steady states fit several puzzling empirical features of currency returns and international portfolio positions. The country with a dominant currency earns a liquidity premium, derived from the fees paid by firms who use the asset as collateral. This premium generates a realistic failure of uncovered interest parity, as the non-dominant, high-interest currency earns a higher return.<sup>5</sup> Moreover, holdings of the dominant asset are more widely distributed around the world, implying a permanent negative net foreign asset position for the issuing country. The aggregate portfolio holdings of countries also exhibit significant home bias, with the dominant country's portfolio being substantially less biased than that of the other major country. Qualitatively, these results describe the status quo in the data quite well.

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<sup>5</sup>Jiang et al. (2018), Engel and Wu (2018) and Valchev (Forthcoming) all provide empirical evidence linking bond convenience yields with UIP violations.

To quantify our mechanism, we embed the framework within a fully-fledged open economy model consisting of two big countries, the ‘US’ and the ‘EU’, and a continuum of small open economies that form the ‘rest-of-the-world’ (RW). Each country receives a regular endowment of a differentiated tradable good, and the governments of the two big countries issue otherwise ex-ante identical safe (collateral-eligible) assets denominated in their respective good. Households consume both domestic and foreign goods, sold domestically by importing firms, and freely trade the bonds issued by the two big countries’ governments.

Though more realistic, our quantitative model incorporates the same feedback effect between firms’ currency choice and the asset positions of households. To quantify our mechanism, we select parameters to match several target moments from the data on the size of government debt, trade, invoicing currency, and import markups, as well as a modest cost of currency mismatch. We fix these parameters so that the model achieves its target values at the empirically relevant, dollar-dominant steady state and show the model has realistic implications for several non-targeted moments.

To study dynamics away from steady state, we introduce modest portfolio adjustment costs. These costs prevent conjectured deviations in currency choice from driving instantaneous jumps in asset holdings. At the benchmark calibration, we find that the two dominant-currency steady states are dynamically stable, and lie within large regions of the state space that uniquely converge to their respective dominant-currency steady state. Within those regions, the equilibrium paths of the economy are determinate (i.e. not subject to sunspot shocks) and the currency regime is uniquely determined by initial economic states.

Using the calibrated model, we compute the welfare implications of owning the dominant asset. The steady-state welfare gain of the dominant country, relative to the large non-dominant country, is small: only 0.03 percent of permanent consumption at our benchmark calibration. This happens despite the dominant country’s “exorbitant privilege,” because foreign demand for the dominant asset means its issuer has a *negative* net foreign asset position. Thus, in steady state, the benefits of the lower interest rate are offset by the dominant country’s large external debt. However, factoring the transition path from a multipolar to a dominant-currency steady state changes the result by an order of magnitude. Overall, the dominant country gains the equivalent of 0.75 percent of permanent consumption, since it runs temporary trade deficits as external demand for its assets increases. Thus, understanding transition dynamics is crucial for assessing the benefits of owning a dominant asset.

We conclude the paper with two experiments that highlight the role of economic policy in determining the dominant currency. First, we consider a trade war between the US and the

other countries, initializing the economy at the dollar-dominant steady state. We find that a 15% permanent increase in tariffs is enough to threaten the dominance of the dollar, while a tariff rate of 30% is certain to end that dominance. In the latter case, the welfare cost to the US is substantial — more than 3% of permanent consumption, about 1% of which is a direct consequence of losing currency dominance. This suggests that trade barriers impose additional costs on the dominant country, beyond the usual destruction of consumer surplus. Moreover, in some circumstances, a non-dominant country could have an incentive to incite (or fail to avert) a trade war if it expects the equilibrium to shift in its favor as a result.

In our second experiment, we ask under what circumstances an emerging safe asset may unseat the reigning dominant currency. Using the pre-Euro status quo as a baseline, we find that the emergence of Euro safe assets, in supply roughly equivalent to the dollar, is not enough to precipitate a switch away from the dollar-dominant steady state. However, if the Eurozone had continue to grow to become moderately larger than the US, this could have eliminated the dominance of the dollar. The effects of asymmetric supply of safe assets can similarly explain the puzzling fact that dollar dominance increased after the fall of Bretton Woods.<sup>6</sup> Thus, the model can explain both strong persistence and occasional transitions in the dominant asset.

## Relation to existing literature

In this paper, we provide a model where the dominant position of US assets in the international monetary system is due to their endogenous role as the main *medium of exchange*. Several previous authors have explored other explanations of the special role of the dollar, focusing on the implications of the unit of account and store of value roles of money. [Gourinchas et al. \(2019\)](#) provide an overview of related literature, and analyze how the various functions of a dominant global currency interact.

Among the papers that focus on the unit of account role of money, concurrent work by [Gopinath and Stein \(2018\)](#) is most closely related to our own. To our knowledge, it is the only other paper that explicitly connects the dollarization of international trade and exorbitant privilege. The unit of account function is also highlighted by [Casas et al. \(2016\)](#), who consider the effects of dollar-denomination on shock pass-through and expenditure switching.

A larger literature focuses on the capacity of countries to generate good store-of-value assets. [Caballero et al. \(2008\)](#) argue that the United States' superior ability to produce store-

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<sup>6</sup>See page 3 of [Gourinchas et al. \(2019\)](#): “Paradoxically, once free from the shackle of a fixed gold parity, the use of the US dollar as an international currency soared to unprecedented levels.”

of-value assets can explain the US experience of persistent trade deficits, falling interest rates, and rising portfolio share of US assets in developing countries. [Mendoza et al. \(2009\)](#) and [Maggiore \(2017\)](#) focus on differences in financial development as a driver of global imbalances and the emergence of a dominant currency. [Brunnermeier and Huang \(2018\)](#) explore the impact of this mechanism in emerging market crises. [Gourinchas et al. \(2017\)](#) propose a different insurance framework, where US households have lower risk-aversion than foreign households and, thus, end up holding most of the world’s risky assets. [Bocola and Lorenzoni \(2017\)](#) also provide a framework where dollarization of world financial markets occurs because of the dollar’s unique risk profile. Meanwhile, [Farhi and Maggiore \(2016\)](#) consider the positive and normative implications of a single dominant reserve asset versus a multipolar system. [He et al. \(2016\)](#) use a global-game in a world with two ex-ante identical assets to model how a single safe asset emerges in equilibrium.

Several features differentiate our work from the preceding literature cited above. First, compared to models with ex-ante asymmetries, currency dominance emerges endogenously here in an otherwise symmetric world. Second, currency choice is uniquely determined by economic states, as asset availability enforces continued coordination on a dominant currency once it is established. Third, we model a fully dynamic economy and consider transitions between currency regimes. We view the last two features as an especially desirable combination, because (i) historically currency regimes have change only infrequently (i.e the equilibrium seems to be stable) and (ii) accounting for transitions changes welfare implications substantially. Previous work has included some of these features, but not all together.

This paper is also related to the long literature on search-based theories of money as surveyed, for example, by [Lagos et al. \(2017\)](#). Many such papers examine equilibrium multiplicity and the co-existence of multiple currencies in an international setting (e.g. [Matsuyama et al. \(1993\)](#), [Zhou \(1997\)](#), [Wright and Trejos \(2001\)](#), [Rey \(2001\)](#), [Ravikumar and Wallace \(2002\)](#), [Kannan \(2009\)](#), [Devereux and Shi \(2013\)](#), [Zhang \(2014\)](#), [Doepke and Schneider \(2017\)](#)). A central difference in our work, relative to this literature, is that the endogenous asset availability channel uniquely determines which asset emerges as “money.” Hence, the environment we develop in this paper is especially well-suited to explain the observed persistence in the dominant international medium of exchange.

Lastly, the paper relates to the literature on global imbalances ([Obstfeld and Rogoff, 2007](#)), particularly the work that has emphasized the importance of differential returns for determining their sustainability ([Gourinchas and Rey, 2007b](#)). Our theory shows how the endogenous adjustment of liquidity premia can sustain some imbalances indefinitely.

## 2 A Simple Model

In this section, we develop the key insights of our mechanism in the context of a simple, analytically tractable model. For these analytical results, we make several simplifying assumptions. We relax many of these assumptions in Section 3, where we embed our mechanism in a dynamic general equilibrium model. In Section 4, we discuss quantitative implications.

### The environment

The world consists of two symmetric big countries, the US and the EU, of size  $\mu_{us} = \mu_{eu}$ , and a continuum of small open economies with total mass  $\mu_{rw}$  making up the *rest of the world* (RW). Countries are indexed by  $j \in \{us, eu, [0, \mu_{rw}]\}$ . Within each country  $j$ , there exists a continuum of trading firms,  $i \in [0, 1]$ , that seek to engage in a profitable transaction with a foreign firm. For tractability, in this section we restrict RW firms to trade only with other RW firms.

Contract enforcement across borders is imperfect, so each firm must post collateral to guarantee their side of any international transaction. Two ex-ante identical and universally-recognized safe assets can serve this collateral role: US and EU government bonds. To obtain this collateral, firms seek an intra-period loan of one of the two safe assets from the portfolio holdings of their domestic (country  $j$ ) households, in local bond-specific search and matching markets. We assume that firms look for a fixed amount of funding, which we normalize to one, and that firms make the binary choice of either seeking dollars or euros.

The probability that a country- $j$  trading firm seeking to borrow a US asset is successful is given by  $p_j^{\$}$ , and the probability that an euro-seeking firm finds funding is  $p_j^{\text{€}}$ . If a firm finds collateral, it pays a fee  $r > 0$  to the household for the use of the asset, and proceeds to the international transactions market.<sup>7</sup> If the firm does not find collateral, it exits.

Conditional on finding collateral, the trading firm seeks a trading partner across all RW countries  $j' \in [0, \mu_{rw}]$ , where  $j' \neq j$ . For simplicity, we assume that the trading partner search is undirected and all trading firms match with exactly one trading partner. Upon matching with a partner, the matched pair transacts using their collateral to clear any payments needed and splits the resulting transaction surplus, totaling  $2\pi$ , equally.

In the event that the transacting firms' type of collateral is mismatched — i.e. that one side of the match arrives with dollar assets and the other side with euro assets — the

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<sup>7</sup>We fix  $r$  as a parameter for simplicity, but it could be endogenized, e.g. by assuming Nash bargaining.

transaction's surplus is reduced by a currency mismatch cost of  $2\kappa$ . We assume throughout that  $0 \leq \kappa < \pi$ . This mismatch cost captures, in reduced-form, several potential reasons that firms may wish to coordinate the currency in which they transact, including (i) the risk that collateral on one side of the exchange may not be sufficient ex-post, (ii) the risk of mismatched incoming and outgoing cash flows, and (iii) the added complication of negotiating a contract that includes a currency conversion or is conditioned on nominal exchange rates. While the existence of *some* incentive for currency coordination plays an important role in our story, we later demonstrate that our mechanism works even when this force is small.

While stylized, this framework most closely resembles the letter of credit mode of financing imports and exports, and serves as tractable abstraction for a wide range of trade finance arrangements used in practice. Over 90% of international trade involves some form of credit, guarantee or insurance. Importantly, the burden of providing financing may fall on the importer, the exporter or both. Moreover, most internationally active firms are both importing and exporting, and hence are rarely exclusively an importer or an exporter. Hence, the bulk of firms engaged in international trade require some form of financing.

### Equilibrium with exogenous finding probabilities

In deciding which asset to seek as collateral, trading firms compare the expected payoffs of seeking US and EU assets. Since trading firms on both sides of a transaction split profits equally, each side realizes gross profits of  $\pi$  regardless of their collateral type. The firm's currency choice, however, affects (i) its probability of obtaining funding and (ii) its probability of facing a currency mismatch.

Let  $X_j$  be the fraction of country  $j$  firms that choose to operate in dollars. Then, the average dollar use in the rest of the world is:

$$\bar{X} \equiv \frac{1}{\mu_{rw}} \int_0^{\mu_{rw}} X_j dj.$$

This is also the probability that a firm will be matched with a trade partner who uses dollars. Hence, a trading firm's expected mismatch cost is  $\kappa(1 - \bar{X})$  if it operates in dollars and  $\kappa\bar{X}$  if it operates in euros.

Accounting for mismatch costs, the benefit to a firm choosing dollars over euros is:

$$V_j^{\$} = p_j^{\$} [\pi - \kappa(1 - \bar{X})] - p_j^{\text{€}} [\pi - \kappa\bar{X}]. \quad (1)$$

The first term on the right-hand side of equation (1) is expected profits when operating in dollars, and the second term is expected profits when operating in euros. If  $V_j^{\$} > 0$ , a firm in country  $j$  strictly prefers to seek dollar funding, while if  $V_j^{\$} < 0$  it strictly prefers to seek euro funding. Otherwise, it is indifferent between the two currencies.

When  $p_j^{\$}$  and  $p_j^{\text{€}}$  are exogenous and constant across  $j$ , equation (1) describes the payoff structure of a classic coordination game: when  $\kappa > 0$  firms find dollars more desirable whenever they expect other firms to use dollars and visa-versa for euros. This game can have multiple equilibria, including corner equilibria where either everybody uses dollars ( $\bar{X} = 1$ ) or euros ( $\bar{X} = 0$ ), and a mixed-strategy equilibrium in which both currencies are used.

Proposition 1 provides a necessary and sufficient condition for equilibrium uniqueness.

**Proposition 1.** *Dollar dominance ( $\bar{X} = 1$ ) is an equilibrium of the economy whenever  $\kappa \geq (1 - p^{\$}/p^{\text{€}})\pi$ . Similarly, euro dominance ( $\bar{X} = 0$ ) is an equilibrium whenever  $\kappa \geq (1 - p^{\text{€}}/p^{\$})\pi$ . The economy has a unique equilibrium if and only if*

$$\kappa < (1 - p^*) \pi,$$

where  $p^* \equiv \min\left(\frac{p^{\text{€}}}{p^{\$}}, \frac{p^{\$}}{p^{\text{€}}}\right) \leq 1$ .

*Proof.* Proved in Appendix A. ■

Intuitively, the model has a unique equilibrium so long as the probabilities of finding each type of funding are not too close to each other. In these cases, one candidate currency has a “fundamental” advantage over the other, so that currency choice is uniquely coordinated by credit availability. Conversely, when both funding probabilities are relatively similar, firms’ currency choice will be driven more strongly by their conjecture about other firms’ choices. In this case, the model has several equilibria, including potentially mixed-strategy equilibria. The previous literature has often emphasized similar types of multiplicity, and we use it in what follows as a point of comparison to highlight our key contributions.

## Equilibrium with endogenous finding probabilities

We now characterize equilibrium when the funding probabilities,  $p_j^{\$}$  and  $p_j^{\text{€}}$ , are endogenously determined in search and matching funding markets. On one side of these markets are domestic households, who offer to lend their portfolio holdings of safe assets, and on the other are the domestic firms who seek to borrow these assets and use them in trade. In

each country, there are two markets, one for dollar funding and one for euro funding, and households make all of their assets available for lending in the respective markets.

We begin by assuming that the holdings of assets around the world,  $B_j^{\$}$  and  $B_j^{\text{€}}$ , are exogenously given. The number of matches that emerge in a given country-currency market is governed by the [den Haan et al. \(2000\)](#) matching function, according to which the number of matches formed in a market with  $B$  units of the asset on offer and  $X$  firms searching is<sup>8</sup>

$$M^f(B, X) = \frac{BX}{(B^{\frac{1}{\varepsilon_f}} + X^{\frac{1}{\varepsilon_f}})^{\varepsilon_f}}.$$

We call the parameter  $\varepsilon_f$  the matching function elasticity, and for tractability we set this parameter to unity for the remainder of this section. Thus, the probability that a firm in country- $j$  searching for dollar assets finds them is

$$p_j^{\$} = \frac{M^f(B_j^{\$}, X_j)}{X_j} = \frac{B_j^{\$}}{B_j^{\$} + X_j}.$$

A similar expression describes the probability of obtaining euro funding. Substituting the expressions for finding probabilities into equation (1) yields

$$V_j^{\$} = \frac{B_j^{\$}}{B_j^{\$} + X_j} [\pi - \kappa(1 - \bar{X})] - \frac{B_j^{\text{€}}}{B_j^{\text{€}} + 1 - X_j} [\pi - \kappa\bar{X}]. \quad (2)$$

Equation (2) captures the first key implication of our asset availability channel: the payoff of using dollars in country  $j$  is *decreasing* in  $X_j$ , the intensity of dollar use by other country- $j$  firms. When more domestic firms look for dollar collateral, the local dollar funding market becomes crowded, lowering the probability of a firm obtaining dollar funding. Thus, this currency game captures a within-country strategic *substitutability* in collateral choice that contrasts with the cross-country complementarity embedded in the parameter  $\kappa$ .

The consequences of the substitutability created by limited asset availability are summarized in Proposition 2 and subsequent corollaries.

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<sup>8</sup>This form exhibits constant returns to scale and ensures matching probabilities  $\in [0, 1]$ .

**Proposition 2.** *Suppose that rest-of-world asset holdings are symmetric across  $j$ , i.e.  $B_j^{\$} = B^{\$}$  and  $B_j^{\text{€}} = B^{\text{€}}$  for all  $j$ . Then, dollar dominance ( $\bar{X} = 1$ ) is an equilibrium of the economy whenever  $\kappa \geq \frac{1}{B^{\$}+1}\pi$ . Similarly, euro dominance ( $\bar{X} = 0$ ) is an equilibrium whenever  $\kappa \geq \frac{1}{B^{\text{€}}+1}\pi$ . The economy has a unique equilibrium if and only if*

$$\kappa < \frac{1}{\min(B^{\$}, B^{\text{€}}) + 1}\pi. \quad (3)$$

*Proof.* Proved in Appendix A. ■

The crucial implication of Proposition 2 is that the availability channel serves to limit the scope for equilibrium multiplicity. To see why, the case of perfectly diversified bond holdings ( $B^{\$} = B^{\text{€}} = \bar{B}$ ) is especially instructive. When finding probabilities are exogenous and equal ( $p^{\$} = p^{\text{€}}$ ), Proposition 1 implies multiplicity in the currency choice for *any*  $\kappa$ . In contrast, in the symmetric case of Proposition 2, the condition for multiplicity,  $\kappa \geq (\bar{B} + 1)^{-1}\pi$ , is a stricter requirement for any finite  $\bar{B}$ . In this case, if coordination incentives are modest, dominant currency equilibria cannot be sustained because congestion in the market for the putative dominant currency makes the alternative asset a more attractive funding option. Instead, a unique symmetric mixed-strategy equilibrium emerges.

While symmetric asset holdings reinforce mixed-currency equilibria, asymmetric asset holdings will favor equilibria in which one currency dominates. Corollary 1, which follows from the proof of Proposition 2 in the appendix, summarizes the cases in which the portfolio composition ensures that a single currency intermediates *all* transactions.

**Corollary 1.** *The unique equilibrium is a dominant equilibrium, i.e.  $\bar{X} \in \{0, 1\}$ , whenever*

$$\kappa \in \left[ \frac{1}{\max(B^{\$}, B^{\text{€}}) + 1}\pi, \frac{1}{\min(B^{\$}, B^{\text{€}}) + 1}\pi \right) \quad (4)$$

When holdings are perfectly diversified, the set in (4) is empty: dominant equilibria may exist but they cannot be unique. However, a shift in portfolios towards one assets increases  $\max(B^{\$}, B^{\text{€}})$  and decreases  $\min(B^{\$}, B^{\text{€}})$ , thus expanding set of  $\kappa$  that satisfy equation (4).

Intuitively, higher household holdings of one of the assets makes that asset more attractive to firms for two reasons. First, larger domestic holdings means firms are more likely to be funded if they seek that asset. Second, larger holdings of that asset across the rest of the world means firms' potential trading partners are more likely to use it. As a result, a sufficiently unbalanced position,  $B^{\$} \gg B^{\text{€}}$ , can eliminate the euro-dominant equilibrium ( $\bar{X} = 0$ ) for

any level of  $\kappa$ , and the resulting unique equilibrium will be completely coordinated on the dollar. Thus, concentrated asset positions are a powerful coordination device, as firms realize that few of their trading partners are likely to be funded with the relatively scarce asset.

The *overall level* of asset holdings also has important implications for equilibrium uniqueness, which we derive in Corollary 2.

**Corollary 2.** *Fixing average bond holdings,  $\hat{B} \equiv \frac{B^{\$} + B^{\text{€}}}{2}$ , the lowest  $\kappa$  for which equilibrium multiplicity can emerge is*

$$\kappa^{sunspot}(\hat{B}) \equiv \frac{\pi}{\hat{B} + 1}.$$

*Proof.* The result follows from the fact that  $\min(B^{\$}, B^{\text{€}})$  is maximized, and the right-hand side of (3) minimized, by equalizing holdings of assets,  $B^{\$} = B^{\text{€}} = \hat{B}$ . ■

Corollary 2 states that the threshold for the existence of multiplicity,  $\kappa^{sunspot}(\hat{B})$ , is decreasing in the overall size of the RW household portfolio,  $\hat{B}$ . Smaller asset holdings imply larger congestion effects from other firms' use of an asset, leading to stronger substitutability in currency choice. As a result, firms' currency choices are more tightly anchored to asset availability, increasing its effectiveness as a coordination device. Conversely, when  $B^{\$}$  and  $B^{\text{€}}$  become arbitrarily large, the availability channel essentially vanishes, and the payoffs in equation (2) converge to those in equation (1) when  $p_j^{\$} = p_j^{\text{€}} = 1$ , i.e. the case with multiplicity for any positive  $\kappa$ .

Figure 1 illustrates the anchoring effects of portfolio composition. The solid black lines in Figure 1 plot the firms' optimal currency choice,  $\bar{X}$ , as a function of the share of US bonds held in the portfolios of RW households for different levels of  $\kappa$ . In both panels, the asset availability channel is strong enough to ensure a unique currency choice equilibrium for any portfolio composition. The black line is an increasing function everywhere: a higher share of dollars held by RW households increases the availability of dollar funding and drives a larger share of firms to choose dollars. Though we do not show the case here, multiple equilibria can emerge when portfolios are close enough to evenly distributed and  $\kappa > \kappa^{sunspot}(\hat{B})$ , implying a backwards bend in the black line.

## Equilibrium asset holdings

We complete our model by characterizing households' optimal portfolio choices. There is a single consumption good, and both risk-free bonds promise one unit of that good.

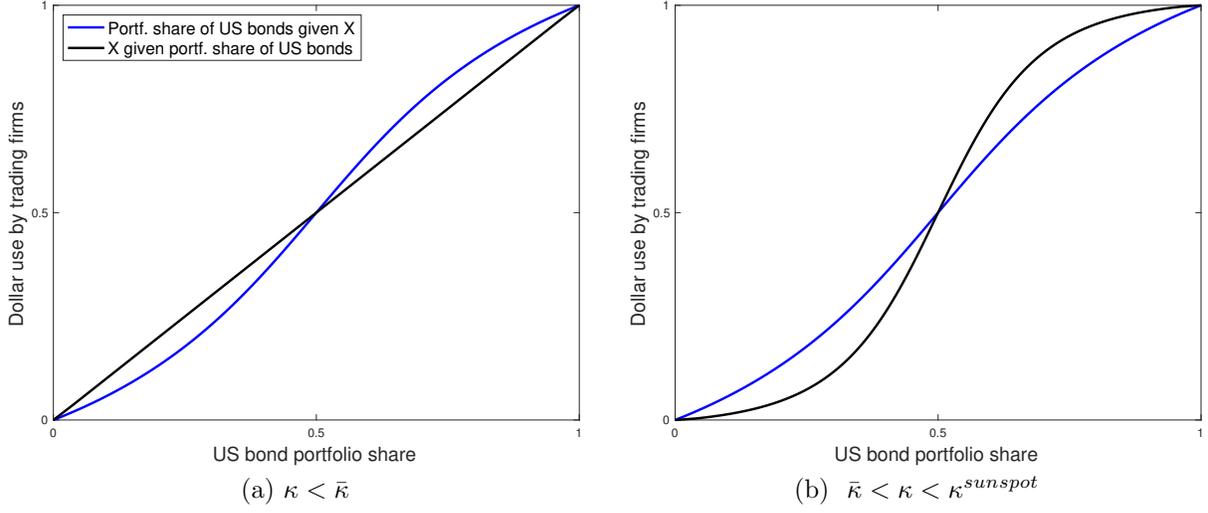


Figure 1: Equilibrium in the simple game.

Households in each country solve

$$\max_{C_{jt}, B_{jt}^{\$, B_{jt}^{\epsilon}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$$

subject to

$$C_{jt} + (Q_t^{\$} - \Delta_{jt}^{\$})B_{jt}^{\$} + (Q_t^{\epsilon} - \Delta_{jt}^{\epsilon})B_{jt}^{\epsilon} = B_{jt-1}^{\$} + B_{jt-1}^{\epsilon} + Y_{jt} + \Pi_{jt},$$

where  $Q^c$  is the price and  $\Delta_j^c$  is the liquidity premium of asset  $c \in \{\$, \epsilon\}$ .

Liquidity premia are endogenous and derive from the fees that borrowing firms pay to lending households. The liquidity premium a given asset earns in country  $j$  is equal to the share of the asset that the country  $j$  household successfully lends in credit markets multiplied by the funding fee  $r$  that it receives when it does so. Thus, the premia can be expressed as

$$\Delta_j^{\$} = r \times \frac{M^f(B_j^{\$}, X_j)}{B_j^{\$}} = \frac{rX_j}{B_j^{\$} + X_j}, \quad (5)$$

$$\Delta_j^{\epsilon} = r \times \frac{M^f(B_j^{\epsilon}, 1 - X_j)}{B_j^{\epsilon}} = \frac{r(1 - X_j)}{B_j^{\epsilon} + (1 - X_j)}, \quad (6)$$

for US and EU bonds respectively. The more firms that use dollars ( $X_j$ ), the higher are the aggregate fees paid on dollar borrowing and thus the higher the liquidity premium on US

safe assets. On the other hand, the higher the number of dollar bonds available in a country ( $B_j^{\$}$ ), the lower is the average liquidity premium each one of them earns.

For the analytical results in this section, we focus on steady-state equilibria, i.e. situations in which both households and firms would find it optimal to maintain their current strategy indefinitely. At the steady state, optimal bond holdings are governed by a set of country-specific Euler equations that equate returns across assets:

$$\frac{1}{\beta} = \frac{1}{Q^{\$} - \Delta_j^{\$}} = \frac{1}{Q^{\text{€}} - \Delta_j^{\text{€}}}. \quad (7)$$

Moreover, since all households have access to frictionless international bond markets, the liquidity premium on each type of asset must be equalized across countries:

$$\Delta_j^c = \Delta^c \text{ for all } j, \text{ for given } c \in \{\$, \text{€}\}.$$

We assume that the supply of US and EU safe assets is exogenous and symmetric across assets, at a value  $\bar{B}$ . Combing Euler equations with bond market clearing (e.g.  $\bar{B} = \int B_j^{\$} dj + \mu_{us} B_{us}^{\$} + \mu_{eu} B_{eu}^{\$}$ ), it follows that the optimal bond holdings in country  $j$  are:

$$B_j^{\$} = \bar{B} \frac{X_j}{\mu_{rw} \bar{X} + \mu_{us} X^{us} + \mu_{eu} X^{eu}} \quad (8)$$

$$B_j^{\text{€}} = \bar{B} \frac{1 - X_j}{\mu_{rw}(1 - \bar{X}) + \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})}. \quad (9)$$

Equations (8) and (9) summarize the link between currency use, including by firms in the large countries, and household portfolios.<sup>9</sup> Crucially, these equations imply that household asset allocations and firm currency choices are strategic complements: higher  $X_j$  implies higher household desire to hold dollar bonds and vice versa. Intuitively, this occurs because an asset that is widely used for intermediating trade will deliver a higher liquidity premium, thereby giving RW households an incentive to increase holdings of this bond. Meanwhile, as described in the previous subsection, firms have incentive to seek the asset that is more widely held across countries, as this predicts the currency choices of potential trading partners.

The blue lines in Figure 1 illustrate the optimal portfolio choice of the households, plotting the implied share of dollar bonds (on the x-axis) as a function of firms' currency decisions (y-

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<sup>9</sup>One advantage of our modeling approach is that it delivers determinate wealth and asset allocations even in a non-stochastic steady state, as endogenous liquidity premia imply that assets are not perfect substitutes out of equilibrium.

axis.) This line is also upward-sloping everywhere, capturing the complementarity between households and firm decisions.

### Steady-state equilibria

With expressions for optimal firm currency choices and household asset holdings, we can now solve for the set of steady-state equilibria. We focus on the case where the large countries are symmetric in the use of their domestic asset, so that the total use of dollars and euros among big-country firms is exogenous and equal, i.e.  $\mu_{us}X_{us} + \mu_{eu}X_{eu} = \mu_{us}(1 - X_{us}) + \mu_{eu}(1 - X_{eu}) = (\mu_{us} + \mu_{eu})/2$ . Lastly, we look for symmetric equilibria where the strategies of the ex-ante identical rest-of-world firms are the same ( $X_j = X$  for all  $j$ ).

As suggested by the intersection points in Figure 1, the model generally has three steady-state equilibria. We characterize the equilibria formally in Proposition 3.

**Proposition 3.** *For any  $\kappa$ , the symmetric economy has three steady-state equilibria: (i) a dollar-dominant steady state with  $\bar{X} = 1$  and  $B^\text{€} = 0$  (ii) a euro-dominant steady state with  $\bar{X} = 0$  and  $B^\text{\$} = 0$ , and (iii) a multipolar steady state with  $\bar{X} = 1/2$  and  $B^\text{\$} = B^\text{€}$ .*

*Moreover, these are the only steady states as long as  $\kappa \neq \bar{\kappa}(\bar{B})$ , where*

$$\bar{\kappa}(\bar{B}) \equiv \frac{\mu_{rw}\pi}{\bar{B} + \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu}} < k^{\text{sunspot}}(\bar{B}),$$

*in which case there is a continuum of equilibria with  $\bar{X} \in [0, 1]$ .*

*Proof.* Proved in Appendix A. ■

The fact that steady-state multiplicity emerges for *any* level of  $\kappa$ , including  $\kappa = 0$ , contrasts with the restricted versions of the game used for Propositions 1 and 2. In those cases, multiplicity arises purely as a consequence of firms' cross-country coordination incentives. Instead, steady-state multiplicity in Proposition 3 arises from the feedback between household and firm choices *within* countries. Hence, any switch between steady-state equilibria requires a discrete shift in both firms' currency choice and household bond positions.

To better understand why steady-state multiplicity does not depend on currency coordination incentives, consider an economy in an initial steady-state equilibrium with equal portfolio shares and equal asset usage in trade. Now suppose that firms exogenously increase their dollar use ( $\bar{X} \uparrow$ ). This encourages households to shift their portfolios towards US assets. But as the holdings of US bonds increase relative to EU bonds, this only increases

firms' incentives to seek dollars, thus strengthening the initial shift in the firms' currency choice. If the initial conjectured shift in  $\bar{X}$  is large enough, the availability channel sustains it regardless of  $\kappa$ , yielding the dominant currency equilibrium where only the dollar is used.

Though it can perpetuate a coordinated outcome, the feedback between households and firms itself does not create any incentive to deviate from the initial multipolar equilibrium, in which both currencies are held and used equally. Because of this, the value of  $\kappa$  is not important for the existence of the different steady states.

### Stability of steady states

A natural question is which (if any) of the steady states described above are stable? The answer to this question does depend on the level of  $\kappa$ , the incentive for currency coordination *across* countries. When this incentive is relatively high, the two coordinated steady states are stable and the symmetric steady state is not. The converse holds when  $\kappa$  is low. This result is stated formally in Proposition 4.

**Proposition 4.** *For  $\kappa < \bar{\kappa}(\bar{B})$  only the symmetric steady state is locally stable, while for  $\kappa > \bar{\kappa}(\bar{B})$  only the two coordinated, dollar and euro-dominant steady states, are locally stable.*

*Proof.* Proved in Appendix A. ■

The key mechanism behind this proposition is that when  $\kappa$  is sufficiently high, the optimal currency choice of firms around coordinated steady states is relatively insensitive to local off-equilibrium shifts in bond holdings. With high  $\kappa$  the asset availability channel is relatively weak, and hence optimal currency choice is less tightly anchored to the portfolio positions. Thus, a local off-equilibrium decrease in US bond holdings has little effect on the use of dollars, so that the liquidity premium on US assets increases since only the denominator in equation (5) changes. This rise in the dollar premium causes households to increase their US bond holdings, thus reversing the conjectured shift in portfolios away from dollar assets. This makes the coordinated steady states locally stable when  $\kappa$  is sufficiently high.

Conversely, if  $\kappa$  is low, the asset availability channel plays a stronger role. In this case, a decrease in  $B_j^{\$}$  will also cause a relatively large decrease in  $X_j$ , as firms respond to the decreased probability of obtaining dollar funding. When  $\kappa$  is sufficiently low, this decrease in  $X_j$  is more than enough to offset the effect of the decrease  $B_j^{\$}$  in equation (5) and thus the dollar premium will fall, leading households to lower dollar holdings even further, unraveling the coordinated equilibrium.

Hence, while  $\kappa$  plays no role in the existence of multiple steady-state equilibria, it is an important determinant of which ones are stable. Importantly, coordinated steady states may be locally stable even when coordination motives are too weak to create sunspot equilibria. For  $\kappa \in [\bar{\kappa}, \kappa^{sunspot})$  the model delivers steady-state multiplicity, but currency choice remains uniquely determined *given* bond holdings. In such cases, dynamics around steady states have unique, locally stable dynamics, and currency regimes are endogenously persistent.

Lastly, Proposition 4 refers to local-stability, but it cannot rule out non-local shifts in equilibria: i.e. cases where asset allocations and asset usage both jump discretely from one steady state to another. In practice, such dramatic shifts can be ruled out by making either bond allocations or currency choices inertial. We find the former more natural, and proceed this way in the enriched model in Section 3.

## Implications

Although our model is stylized, the dollar-dominant steady state ( $\bar{X} = 1$ ) already captures several important features of the real world. First, dollar assets are more widely used to intermediate international transactions, and thus serve as the dominant international medium of exchange. At the dollar steady state, the total international use of dollars and euros, respectively, is  $\mu_{\$} \equiv \mu_{rw} + \frac{1}{2}(\mu_{us} + \mu_{eu})$  and  $\mu_{\text{€}} \equiv \frac{1}{2}(\mu_{us} + \mu_{eu})$ , so that

$$\mu_{\$} - \mu_{\text{€}} = \mu_{rw} > 0.$$

Thus, the dollar dominates international exchange because of its outsize role in facilitating *third-party* transactions.

An implication of the higher demand for US assets is that the dollar liquidity premium is higher than that of the euro. By equations (5) and (6)

$$\Delta^{\$} = \frac{r\mu_{\$}}{B + \mu_{\$}} > \frac{r\mu_{\text{€}}}{B + \mu_{\text{€}}} = \Delta^{\text{€}}.$$

As more firms use dollars for international trade, a larger proportion of funding fees are paid to dollar assets, increasing their equilibrium liquidity premium. As a result, the US bond also has a higher equilibrium price: substituting  $\bar{X} = 1$  into (7), it follows that

$$Q^{\$} - Q^{\text{€}} = \Delta^{\$} - \Delta^{\text{€}} > 0.$$

Differences in bond prices lead to differences in steady-state interest rates, and thus an

unconditional failure of uncovered interest parity in favor of the (high-interest rate) euro. Defining the interest rate  $i^c \equiv \frac{1}{Q^c} - 1$  for currency  $c$ , we have:

$$\frac{1}{Q^\epsilon} - \frac{1}{Q^\$} = \frac{i^\epsilon - i^\$}{(1 + i^\$)(1 + i^\epsilon)} = \Delta^\$ - \Delta^\epsilon > 0.$$

Intuitively, since the dollar liquidity premium is higher, the euro asset must be compensated with a higher interest rate. Thus, the high interest rate currency earns positive excess returns relative to the low interest rate currency, a pattern that is well documented by [Hassan and Mano \(2018\)](#) among others.

A second important implication is that dollar assets are also more broadly *held* around the world than euro assets: substituting  $X_j = 1$  into (8) and (9) it follows that  $B_j^\$ > B_j^\epsilon$  for all  $j$  in RW. This is due to the feedback between household and firm choices — the currency choice of firms gives RW households an incentive to concentrate their portfolios in US assets.

This large external demand for US assets leads the US to hold a negative net foreign asset (NFA) position in steady state. Using the closed form solutions for bond holdings (8) and (9) at the dollar steady state, we have:

$$NFA_{us} = \underbrace{\mu_{us} B_{us}^\epsilon}_{\text{US foreign assets}} - \underbrace{(\mu_{eu} B_{eu}^\$ + \mu_{rw} B_{rw}^\$)}_{\text{US foreign liab.}} = -\bar{B} \mu_{rw} \frac{\mu_{us} X_{us}}{\mu_{rw} + \mu_{us} X_{us} + \mu_{eu} X_{eu}} < 0.$$

This underscores the tight link between international asset positions and currency dominance: coordination on the dominant currency is sustained by large holdings of the asset outside of its issuing country.

Lastly, due to the failure of uncovered interest parity (UIP) described above, US foreign asset holdings (i.e. euro bonds) earn higher returns than the interest rate it pays on its foreign liabilities. Thanks to the “exorbitant privilege” of this differential return, the US could in principle support its negative NFA position even while running a trade deficit. Expressing the US trade balance as the negative of the capital account, we have

$$TB_{us} = \underbrace{(\mu_{eu} B_{eu}^\$ + \mu_{rw} B_{rw}^\$ - \mu_{us} B_{us}^\epsilon)}_{-NFA} i^\epsilon - \underbrace{(\mu_{eu} B_{eu}^\$ + \mu_{rw} B_{rw}^\$)}_{\text{exorbitant privilege}} (i^\epsilon - i^\$).$$

Thus to the extent to which the US has (i) large *gross* external asset positions and (ii) the foreign asset earns a higher return ( $i^\epsilon > i^\$$ ), a small US trade surplus (and possibly even a trade deficit) can coexist indefinitely with large negative net positions.

### 3 Dynamic General Equilibrium Model

In this section, we embed our asset availability mechanism in a full dynamic general equilibrium endowment economy that we use for the quantitative analysis of Section 4. We describe the key elements of the model here, and leave detailed derivations to Appendix B.

#### Households

The household sector in country  $j$  consists of a representative consumer who seeks to maximize the present discounted value of utility,

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}.$$

The consumption basket  $C_{jt}$  is a Cobb-Douglas aggregator of home and foreign goods, with consumption share  $\alpha_h$  on the domestic good. For  $j \in \{us, eu\}$ , foreign imports consist of the good of the other big country and an aggregate of RW goods, while rest-of-world countries,  $j \in [0, \mu_{rw}]$ , consume imports from both big countries and the other RW small countries. Aggregate consumption price indexes,  $P_{jt}$ , are standard and derived in the appendix.

In addition to choosing consumption each period, households choose how much to save and how to allocate savings among US and EU bonds, which are risk-free in the units of their denomination. A US bond returns a payment equal to the price of the US good — i.e. one unit of US bonds purchased at time  $t - 1$  yields a payment of  $P_{us,t}^{us}$  at time  $t$ . Thus, households in any country  $j$  face the budget constraint:

$$P_{jt} C_{jt} + (1 - \Delta_{jt}^{\$}) P_{us,t}^{us} Q_t^{\$} B_{jt}^{\$} + (1 - \Delta_{jt}^{\text{€}}) P_{eu,t}^{eu} Q_t^{\text{€}} B_{jt}^{\text{€}} + P_{us,t}^{us} Q_t^{\$} \tau(B_{jt}^{\$}, \underline{B}_{j,t-1}^{\$}) + P_{eu,t}^{eu} Q_t^{\text{€}} \tau(B_{jt}^{\text{€}}, \underline{B}_{j,t-1}^{\text{€}}) = P_{us,t}^{us} B_{jt-1}^{\$} + P_{eu,t}^{eu} B_{jt-1}^{\text{€}} + P_{jt}^j Y_{jt} + \Pi_{jt}^T + T_{jt}, \quad (10)$$

where  $Y_{jt}$  is the household's endowment of the domestic good,  $\Pi_{jt}^T$  is the total profit of country  $j$ 's trading sector, which is returned lump-sum to the representative household,  $Q_t^{\$}$  and  $Q_t^{\text{€}}$  are the prices of the US and the EU bonds respectively,  $\Delta_{jt}^{\$}$  and  $\Delta_{jt}^{\text{€}}$  represent the endogenous liquidity premia (i.e. lending fees) earned by the bonds lent within the period to trading firms, and  $T_{jt}$  are lump-sum taxes.

Lastly, we assume that households face external portfolio adjustment costs, parameterized by the function  $\tau(B, \underline{B}) \equiv \frac{\tau}{2} \left( \frac{B - \underline{B}}{\underline{B}} \right)^2 \underline{B}$ , which are quadratic in terms of percent deviations

from country-wide bond holdings entering the period,  $\underline{B}_{j,t-1}^{\$}$  and  $\underline{B}_{j,t-1}^{\epsilon}$ . These adjustment costs are zero at (any) steady state, and serve to prevent excess volatility of capital flows without affecting the average level of bond holdings.

Intertemporal optimality implies the following household Euler equations:

$$1 = \beta E_t \left[ \left( \frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{us,t+1}^{us}}{P_{us,t}^{us}} \frac{1}{Q_t^{\$} (1 - \Delta_{jt}^{\$} + \tau'(B_{jt}^{\$}, \underline{B}_{j,t-1}^{\$}))} \right] \quad (11)$$

$$1 = \beta E_t \left[ \left( \frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{eu,t+1}^{eu}}{P_{eu,t}^{eu}} \frac{1}{Q_t^{\epsilon} (1 - \Delta_{jt}^{\epsilon} + \tau'(B_{jt}^{\epsilon}, \underline{B}_{j,t-1}^{\epsilon}))} \right]. \quad (12)$$

## Government

We assume that government expenditures on real goods are zero each period. The large countries  $j \in \{us, eu\}$  issue bonds in fixed supply  $\bar{B} = B^{\$} = B^{\epsilon}$  and set the level of lump-sum taxes so as to keep their stock of debt constant at  $\bar{B}$ :

$$\bar{B} = T_{jt} + Q_t^j \bar{B}.$$

The small rest-of-world countries  $j \in [0, \mu_{rw}]$  do not issue debt and set  $T_{jt} = 0$ .

## The Import-Export Sector

Our model of trade enriches our baseline model from Section 2 with endogenous firm entry, a more rigorous description of the matching process in which firms find trading partners, and additional details to ensure consistency with full general equilibrium in the world economy. We model trading firms in the goods market, with exporting firms from one country looking to match with importers from another. Each period, new import-export firms are formed, operate, return profits to the local household, and are then disbanded. This market structure aligns well with micro data, which shows significant churn of bilateral trade relationships (e.g. [Eaton et al., 2016](#)).

The problem of trading firms is static, but there are three stages to the life of each firm. In stage one, prospective firms choose whether or not to pay a fixed cost  $\phi$  in domestic output units and become operational for the period. A firm does not know whether it is going to be an importer or an exporter, or the country with which it will eventually trade, but optimally chooses the probability of each one of these things ex-ante.<sup>10</sup> Intuitively,

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<sup>10</sup>This assumption is convenient because it us allows to analyze a game with a single player type, but it

importing or exporting opportunities with any partner country arrive stochastically, and firms can choose how hard to look for each type of opportunity.

In stage two, firms look for funding and have the choice of whether to seek funding in either dollars or euros (i.e. to seek either US or EU bonds as collateral). The firms face a search friction in obtaining funding, hence a probability less than one of being funded. We assume that the firms look for a fixed amount of funding, which we normalize to one unit of the numeraire.

If a firm obtains funding, it proceeds to stage three, where it discovers whether it is an importer or an exporter and where from/to, and then searches for an appropriate foreign trading partner. If that search is successful, a trading match is formed, the importer buys goods from the exporter and sells them in its domestic market, and the resulting surplus is split between the two. If the two counter-parties to the transaction have mismatched collateral, the firms must pay an additional transaction cost before the trade is settled.

We consider the stages of the firm problem in reverse.

### Stage 3: Trading Round and Profits

When a match between an importer and exporter is formed, the two parties split the surplus that emerges from their trade. For example, the expected surplus for a transaction between a RW importer who uses dollars and an US exporter is:

$$\pi_{(rw,us),t}^{\$,im} = \frac{(1 - \alpha)}{P_{(us,rw),t}^{whol}} \left[ P_{rw,t}^{us} - P_{us,t}^{us} - \kappa P_{(us,rw),t}^{whol} (1 - \tilde{X}_{us,t}) \right]. \quad (13)$$

In the above,  $\alpha$  is the Nash bargaining share of the exporter,  $P_{rw,t}^{us}$  is the price of the US good when sold in RW,  $P_{us,t}^{us}$  is the exporter's cost of the US good (the price at which it can be procured in the US).  $P_{(us,rw),t}^{whol}$  is the effective transaction price between the US exporter and the RW importer (determined by the Nash bargain over the surplus),  $\tilde{X}_{us,t}$  is the fraction of *funded* US trading firms using dollars, and  $\kappa P_{(us,rw),t}^{whol} (1 - \tilde{X}_{us,t})$  is the expected currency mismatch cost which is proportional to the transaction value. General formulas for all bilateral matches are provided in the appendix.

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is not necessary for our mechanism. If, for example, we assumed firms knew whether they were importers or exporters, we would then need to analyze the game with both importer and exporter types.

## Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. Supply in each of these markets is furnished by the domestic household, which lends its bond holdings of a particular currency. On the demand side are the domestic firms who choose to search for that currency. Matches in the funding market are again generated by the [den Haan et al. \(2000\)](#) matching function, with an additional parameter  $\nu$  that corresponds to an exogenous velocity component in the market for collateral. For example, the probability that a country  $j$  trading firm seeking US bonds finds a funding match is

$$p_{jt}^{\$} = \frac{M^f(m_{jt}X_{jt}, \nu P_{us,t}^{us} B_{jt}^{\$} Q_t^{\$})}{m_{jt}X_{jt}},$$

where  $m_{jt}$  is the equilibrium mass of trading firms operating in country  $j$ . The matching probabilities for other markets are computed in the appendix.

In order to make their currency choice, firms compare the expected profits of being funded either with dollars ( $\Pi_{jt}^{\$}$ ) or euros ( $\Pi_{jt}^{\text{€}}$ ). The net benefit to firm  $i$  in country  $j$  of choosing dollars is then given by

$$V_{jt}^{\$} = \Pi_{jt}^{\$} - \Pi_{jt}^{\text{€}} + \theta_{it},$$

where  $\theta_{it} \sim N(0, \sigma_{\varepsilon}^2)$  and iid across firms. The shock  $\theta_{it}$  captures firms' idiosyncratic preferences for one currency over the other, and ensures that currency choice and asset holdings are interior.

Given that the expected payoff of seeking dollar funding is increasing in  $\theta_{it}$ , we restrict our analysis to the space of monotone strategies in which trading firms adopt the US asset so long as their private shock exceeds the threshold  $\bar{\theta}_{jt}$ . Given a value for this cutoff, the resulting fraction of each country's trading firms seeking the US asset are

$$X_{jt} \equiv \int_0^1 \mathbb{1}(\theta_{it} \geq \bar{\theta}_{jt}) di = 1 - \Phi\left(\frac{\bar{\theta}_{jt}}{\sigma_{\varepsilon}}\right),$$

where  $\Phi(\cdot)$  denotes the standard normal CDF.

The equilibrium cutoff  $\bar{\theta}_{jt}$  is the value of the idiosyncratic preference shock that leaves an RW importer-exporter indifferent between choosing one asset or the other, given everyone else's strategy. We focus on symmetric equilibria, so that the equilibrium cutoff value solves

$$\Pi_t^{\$} - \Pi_t^{\text{€}} + \bar{\theta}_t = 0. \tag{14}$$

## Stage 1: Firm Formation

In the initial formation stage, new firms choose a probability of importing or exporting to each potential trading partner country, and hence in equilibrium are indifferent among all possible outcomes. Finally, given all of the above choices, prospective firms must decide whether or not to pay the fixed cost  $\phi > 0$  in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi_{jt}^{\$}, \Pi_{jt}^{\epsilon}\} - \phi = 0,$$

is satisfied. The entry condition thus determines the equilibrium size,  $m_{jt}$ , of the import-export sector in each country.

## Market Clearing

Market clearing in the goods market requires that the endowment of each good is either consumed at home or exported abroad via the export sector. Since RW countries are symmetric, we treat it as a single country for aggregation purposes. Then we have,

$$\mu_j Y_{jt} = \sum_{j' \in \{us, eu, rw\}} \mu_{j'} C_{j',t}^j \quad (15)$$

Here, we have assumed for simplicity that bond adjustment costs, fixed costs, and currency-mismatch costs are transferred in lump-sum back to the households located where they are incurred within each period.

Bond market clearing requires that the foreign and domestic holding of bonds combine to equal the fixed aggregate supply of government debt:

$$\bar{B}^{\$} = \sum_{j \in \{us, eu, rw\}} \mu_j B_{j,t}^{\$} \quad (16)$$

$$\bar{B}^{\epsilon} = \sum_{j \in \{us, eu, rw\}} \mu_j B_{j,t}^{\epsilon}. \quad (17)$$

Finally, note that because of the frictions in cross-border trade, the law of one price does not hold across countries. Specifically,

$$P_{j',t}^j = \Delta_{j',t}^j P_{j,t}^j \quad (18)$$

Parameter	Concept	Value
$\beta$	Time preference	0.960
$\mu_{us} = \mu_{eu}$	Big country measure	0.200
$\kappa$	Mismatch cost	0.010
$r$	Funding fee	0.005
$v$	Exog. velocity	8.000
$X_{us}$	US dollar share	0.900
$X_{eu}$	EU dollar share	0.100
$\alpha$	Exporters bargaining parameter	0.500
$\sigma$	Risk aversion	1.000
$\varepsilon_T$	Elasticity of trade matching function	0.010
$\sigma_\varepsilon^2$	Variance of idio. shock	1e-06
$\tau$	Portfolio adj. costs	0.040

Table 1: Exogenously fixed parameters

for all country pairs  $j, j'$  such that  $j' \neq j$ . The  $\Delta_{j',t}^j$  constitute a total of six country specific wedges capturing the markups of imported goods over their production cost in the originating country. The  $\Delta_{j',t}^j$  are equilibrium objects pinned down by free entry of trading firms and the equilibrium of the coordination game played by trading firms.

## 4 Quantifying the Mechanism

We now calibrate our model in order to examine its quantitative implications. We fix a set of parameters to standard values, then use the remaining parameters for which we have relatively weak priors to target four steady-state moments, computed at the empirically relevant dollar-dominant steady state. The model is able to exactly replicate our target moments. We analyze the non-targeted characteristics of the benchmark model, compare welfare under different scenarios, and perform two counter-factual policy exercises.

### Steady State

The set of exogenously-fixed parameters is listed in Table 1. We always parameterize the two big countries symmetrically. We set the size of each of the two big countries equal to 20% of the world economy, which is consistent with the size of the US and the EU in world GDP. We set the currency use in the big countries ( $X_{us}$  and  $X_{eu}$ ) so that 90% of their firms use the domestic currency, to match the evidence of [Gopinath \(2015\)](#). One model period

Concept	Target Value	Parameter	Concept	Value
Gross debt/GDP	0.60	$\bar{B}$	US/EU asset supply	1.471
ROW trade/GDP	0.55	$a_j^h$	Home bias	0.718
ROW USD invoice shr.	0.80	$\varepsilon^f$	Funding match. elas.	0.294
Import markup	1.10	$\phi$	Fixed cost of entry	0.038

(a) Calibration Targets

(b) Implied Parameter Values

Table 2: Calibration targets and parameters

represents a year, hence we set  $\beta = 0.96$  to imply annual risk-free rate of 4%, and assume log preferences ( $\sigma = 1$ ). We minimize real trading frictions by setting a low value of the elasticity of the trade matching function,  $\varepsilon_T = 0.01$ , which ensures that firms on the less crowded side of trade markets are virtually guaranteed to find a trading partner. Furthermore, we fix  $\alpha = 0.5$ , implying that importers and exporters have equal bargaining power. Finally, we assume that the currency mismatch cost is small, and set  $\kappa = 0.01$  so that it represents just 1% of the transaction surplus earned by each firm.

To match the observed maturity and cost of a typical letter of credit contract in the data, we set  $\nu = 8$ , implying a typical funding relationship lasts about 45 days, and  $r = 0.005$ .<sup>11</sup> We also make the variance of the firms' currency preference shock very small ( $\sigma_e^2 = 1e-06$ ), so that it ensures the model is numerically smooth but plays a small quantitative role. Lastly, we parameterize the bond adjustment cost so that it has minimal effects on local dynamics: we set  $\tau = 0.04$ , implying that a 10% change in bond positions incurs a cost of 20 basis points on the total transaction.

We calibrate the remaining parameters of our model to match a set of target moments at the empirically-relevant dollar-dominant steady state. Panel (a) of Table 2 summarizes the target moments. They are (1) government debt of 60% of GDP, consistent with the US average since 1984; (2) rest-of-world trade share ( $\frac{\text{Imports} + \text{Exports}}{\text{GDP}}$ ) of 55%, consistent with trade data from the World Bank since 1984; (3) RW dollar invoicing share of 80%, to match the evidence of Gopinath (2015); and (4) import markups of 10%, consistent with micro-level estimates on import markups in Coşar et al. (2018).

We target these four moments with the four remaining free parameters  $\{\bar{B}, a^h, \varepsilon^f, \phi\}$ . These parameters are (1) the supply of government debt  $\bar{B}$ ; (2) the home bias parameter

<sup>11</sup>Letters of credit tend to be for one to two months, and the typical letter of credit includes a substantial fixed cost, estimated at about 40 basis points of the nominal amount, plus a small spread on top of the LIBOR rate. See, for example, the guidelines issues by the US Commerce Department: <https://acetoool.commerce.gov/cost-risk-topic/trade-financing-costs>.

Moments	USD Coord.			Symmetric			EUR Coord.		
	US	EU	RW	US	EU	RW	US	EU	RW
Dollar Share	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100 \times (i^{\$} - i^{\text{€}})$	1.07	-	-	0.00	0.00	-	-	-1.07	-
100×Implied rev./GDP	0.88	0.23	-	0.56	0.56	-	0.23	0.88	-
100×Trade bal./GDP	0.87	0.86	-0.45	1.01	1.01	-0.52	0.86	0.87	-0.45
Gross debt/GDP	0.60	0.59	0.00	0.60	0.60	0.00	0.59	0.60	0.00
NFA/GDP	-0.42	-0.26	0.18	-0.38	-0.38	0.19	-0.26	-0.42	0.18
Home Bias	0.61	0.90	0.00	0.80	0.80	0.00	0.90	0.61	0.00
Consumption	11.278	11.275	12.387	11.250	11.250	12.402	11.275	11.278	12.387

Table 3: Steady state values for baseline model.

in consumption preferences ( $a_h$ ) which determines the trade share; (3) the elasticity of the funding matching function ( $\varepsilon^f$ ) which helps determine the equilibrium level of currency coordination; and (4) the fixed cost of entry in the trading sector ( $\phi$ ) which helps determine the steady-state markups. We find the model can exactly match the targeted moments, with the implied parameter values given in Panel (b) of Table 2.

Table 3 summarizes several key moments for each of the three steady states in the calibrated economy. Similar to the stylized model in Section 2, the resulting dollar-dominant steady state matches a number of (untargeted) empirical regularities. First, since the rest-of-world primarily uses dollars for trade finance, foreign households hold a substantial portion of the supply of US safe assets. This generates a negative net foreign asset position for the US of 42% of GDP (a value that is very close to the current US position), and a realistically large positive foreign asset position in the rest-of-world. Since a substantial portion of RW firms still seek euro funding, RW households also hold a significant quantity of EU assets, leading the EU to have a (smaller) negative foreign asset position as well.

Despite the fact that the US has a less favorable asset position than the EU, its steady-state trade surplus is essentially the same. This is a consequence of the “exorbitant privilege” of the country issuing the dominant medium of exchange: the interest rate the US pays on its obligations is 1.07% lower than the correspond euro interest rate. This violation of uncovered interest parity earns the US a significant premium on its external position, funding a substantial portion of its NFA liabilities. As a result, the US can *indefinitely* support its more negative foreign asset position with virtually the same real trade outlay as the EU.

The third line of the table (implied revenue) provides a simple measure of the benefit the US receives from the liquidity premia earned by US assets. This measure computes the additional interest outlay the US would face, if it paid an interest rate equal to the inverse of the time discount, holding asset positions fixed. For the US, this counterfactual cost corresponds to 0.88% of GDP, roughly five times the size of the corresponding benefit to the EU. Notice this difference is a function both of exorbitant privilege ( $i^{US} < i^{EU}$ ) and the size of the *gross* external asset and liabilities positions of the US to which this interest rate differential is applied. Thus, the larger is foreign demand for US assets, the larger is the base on which the exorbitant privilege applies.

From the seventh line we can see that both the US and EU portfolios display significant home bias, defined as  $1 - \frac{\text{share of foreign bonds in HH portfolio}}{\text{share of foreign bonds in world supply}}$ , but the bias is especially large for the EU. Both the level and relative strength of the home bias are realistic. In the model, home bias is positive because each large country primarily uses its own asset for trade financing, creating high domestic demand for the home asset. US home bias is lower than EU home bias, however, because the dollar's central role in international trade creates stronger external demand for US assets, leading RW households to concentrate their portfolios in US assets.

The last row of the table shows steady-state consumption, which corresponds to welfare in our endowment economy. Whether US consumption is higher than EU consumption at the dollar steady state is ex-ante ambiguous, as the interest rate benefits of being dominant are offset by the need to support a more negative NFA position in perpetuity. Indeed, in our calibration, steady-state US consumption is just 0.03% higher than EU consumption.<sup>12</sup>

Lastly, similar to the consumption finding above, the US-EU real exchange rate,  $\frac{P_{eu}}{P_{us}}$ , equals unity up to three decimals. This happens because, even though foreign demand for US assets is high, US wealth is relatively low, leading to lower world demand for US goods since the US consumption basket is home biased. The two effects offset each other, leaving the real exchange rate almost perfectly balanced. This result contradicts a common view that the central country in the international monetary system should have an appreciated real exchange rate. Yet, the result broadly matches the empirical evidence: in real terms, the dollar has not been consistently overvalued relative to other advanced currencies.

We conclude by noting that, naturally, the euro-dominant steady state is the mirror image of the dollar-dominant steady state, with the EU earning the exorbitant privilege.

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<sup>12</sup>In the appendix, we consider a version of the model in which the rest-of-world also issues an asset in positive net supply. This change moderates the absolute size of US and EU NFA positions, but not their difference, leaving other implications, including long-run relative consumption, essentially unchanged.

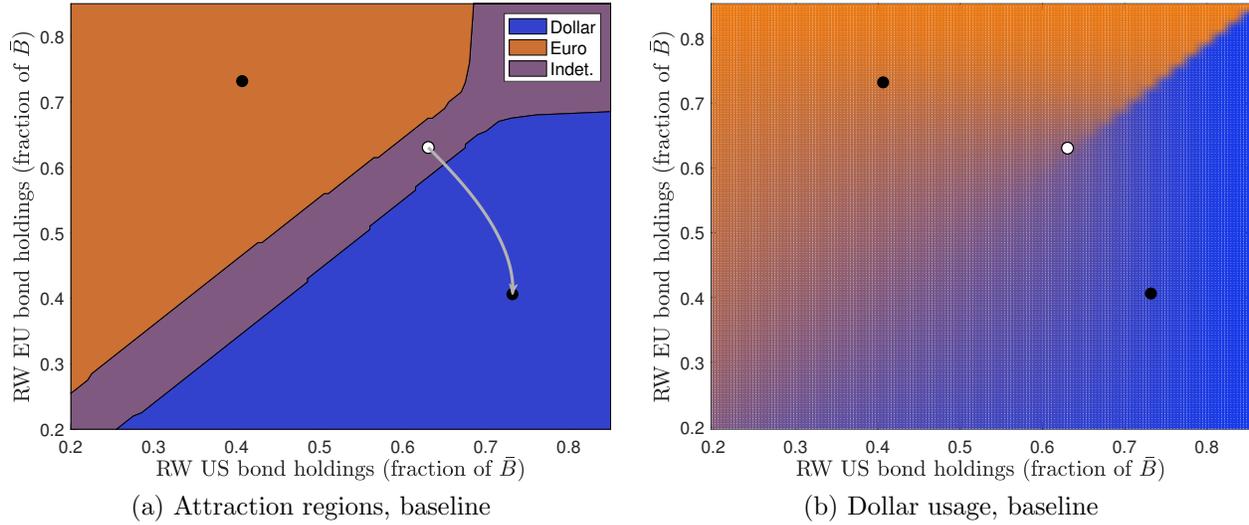


Figure 2: Attraction regions and asset usage in the dynamic model.

Meanwhile in the symmetric steady state (middle of Table 3), both assets are used equally, and thus earn the same liquidity premium and are equally distributed around the world.

### Dynamic Stability and Regions of Attraction

One advantage of our fully dynamic approach is that we can consider questions related to the stability or sustainability of a dominant currency equilibrium.<sup>13</sup> We evaluate these issues at our baseline parameterization in Figure 2.

Panel (a) of the Figure plots the respective attraction regions of the models' three steady states. We compute these regions by first defining a fine grid of initial asset positions covering the axes depicted in the picture.<sup>14</sup> For each grid point, we then solve a perfect foresight version of the economy, attempting to shoot from the initial vector of states to a candidate steady state. For points in the blue region, we found that the only solutions that exist converge to the dollar-dominant steady state, while for points in the orange region the only feasible outcome is the euro-dominant steady state. Finally, the purple region corresponds to points where we found perfect foresight paths that arrive at both coordinated steady states.

A first observation from the Figure is that only the coordinated steady states are dynamically stable, i.e. dynamic paths that are initialized away from the symmetric steady state

<sup>13</sup>Our approach to steady-state multiplicity is similar to that of [Schaal and Taschereau-Dumouchel \(2015\)](#).

<sup>14</sup>The model has four state variables: RW holding of US and EU, as well as US holdings of US bonds, and EU holdings of EU bonds (with US and EU foreign holdings determined by market clearing.) For all points in Figure 2, we initialize US and EU portfolios shares at their symmetric steady-state level.

never converge there.<sup>15</sup> Moreover, both dominant steady states are contained within large regions from which the economy uniquely converges to them. For example, whenever the RW households' initial portfolio positions are sufficiently biased towards US assets, the unique equilibrium path converges to the dollar-dominant steady state. In this respect, the model implies that currency regimes are endogenously persistent and sustainable indefinitely, so long as no large shocks push the economy out of the respective regions of attraction.

Nevertheless, we also find there is a portion of the state-space where the eventual steady-state currency is not uniquely determined by the initial asset allocations. In this region, it is possible for sunspots or short-term policy choices to have long-run effects on the currency regime. We illustrate this possibility when we consider a temporary trade war in Section 4.1.

While attraction regions tell us to where the economy *eventually* converges, not all points in the blue region correspond to high contemporaneous dollar use. As the economy transitions through the state space, equilibrium currency use may remain close to symmetric for extended periods of time. Panel (b) of Figure 2 presents a heat map of equilibrium usage of dollars for different initial distributions of assets, with darker orange representing strong euro usage, blue representing strong dollar usage, and purple representing cases where both currencies are in relatively equal use.

As our mechanism suggests, dollars tend to dominate in the bottom right corner of the Figure, when RW portfolios are strongly dollar biased. Currency usage is more mixed along the diagonal, where portfolios are roughly balanced. Importantly, currency choice is more sensitive to asset positions when total holdings in RW are low, captured by the larger purple region in the bottom left corner of the figure, where a balanced bond position is closely associated with mixed currency use. This is consistent with our observation in Corollary 2 that the anchoring effect of asset availability increases when total bond holdings are smaller.

Figure 3 plots the transition paths of several endogenous variables as the economy starts at the symmetric steady state and converges to the dollar-dominant steady state. The top right panel shows the evolutions of currency use, which starts close to equally balanced and then gradually converges to dominant dollar use over the subsequent 15-20 years. Along this transition path, the exorbitant privilege of the US gradually builds as RW portfolios and trade financing shift towards US assets. The shift towards US assets, in turn, drives the fall in the US net foreign asset position and the corresponding rise in the EU's NFA position. The evolution of the RW portfolio over this transition is depicted by the gray line Panel (a)

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<sup>15</sup>We have also confirmed via linearization that the dominant equilibria are locally-stable, while a locally-stable equilibrium does not exist for the symmetric steady state.

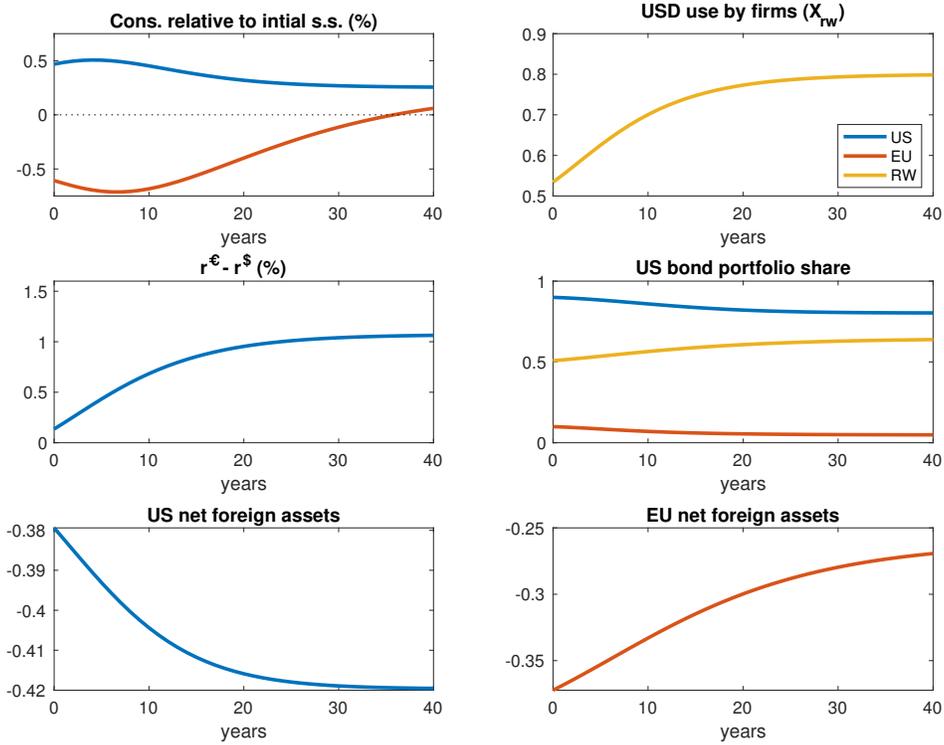


Figure 3: Transition from the symmetric steady state to dollar-dominant steady state.

of Figure 2.

Lastly, the top-left panel shows the paths of US and EU consumption. During this transition period US consumption is elevated for an extended period of time. This is a result of the *increasing* foreign demand for US assets and their concomitant liquidity services, which allows the US to steadily increase its borrowing from the rest of the world. Meanwhile, EU consumption is significantly depressed, as the EU increases savings in order to repatriate its assets, which are no longer in high demand externally. Thus, while consumption in the US and the EU eventually converges to roughly the same steady-state level, the two countries' experiences are quite different during the transition period.

Table 4 summarizes the welfare effects of this transition. Interestingly, both large countries enjoy a higher long-run level of consumption under the dollar-dominant steady state compared to their consumption in the symmetric steady state. The benefit to the US derives primarily from the higher seignorage it earns on its foreign liabilities. On the other hand, the EU enjoys higher consumption because of a more favorable net foreign asset position. Thus a comparison of steady states alone would suggest both US and EU should be roughly indifferent about which country's asset becomes dominant.

	US	EU	RW
steady-state	0.25%	0.22%	-0.12%
incl. transition	0.37%	-0.37%	-0.00%

Table 4: Gain/loss as percentage of symmetric steady state consumption.

While the long-run implications suggest no harm of dollar dominance for the EU, incorporating the EU’s lower consumption during the transition reverses this conclusion, while increasing the benefits to the US. Incorporating transitional effects, US permanent consumption is 0.75% higher than the EU’s, more than one order of magnitude larger than the 0.03% welfare benefit implied by steady-state consumption difference. Our subsequent results further reinforce the conclusion that considering the transitions periods is crucial for assessing the welfare implications of owning the dominant currency.

#### 4.1 Policy Experiment: Consequences of Trade Barriers

Since our main mechanism operates through trade flows, a natural question is whether trade barriers can affect the currency regime or the benefits of issuing the dominant currency. In this section, we consider two scenarios motivated by recent events in US trade policy. In the first scenario, which we call a moderate trade war, the US permanently raises tariffs on all imports by 15% and the EU and the RW respond in kind. In the second scenario, which we refer to as an acute trade war, we assume that tariffs rise to 30%, and we consider the effects of both a temporary and permanent imposition of these larger tariffs.

##### Moderate Trade War

Panel (a) of Figure 4 depicts the consequences of a permanent trade war between the US and the EU/RW with tariffs of 15%. The tariffs change the position of the steady states (we mark the old steady states with an  $\times$  and the new steady states with a dot) and also their respective attraction regions. In particular, the figure shows that the region of unique attraction to the dollar-dominant steady state is eliminated, and both the old and new dollar-dominant steady states lie within the region of equilibrium indeterminacy. Hence, even a moderate trade war potentially endangers the position of the dollar.<sup>16</sup>

<sup>16</sup>A 10% tariff leaves the old and new dollar equilibria just inside a shrinking dollar attraction region.

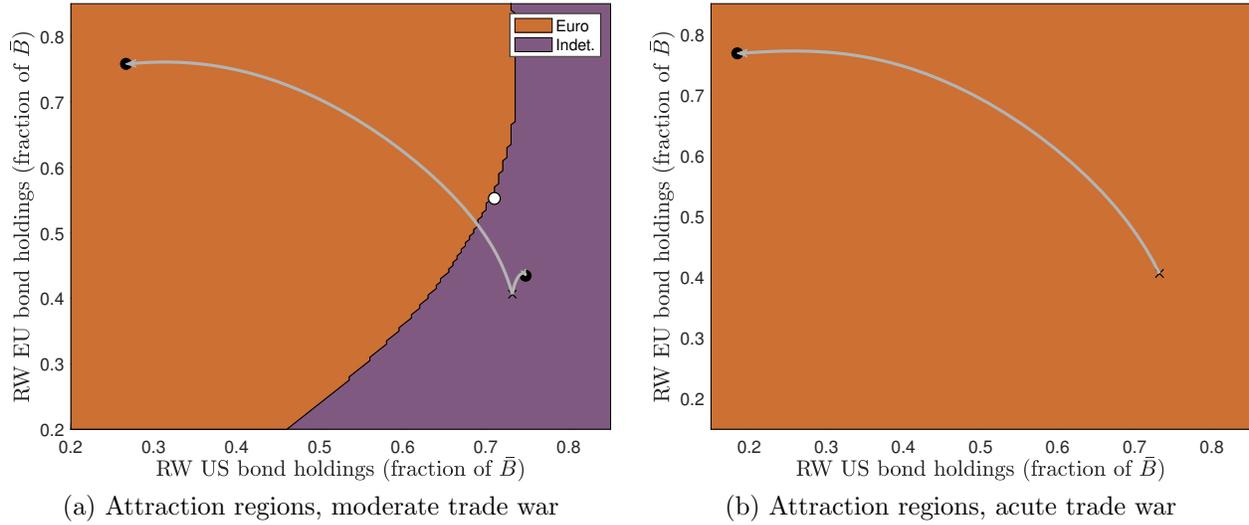


Figure 4: Attraction regions under trade war scenarios.

The trade war weakens dollar dominance for two reasons. First, it diverts RW trade away from the US towards the EU. Since EU firms are far more likely to use EU assets in their trade, RW firms become more likely to encounter euro trading partners and, hence, to prefer euros. Second, the overall world trade level falls, decreasing trade quantities relative to total asset supply. As a result, the equilibrium anchoring effects of asset availability become weaker, and multiple equilibria become more likely, both in the static sense of Corollary 2 and dynamically, as a smaller conjectured shift in asset holdings can switch trading firms’ choices. Hence, the purple multiplicity region grows as well.

Panel (a) of Table 5 summarizes the welfare implications of the moderate trade war, *assuming the dollar remains dominant*. The US is disproportionately hurt by the trade war for two reasons. First, the distortions created by the tariffs hit all of its exports, while the EU and RW face tariffs only on the relatively small portion of trade with the US. Second, as world trade levels fall, the size of seignorage revenue decreases as fewer firms overall require liquidity and a smaller portion of firms choose dollars. Quantitatively, however, the size of the seignorage effect is modest, with the US “privilege” falling from 1.07% in our baseline to 0.99% with a moderate trade war. Moreover, the effect of including transitions is negligible, which is not surprising given the very short distance of the transition, depicted by the shorter gray line in Panel (a) of Figure 4.

Panel (b) of Table 5 summarizes the welfare implications of the same trade war scenario, but *assuming the dollar loses dominance*, and the economy transitions to the euro steady

	US	EU	RW		US	EU	RW
steady-state	-1.42%	-0.01%	0.13%	steady-state	-1.06%	0.21%	-0.03%
incl. transition	-1.34%	0.07%	0.09%	incl. transition	-2.13%	0.79%	0.10%

(a) Moderate trade war, USD remains dominant      (b) Moderate trade war, USD loses dominance

Table 5: Gain/loss as percentage of dollar-dominant steady state consumption.

state. In this case, the US is hurt by the trade war while the EU is substantially helped. On the one hand, at the new steady state the US loses all of its seignorage, as the world currency use and the resulting exorbitant privilege shift to the EU. On the other hand, the transition to this new steady state, depicted by the long gray line in Panel (a) of Figure 4, further hurts the US (to the tune of an additional permanent consumption loss of more than 1%) because along the transition path it runs significant trade surpluses to reduce its NFA position as external demand for dollars dries up. Overall, the US loses 2.13% of permanent consumption.

### Acute Trade War

Panel (b) of Figure 4 depicts the implications of a permanent 30% tariff between the US and EU/RW, a scenario we call an “acute” trade war. In this case, the effects of the trade barriers are strong enough to eliminate both the symmetric and the dollar-dominant steady state, thereby guaranteeing a transition to the now unique euro-dominant steady state.

Since presumably a permanent 30% tariff is implausible, we also consider the effects of a temporary such trade war. Figure 5 depicts the welfare cost of a temporary 30% tariff for different possible durations, ranging from one to 40 years. The figure shows that for trade wars lasting under 10 years, the economy cannot transition to the euro steady state absent other shocks. Within this range, longer trade wars are worse, but not discretely so.

For trade wars lasting 10 year or longer, however, transition to a euro-dominant steady state becomes possible. If such a transition does occur, the US is discretely worse off, with the transition costing nearly an additional 1% of permanent consumption. If the trade war lasts longer than 28 years, the *unique* outcome is a transition to the euro-dominant steady state, despite the fact that policy reverts to zero tariffs in the long run. Thus, temporary shocks can have large permanent effects.

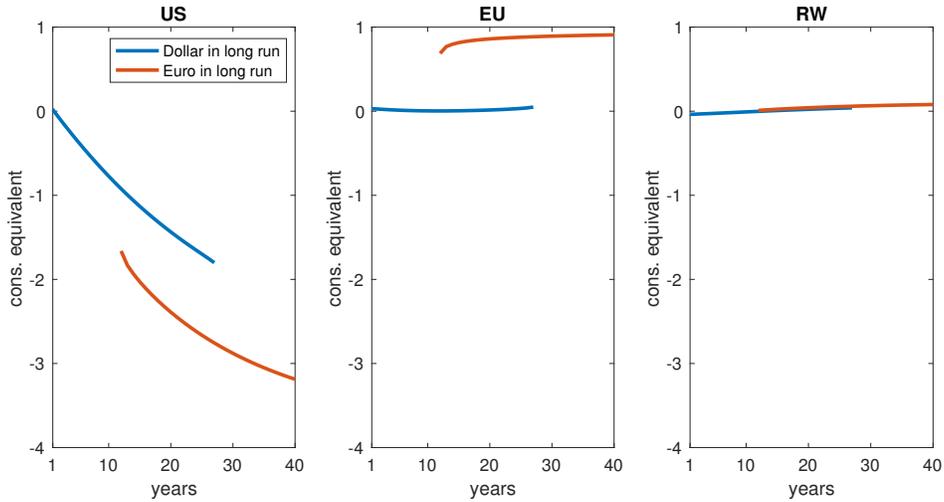


Figure 5: Welfare costs of an acute trade war as a function of duration.

## 4.2 Introduction of the Euro

At the time of the introduction of the euro, many policy makers and academics speculated about the potential of the euro to “unseat” the dollar as the sole international currency (see [Chinn and Frankel, 2007](#), for example.) One reason this seemed possible was that a unified Euro area would provide a large and effectively-homogeneous stock of safe assets that could rival the dollar. In [Figure 6](#), we model this side of the euro introduction by considering an economy in which the stock of euro assets, initially only 60% of the stock of US assets, grows quickly over time.

In the first scenario, we assume that the total supply of EU safe assets converges to the same level as that of the US asset over a 10 year period (i.e. our baseline calibration).<sup>17</sup> Initially, when the supply of EU assets is only 60% of the supply of US assets, the model has a unique, dollar-dominant steady state. We initialize the economy at this steady state, and consider the resulting transition path as the supply of EU assets grows. We find that the unique equilibrium path converges to the dollar steady state. Along the transition, dollar use, the interest rate differential, and the US net foreign asset position are essentially unchanged, rather consistent with the continued dominance of the dollar since 2000. This result highlights two key implications of our model: (i) the existence of two ex-ante equivalent safe assets does not guarantee a multipolar world and (ii) initial conditions matter.

<sup>17</sup>This scenario is motivated by the fact that the interest rates on euro-area sovereigns did not collapse to the interest rate of the German Bund at the moment of the introduction of the euro, but took several years to converge. This suggests markets only gradually accepted euro bonds as a homogeneous safe asset.

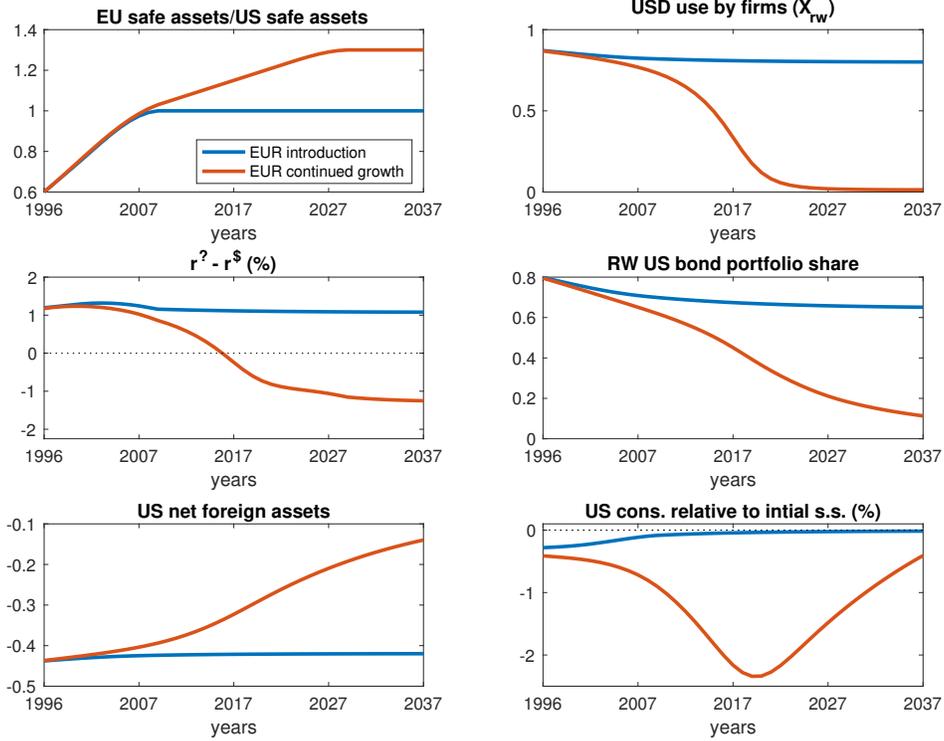


Figure 6: Introduction of euro

In the alternative scenario, we consider the unrealized possibility that the set of euro countries continued to grow as anticipated by, for example, the inclusion of Turkey, the UK and other eastern European nations. In this case, we assume the long-run stock of EU assets grows to exceed that of the US by 30%. The higher long-run availability of EU safe assets implies a unique euro-dominant steady state.<sup>18</sup> Along the path to this new steady state, the US net foreign asset position shrinks towards zero, and the US's exorbitant privilege benefits disappears as the US assets leaves international markets and return to US portfolios. The figure shows that anticipation effects are important, as much of the shift in portfolios and usage occurs before the end of the rise in EU asset supply. The welfare impact in this case is also considerable: the US loses 0.77%, while the EU gains 0.66% in permanent consumption.

## 5 Conclusions

This paper presents a new model of currency coordination that is qualitatively appealing, quantitatively realistic, and tractable enough to use for standard macroeconomic analysis. In

<sup>18</sup>An increase of EU assets to 125% of the US total is not sufficient to guarantee a switch to the euro.

doing so, we have abstracted from risk: both the potential for short run shocks that perturb the economy around a given steady state and possible longer-run stochastic transitions between currency regimes. Both of these extensions are rather straightforward: Business-cycle analysis can be easily conducted using linearized policy functions around a given steady state, while global analysis requires global solution techniques, for example of the sort advocated by [Richter et al. \(2013\)](#). Such extensions could help the model address the observation of [Gourinchas et al. \(2017\)](#) that an “exorbitant duty” coincides with the privilege of being the dominant currency. We leave exploration of this issue to future work.

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# Appendix

## A Proof of Propositions

Proof of Proposition 1. When  $\bar{X} = 1$ , we have

$$V_j^{\$}(1) = (p^{\$} - p^{\text{€}})\pi + p^{\text{€}}\kappa,$$

which is weakly positive whenever  $\kappa \geq (1 - p^{\$}/p^{\text{€}})\pi$ . The corresponding condition for  $\bar{X} = 0$  is straightforward. Hence, corners are equilibria whenever  $\kappa \geq (1 - p^*)\pi$ .

To see that the economy has a unique equilibrium otherwise, note that  $V_j^{\$}(\bar{X})$  is linear and increasing in  $\bar{X}$ . An interior solution  $V_j^{\$}(\bar{X}) = 0$  requires  $V_j^{\$}(0) \leq 0$  and  $V_j^{\$}(1) \geq 0$ , which is true if and only if  $\kappa \geq (1 - p^*)\pi$ . Hence, if  $\kappa < (1 - p^*)\pi$ , there can be no interior solution and only one corner is an equilibrium. ■

Proof of Proposition 2. When  $\bar{X} = 1$ , we have

$$V^{\$} = \left( \frac{B^{\$}}{B^{\$} + 1} - 1 \right) \pi + \kappa,$$

which is weakly positive whenever  $\kappa \geq (B^{\$} + 1)^{-1}\pi$ . The corresponding condition for  $\bar{X} = 0$  is straightforward.

To prove the uniqueness condition, note first that for  $\kappa \geq \frac{1}{\min\{B^{\$}, B^{\text{€}}\}}\pi$  both corner solutions are equilibria, implying there is equilibrium multiplicity in this range. Below, we show that for  $\kappa \in [\frac{1}{\max\{B^{\$}, B^{\text{€}}\}}\pi, \frac{1}{\min\{B^{\$}, B^{\text{€}}\}}\pi)$  there exists a unique corner equilibrium and for  $\kappa < \frac{1}{\max\{B^{\$}, B^{\text{€}}\}}\pi$  there exists a unique interior equilibrium (and no corner equilibria).

Suppose first that  $B^{\$} \neq B^{\epsilon}$ . For  $\kappa \in [\frac{1}{\max\{B^{\$}, B^{\epsilon}\}}\pi, \frac{1}{\min\{B^{\$}, B^{\epsilon}\}}\pi)$  clearly only one of the corner equilibria exists, but we need to show that there is no interior equilibrium. Without loss of generality, assume that  $B^{\$} > B^{\epsilon}$ .

An interior symmetric equilibrium solves:

$$V^{\$}(\bar{X}) = \frac{B^{\$}}{B^{\$} + \bar{X}}(\pi - \kappa(1 - \bar{X})) - \frac{B^{\epsilon}}{B^{\epsilon} + (1 - \bar{X})}(\pi - \kappa\bar{X}) = 0.$$

Re-arranging, this simplifies to a quadratic equation,  $P(\bar{X}) = 0$ , where

$$P(\bar{X}) \equiv (B^{\$} - B^{\epsilon})\kappa\bar{X}^2 + [(B^{\$} + B^{\epsilon})\pi - 2B^{\$}(B^{\epsilon} + 1)\kappa] \bar{X} + B^{\$}(B^{\epsilon} + 1)\kappa - B^{\$}\pi.$$

$P(\bar{X})$  is a convex quadratic polynomial, since  $B^{\$} > B^{\epsilon}$ . To show there are no solutions  $\bar{X} \in (0, 1)$  it is enough to show  $P(0) < 0$  and  $P(1) < 0$ . Indeed,

$$P(0) = B^{\$}(B^{\epsilon} + 1)\kappa - B^{\$}\pi < 0$$

since  $\kappa < \frac{1}{B^{\epsilon} + 1}\pi$ , and

$$\begin{aligned} P(1) &= (B^{\$} - B^{\epsilon})\kappa + [(B^{\$} + B^{\epsilon})\pi - 2B^{\$}(B^{\epsilon} + 1)\kappa] + B^{\$}(B^{\epsilon} + 1)\kappa - B^{\$}\pi \\ &= B^{\epsilon}(\pi - (B^{\$} + 1)\kappa) \leq 0 \end{aligned}$$

since  $\kappa \geq \frac{1}{B^{\$} + 1}\pi$ , with equality only when  $\kappa = \frac{1}{B^{\$} + 1}\pi$ , in which case the interior solution corresponds with the corner solution  $\bar{X} = 1$ . Thus, for  $\kappa \in [\frac{1}{\max\{B^{\$}, B^{\epsilon}\}}\pi, \frac{1}{\min\{B^{\$}, B^{\epsilon}\}}\pi)$  there is a unique, corner equilibrium of  $\bar{X} = 1$  when  $B^{\$} > B^{\epsilon}$  and  $\bar{X} = 0$  when  $B^{\$} < B^{\epsilon}$ .

In case  $\kappa < \frac{1}{\max\{B^{\$}, B^{\epsilon}\}}\pi$  corners cannot be equilibria. Thus, we need to show that there exists a unique interior equilibrium. To show uniqueness, define  $\phi^{\$}(\bar{X}) \equiv \frac{B^{\$}}{B^{\$} + \bar{X}} [\pi - \kappa(1 - \bar{X})]$  and  $\phi^{\epsilon}(\bar{X}) \equiv \frac{B^{\epsilon}}{B^{\epsilon} + 1 - \bar{X}} [\pi - \kappa\bar{X}]$ . The interior equilibrium must satisfy:

$$V^{\$}(\bar{X}) = \phi^{\$}(\bar{X}) - \phi^{\epsilon}(\bar{X})$$

Taking derivatives,

$$\frac{\partial V^{\$}}{\partial \bar{X}} = \frac{\partial(\phi^{\$}(\bar{X}))}{\partial \bar{X}} - \frac{\partial(\phi^{\epsilon}(\bar{X}))}{\partial \bar{X}}$$

Now notice that

$$\frac{\partial(\phi^{\$}(\bar{X}))}{\partial \bar{X}} = \frac{\kappa B^{\$}}{B^{\$} + \bar{X}} - [\pi - \kappa(1 - \bar{X})] \frac{B^{\$}}{(B^{\$} + \bar{X})^2} < 0$$

if  $\kappa < (B^{\$} + 1)^{-1}\pi$  and

$$\frac{\partial(\phi^{\epsilon}(\bar{X}))}{\partial\bar{X}} = -\frac{\kappa B^{\epsilon}}{B^{\epsilon} + 1 - \bar{X}} + [\pi - \kappa\bar{X}] \frac{B^{\epsilon}}{(B^{\epsilon} + 1 - \bar{X})^2} > 0$$

if  $\kappa < (B^{\epsilon} + 1)^{-1}\pi$ . Thus, for  $\kappa < \frac{1}{\max\{B^{\$, B^{\epsilon}}\}}\pi$  we know that  $\phi^{\$}(\bar{X})$  is downward sloping and  $\phi^{\epsilon}(\bar{X})$  is upward sloping, hence  $V^{\$}(\bar{X})$  is itself downward sloping. Thus,  $V^{\$}(\bar{X})$  can cross zero at most once. To prove that a crossing does occur, notice that when  $\kappa < (\max\{B^{\$, B^{\epsilon}}\} + 1)^{-1}\pi$ ,

$$\begin{aligned} V^{\$}(0) &= \phi^{\$}(0) - \phi^{\epsilon}(0) = \frac{1}{B^{\epsilon} + 1}\pi - \kappa > 0 \\ V^{\$}(1) &= \phi^{\$}(1) - \phi^{\epsilon}(1) = \kappa - \frac{1}{B^{\$} + 1}\pi < 0. \end{aligned}$$

Thus, we have proven uniqueness for  $B^{\$} \neq B^{\epsilon}$ .

When  $B^{\$} = B^{\epsilon} = B$ , we need to show that the interior solution is unique whenever  $\kappa < \frac{1}{B+1}\pi$ . In this case  $P(\bar{X})$  is linear, and has the unique solution zero

$$X = \frac{B\pi - B(B+1)\kappa}{2B\pi - 2B(B+1)\kappa} = \frac{1}{2}.$$

■

*Proof of Proposition 3.* To show that the dollar-dominant equilibrium exists, conjecture  $B^{\epsilon} = 0$ , implying that  $p_j^{\epsilon} = 0$ . In this case

$$V^{\$}(X) = \frac{\bar{B}}{\bar{B} + \mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}\pi - 0 > 0,$$

for all  $\bar{X} \in [0, 1]$ . Hence,  $\bar{X} = 1$  is optimal from the trading firms' perspective. Since  $\mu_{eu}(1 - X^{eu}) > 0$ , returns cannot be equalized for any positive holdings of  $B^{\epsilon}$ , so  $B^{\epsilon} = 0$  is sustained. A similar argument establishes that same for the euro-dominant equilibrium.

When both  $B^{\$}$  and  $B^{\epsilon}$  are positive, we have

$$\begin{aligned} V^{\$}(X) &= \frac{\bar{B}}{\bar{B} + \mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}} [\pi - \kappa(1 - \bar{X})] - \\ &\quad \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - \bar{X}) + \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})} [\pi - \kappa\bar{X}]. \end{aligned} \quad (19)$$

Since  $\mu_{us}X^{us} = \mu_{eu}(1 - X^{eu})$  and  $\mu_{eu}X^{eu} = \mu_{us}(1 - X^{us})$  by assumption, clearly  $V^{\$} = 0$  when  $\bar{X} = 1/2$ .

To prove these are the only steady states (except when  $\kappa = \frac{\mu_{rw}\pi}{\bar{B} + \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}$ ), redefine

$$\begin{aligned}\phi^{\$}(\bar{X}) &= \frac{\bar{B}}{\bar{B} + \mu_{rw}\bar{X} + \mu_{us}X^{us} + \mu_{eu}X^{eu}} [\pi - \kappa(1 - \bar{X})] \\ \phi^{\epsilon}(\bar{X}) &= \frac{\bar{B}}{\bar{B} + \mu_{rw}(1 - \bar{X}) + \mu_{us}(1 - X^{us}) + \mu_{eu}(1 - X^{eu})} [\pi - \kappa\bar{X}]\end{aligned}$$

so that  $V^{\$}(\bar{X}) = \phi^{\$}(\bar{X}) - \phi^{\epsilon}(\bar{X})$ . Moreover, notice that  $\frac{\partial\phi^{\$}(\bar{X})}{\partial\bar{X}}$  is negative so long as  $\kappa$  falls below the threshold  $\frac{\mu_{rw}\pi}{\bar{B} + \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}$ , is zero when  $\kappa$  equals this value, and is positive otherwise.

Conversely,  $\frac{\partial\phi^{\epsilon}(\bar{X})}{\partial\bar{X}}$  is positive when  $\kappa$  is below this threshold, zero at it, and negative otherwise. Hence, except in the knife-edge case, the two functions cross exactly once. In the special case that  $\kappa = \frac{\mu_{rw}\pi}{\bar{B} + \mu_{rw} + \mu_{us}X^{us} + \mu_{eu}X^{eu}}$ ,  $\phi^{\$}(\bar{X}) = \phi^{\epsilon}(\bar{X})$ , and the economy exhibits a continuum of equilibria.  $\blacksquare$

Proof of Proposition 4. Define the total mass the two large countries,  $\mu^* \equiv \mu_{us} + \mu_{eu}$ , and let

$$X^* \equiv \frac{\mu_{us}X_{us} + \mu_{eu}X_{eu}}{\mu_{us} + \mu_{eu}}$$

be average dollar use among them. Thus,  $\mu^*X^* = \mu_{us}X_{us} + \mu_{eu}X_{eu}$  is the total dollar use among the big countries. Given our symmetry assumptions  $X^* = \frac{1}{2}$ .

To prove local stability, we want to show that best-response functions define a contraction in the neighborhood of a given steady state. Define the vector of best response functions of trading firms and households in country  $j$ , given the actions of all other firms,  $\bar{X}$ , and households in the rest of the world  $B^{\$}$  and  $B^{\epsilon}$ :

$$\begin{aligned}\varphi_X(\bar{X}, B^{\$}, B^{\epsilon}) &= \frac{B^{\$}(\pi - \kappa(B^{\epsilon} + 1) + \bar{X}\kappa(2B^{\epsilon} + 1))}{(B^{\$} + B^{\epsilon})\pi + \kappa(\bar{X}(B^{\$} - B^{\epsilon}) - B^{\$})} \\ \varphi_{B^{\$}}(\bar{X}, B^{\$}, B^{\epsilon}) &= \bar{B} \frac{\bar{X}}{\mu_{rw}\bar{X} + \mu^*X^*} \\ \varphi_{B^{\epsilon}}(\bar{X}, B^{\$}, B^{\epsilon}) &= \bar{B} \frac{1 - \bar{X}}{\mu_{rw}(1 - \bar{X}) + \mu^*(1 - X^*)}\end{aligned}$$

Stacking these in the vector  $\Phi \equiv [\varphi_X, \varphi_{B^{\$}}, \varphi_{B^{\epsilon}}]$ , we want to show that  $\Phi$  is a local contraction map, which is the case whenever the eigenvalues of the Jacobian  $\nabla\Phi$  lie inside the unit circle.

The Jacobian has the form

$$\nabla\Phi = \begin{bmatrix} \frac{\partial\varphi_X}{\partial\bar{X}} & \frac{\partial\varphi_X}{\partial B^{\$}} & \frac{\partial\varphi_X}{\partial B^{\epsilon}} \\ \frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} & 0 & 0 \\ \frac{\partial\varphi_{B^{\epsilon}}}{\partial\bar{X}} & 0 & 0 \end{bmatrix}$$

hence its eigenvalues are given by the roots of the characteristic polynomial

$$\lambda \left( \lambda^2 - \lambda \frac{\partial \varphi_X}{\partial \bar{X}} - \frac{\partial \varphi_{B^s}}{\partial \bar{X}} \frac{\partial \varphi_X}{\partial B^s} - \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}} \frac{\partial \varphi_X}{\partial B^\epsilon} \right) = 0.$$

Clearly, one of the solutions is  $\lambda = 0$ , so we just need to ensure that the roots of the quadratic expression in the parenthesis are inside the unit circle. We proceed to check this condition for each steady state.

### Case I: Symmetric Steady State

At the symmetric steady state we have that  $\frac{\partial \varphi_X}{\partial B^s} = -\frac{\partial \varphi_X}{\partial B^\epsilon}$ . Hence, the relevant condition for the eigenvalues reduces to

$$\lambda^2 - \lambda \frac{\partial \varphi_X}{\partial \bar{X}} - \frac{\partial \varphi_X}{\partial B^s} \left( \frac{\partial \varphi_{B^s}}{\partial \bar{X}} - \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}} \right) = 0$$

with roots

$$\lambda^* = \frac{1}{2} \left( \frac{\partial \varphi_X}{\partial \bar{X}} \pm \sqrt{\left( \frac{\partial \varphi_X}{\partial \bar{X}} \right)^2 + 4 \frac{\partial \varphi_X}{\partial B^s} \left( \frac{\partial \varphi_{B^s}}{\partial \bar{X}} - \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}} \right)} \right).$$

At the symmetric steady state.

$$\frac{\partial \varphi_X}{\partial \bar{X}} = \frac{\kappa (4X^* \mu^* (1 - X^*) + \mu_{rw} (\mu_{rw} + 2\mu^*) + 2\bar{B} (\mu_{rw} + \mu^*))}{(\mu_{rw} + \mu^*)^2 (2\pi - \kappa)} > 0$$

since  $\kappa < 2\pi$  (recall  $\kappa$  is not bigger than the gross surplus of transactions). Hence, the bigger root (in absolute value) is

$$\lambda^* = \frac{1}{2} \left( \frac{\partial \varphi_X}{\partial \bar{X}} + \sqrt{\left( \frac{\partial \varphi_X}{\partial \bar{X}} \right)^2 + 4 \frac{\partial \varphi_{B^s}}{\partial \bar{X}} \left( \frac{\partial \varphi_X}{\partial B^s} - \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}} \right)} \right).$$

Lastly, since we also have that

$$\frac{\partial \frac{\partial \varphi_{B^s}}{\partial \bar{X}}}{\partial \kappa} = \frac{\partial \frac{\partial \varphi_X}{\partial B^s}}{\partial \kappa} = \frac{\partial \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}}}{\partial \kappa} = 0,$$

the root is growing in  $\kappa$ . The threshold  $\bar{\kappa}$  that ensures roots within the unit circle by solves

$$1 - \frac{\partial \varphi_X}{\partial \bar{X}} - \frac{\partial \varphi_{B^s}}{\partial \bar{X}} \left( \frac{\partial \varphi_X}{\partial B^s} - \frac{\partial \varphi_{B^\epsilon}}{\partial \bar{X}} \right) = 0.$$

This value is

$$\bar{\kappa} = \frac{\pi(\mu_{rw}(\mu_{rw} + \mu^*) + (\mu^*)^2(1 - 4X^*(1 - X^*)))}{(\mu_{rw} + \mu^*)(\bar{B} + \mu_{rw} + \mu^*\frac{1}{2})} \Big|_{X^*=\frac{1}{2}} = \frac{\pi\mu_{rw}}{\bar{B} + \mu_{rw} + \mu^*\frac{1}{2}}.$$

Hence, in the neighborhood of the symmetric steady state, the roots of the characteristic polynomial are inside the unit circle so long as  $\kappa < \bar{\kappa}$ .

### Case II: Dollar-dominant Steady State

At the dollar dominant steady state  $\bar{X} = 1$  and

$$\frac{\partial\varphi_X}{\partial\bar{X}} = \frac{\partial\varphi_X}{\partial B^{\$}} = 0.$$

We need to ensure that

$$\lambda^2 = \frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} \frac{\partial\varphi_X}{\partial B^{\$}} < 1$$

where

$$\frac{\partial\varphi_{B^{\$}}}{\partial\bar{X}} \frac{\partial\varphi_X}{\partial B^{\$}} = \frac{\bar{B}}{\mu^*(1 - X^*)} \frac{\pi(\mu_{rw} + \mu^*) - \kappa(\mu_{rw} + \mu^*X^* + \bar{B})}{\bar{B}\pi}.$$

If  $\kappa < \frac{\pi(\mu_{rw} + X^*\mu^*)}{\mu_{rw} + X^*\mu^* + \bar{B}}$  then the above expression is positive. Hence  $|\lambda| < 1$  if and only if the above expression is less than one, which is true if

$$\kappa > \frac{\pi(\mu_{rw} + (2X^* - 1)\mu^*)}{\bar{B} + \mu_{rw} + \mu^*X^*}$$

which is equal to  $\bar{\kappa}$  when  $X^* = \frac{1}{2}$ .

On the other hand, if  $\kappa > \frac{\pi(\mu_{rw} + X^*\mu^*)}{\mu_{rw} + X^*\mu^* + \bar{B}}$  then  $\phi_X = 1$  because it hits the upper bound of  $X \leq 1$ . As a result  $\frac{\partial\varphi_X}{\partial B^{\$}} = 0$  and thus  $\lambda^2 = 0$ . Since all eigenvalues of  $\nabla\Phi$  are zero, the system is stable. Thus, the coordinated steady state is stable for any  $\kappa > \bar{\kappa}$ .

### Case III: Euro-dominant Steady State

Can be proven with identical steps to Case II. ■

# Appendix for Online Publication

## B Model Details

### B.1 Households

For  $j \in \{us, eu\}$ , foreign imports consist of the good of the other big country and an aggregate of rest-of-the world goods. Hence, the big country consumption aggregator is

$$C_{jt} = (C_{jt}^j)^{a_h} \left( (C_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'} + \mu_{rw}}} (C_{jt}^{rw})^{\left(\frac{\mu_{rw}}{\mu_{j'} + \mu_{rw}}\right)} \right)^{1-a_h}, \quad (20)$$

where  $j'$  is the complement of  $j$  and  $C_{jt}^{j'}$  is the consumption in country  $j$  of the good of country  $j'$ , and  $a_h$  controls the degree of home bias in consumption. Rest-of-world consumption goods are aggregated according to  $C_{jt}^{rw} = \left( \int (C_{jt}^i)^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}$ . The corresponding aggregate consumption price index is

$$P_{jt} = \frac{1}{K} (P_{jt}^j)^{a_h} \left( (P_{jt}^{j'})^{\frac{\mu_{j'}}{\mu_{j'} + \mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{j'} + \mu_{rw}}} \right)^{1-a_h}. \quad (21)$$

where  $K \equiv a_h^{a_h} (1 - a_h)^{1-a_h} \left( \frac{\mu_{j'}}{\mu_{j'} + \mu_{rw}} \right)^{(1-a_h)\frac{\mu_{j'}}{\mu_{j'} + \mu_{rw}}} \left( \frac{\mu_{rw}}{\mu_{j'} + \mu_{rw}} \right)^{(1-a_h)\frac{\mu_{rw}}{\mu_{j'} + \mu_{rw}}}$ .

For small countries  $j \in [0, \mu_{rw}]$ , the consumption basket includes imports from both big countries and all other rest-of-world small countries:

$$C_{jt} = C_{jt}^j a_h \left( (C_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} (C_{jt}^{eu})^{\frac{\mu_{eu}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} (C_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} \right)^{1-a_h}. \quad (22)$$

The associated price index is

$$P_{jt} = \frac{1}{K_{rw}} (P_{jt}^j)^{a_h} \left( (P_{jt}^{us})^{\frac{\mu_{us}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} (P_{jt}^{eu})^{\frac{\mu_{eu}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} (P_{jt}^{rw})^{\frac{\mu_{rw}}{\mu_{us} + \mu_{eu} + \mu_{rw}}} \right)^{1-a_h}. \quad (23)$$

where  $K_{rw}$  is defined analogously to  $K$  above.

### B.2 The Import-Export Sector

The following subsection provide additional details on each stage of the trading firm game.

#### Stage 3: Trading Round and Profits

We solve the problem of the trading firms starting with stage three and working backwards. In the final stage, firms discover whether they are importing or exporting this period

and with what country, and then search for an appropriate foreign counterpart. For each sub-market, we again assume that the total number of successful matches is given by the [den Haan et al. \(2000\)](#) matching function  $M^t(u, v) = \frac{uv}{(u^{\frac{1}{\varepsilon_T}} + v^{\frac{1}{\varepsilon_T}})^{\varepsilon_T}}$  with elasticity parameter  $\varepsilon_T$  which may in general be different than  $\varepsilon_f$ .

Let  $c = (j, j')$  be a double index, capturing an arbitrary country pair, and let  $\tilde{m}_{ct}^{im}$  be the mass of *funded* importing firms in country  $j$  seeking trade with funded exporting firms in country  $j'$  at time  $t$ . Then the probability of a country  $j$  importer matching with a country  $j'$  exporter is

$$p_{ct}^{ie} = \frac{\tilde{m}_{c',t}^{ex}}{[(\tilde{m}_{c',t}^{ex})^{1/\xi_t} + (\tilde{m}_{ct}^{im})^{1/\xi_t}]^{\xi_t}},$$

where  $c' \equiv (j', j)$ . Using analogous definitions, the probability of a country  $j$  exporter matching with a country  $j'$  importer is

$$p_{ct}^{ei} = \frac{\tilde{m}_{c',t}^{im}}{[(\tilde{m}_{c',t}^{im})^{1/\xi_t} + (\tilde{m}_{ct}^{ex})^{1/\xi_t}]^{\xi_t}}.$$

Given a successful match, the two parties split the surplus that emerges from their trade. Here, we compute this value in the case of a successful match between a country  $j$  importer and a country  $j'$  exporter; the remaining possibilities can be computed in parallel. For each good that the two firms exchange, they earn a surplus that is equal to the difference in the price of the  $j'$  good in its origin country ( $P_{j',t}^{j'}$ ) and its price in the destination country  $j$  ( $P_{j,t}^{j'}$ ). If there is a currency mismatch between the two counter-parties, however, the trading surplus is reduced by an additional fraction  $\kappa > 0$  of the transaction price.

We assume that the resulting surplus is split via Nash bargaining, with a weight  $\alpha$  for the importer. The effective transaction price is thus:

$$P_{c',t}^{whol} = P_{j',t}^{j'} + (1 - \alpha)(P_{j,t}^{j'} - P_{j',t}^{j'})$$

This is the wholesale price of country  $j'$  exports – the price at which the country- $j$  importer purchases the good from the country- $j'$  exporter. In turn, the country- $j$  importer sells the good at its equilibrium  $j$  retail price  $P_{j,t}^{j'}$ . In equilibrium,  $P_{j,t}^{j'} > P_{c',t}^{whol}$  and hence there is a markup and an associated positive surplus to sustain trading.

Let  $\tilde{X}_{jt}$  be the fraction of funded country  $j$  firms who hold dollar collateral. Then the expected profits of country- $j$  importer importing from  $j'$  who hold dollars is given by

$$\pi_{c,t}^{s,im} = \frac{(1 - \alpha)}{P_{c',t}^{whol}} \left[ P_{j,t}^{j'} - P_{j',t}^{j'} - \kappa P_{c',t}^{whol} (1 - \tilde{X}_{jt}) \right],$$

while if it hold euros, expected profits are

$$\pi_{c,t}^{\epsilon,im} = \frac{(1 - \alpha)}{P_{c',t}^{whol}} \left[ P_{j,t}^{j'} - P_{j',t}^{j'} - \kappa P_{c',t}^{whol} \tilde{X}_{jt} \right].$$

Similar expressions hold for exporters:

$$\begin{aligned}\pi_{c,t}^{\$,ex} &= \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[ P_{j',t}^j - P_{j,t}^j - \kappa P_{c,t}^{whol} (1 - \tilde{X}_{j',t}) \right] \\ \pi_{c,t}^{\€,ex} &= \frac{(1-\alpha)}{P_{c,t}^{whol}} \left[ P_{j',t}^j - P_{j,t}^j - \kappa P_{c,t}^{whol} \tilde{X}_{j',t} \right].\end{aligned}$$

## Stage 2: Funding Stage

At this stage, the trading firms choose what type of funding to seek. We refer to funding with US safe assets as “dollar” funding, and funding with EU safe assets as “euro” funding. Firms seek their funding in matching markets, and not all firms succeed in finding funding each period. Supply in each of these markets is furnished by the domestic household, which lends its bond holdings of a particular currency. On the demand side are the domestic firms who choose to search for that currency.

In order to make their currency choice, firms compare their expected profits conditional on either being funded with dollars or euros. At this stage, they do not yet know whether they will be importers or exporters, with what country they may trade, or whether they will be able to find a successful trading matches in the next stage. Hence, they form expectations over the trading profits that they would receive, conditional on choosing one type of funding over the other.

The expected profit of a country- $j$  trading firm funded with US assets is

$$\tilde{\Pi}_{jt}^{\$} = \sum_c p_{ct}^{im} p_{ct}^{ie} \pi_{ct}^{\$,im} + \sum_c p_{ct}^{ex} p_{ct}^{ei} \pi_{ct}^{\$,ex},$$

where  $p_{ct}^{im}$  is the probability the firm from country  $j$  seeks to import from an exporter in country  $j'$ .

The first of the two terms in the above sum equals the expected profit of being a dollar-funded importer. It equals the probability of being an importer from country  $j'$  times the probability of finding a successful match with a foreign exporter from  $j'$ , times the resulting profits from that match. The second component is the expected profit of being a dollar-funded exporter. The corresponding expected profits of a country  $j$  trading firm funded with EU assets instead is:

$$\tilde{\Pi}_{jt}^{\€} = \sum_c p_{ct}^{im} p_{ct}^{ie} \pi_{ct}^{\€,im} + \sum_c p_{ct}^{ex} p_{ct}^{ei} \pi_{ct}^{\€,ex}.$$

We now compute the probability that firms find the funding they seek. Bonds promise payment of one unit of the issuing country’s tradable good. Thus, the total value of the US bonds available for lending in country  $j$  at time  $t$  is given by  $P_{us,t}^{us} B_{jt}^{\$} Q_t^{\$}$ , where  $B_{jt}^{\$}$  are the holdings of US bonds in the country  $j$  household’s portfolio,  $P_{us,t}^{us}$  is the price of the US tradable good, and  $Q_t^{\$}$  is the real price of the bond that pays off one unit of US consumption tomorrow.

Let  $m_{jt}$  be mass of trading firms operating in country  $j$ , and let  $X_{jt}$  be the fraction of country- $j$  trading firms choosing to seek US bonds. Then, the total mass of country- $j$  trading firms searching the domestic US bond market is  $m_{jt}X_{jt}$ . The total mass of US funds available to intermediate trade is  $P_{us,t}^{us}B_{jt}^{\$}Q_t^{\$}$ . Moreover, we incorporate a parameter,  $\nu$ , that describes the maximum number of times a given bond could intermediate trade per period of time. This parameter allows us to capture an exogenous notion of velocity for assets lent for trade finance, making it possible to consider an annual calibration in our quantitative results without unduly limiting the availability of trade funding. In our quantitative results, we fix  $\nu = 8$  throughout.

The probability that a country  $j$  trading firm seeking US bonds finds a supplier is

$$p_{jt}^{\$} = \frac{M^f(m_{jt}X_{jt}, \nu P_{us,t}^{us}B_{jt}^{\$}Q_t^{\$})}{m_{jt}X_{jt}}.$$

Similarly, the probability that a country  $j$  trading firm seeking EU bonds finds a match is

$$p_{jt}^{\text{€}} = \frac{M^f(m_{jt}(1 - X_{jt}), \nu P_{eu,t}^{eu}B_{jt}^{\text{€}}Q_t^{\text{€}})}{m_{jt}(1 - X_{jt})}.$$

It is now straightforward to evaluate

$$\tilde{X}_{jt} = \frac{p_{jt}^{\$}X_{jt}}{p_{jt}^{\$}X_{jt} + p_{jt}^{\text{€}}(1 - X_{jt})}.$$

In the event that the trading firm finds the funding it seeks, it pays a fee  $r$  for the funding services of dollars or euros. Thus, the expected profit of a country- $j$  firm seeking dollar funding is given by

$$\Pi_{jt}^{\$} = p_{jt}^{\$}(\tilde{\Pi}_{jt}^{\$} - r), \quad (24)$$

which is simply the probability of obtaining dollar funding,  $p_{jt}^{\$}$ , times the expected profit net of the dollar funding costs. Similarly, we can compute the expected profit of a country- $j$  firm seeking Euro funding:

$$\Pi_{jt}^{\text{€}} = p_{jt}^{\text{€}}(\tilde{\Pi}_{jt}^{\text{€}} - r). \quad (25)$$

The only equilibrium requirement for the funding fee  $r$  is that it leaves firms with a positive surplus relative to the alternative of declining funding and doing no trade. In parallel with the labor match and searching literature, these prices can be fixed exogenous parameters — so long as they fall within the surplus range of the trading firms — or they could be endogenously determined by assuming some bargaining paradigm, like Nash bargaining. For simplicity, we follow the first of these paths and fix the funding prices to a common value.

## Stage 1: Firm Formation

Equilibrium with interior  $p_{ct}^{im}$  and  $p_{ct}^{ex}$  requires that, prior to learning their private currency choice, firms are ex post indifferent between importing and exporting to the various countries. Hence, for example in the US, we must have

$$X_{us}p_{(us,j),t}^{ie}\pi_{(us,j),t}^{\$,im} + (1 - X_{us})p_{(us,j),t}^{ie}\pi_{(us,j),t}^{\$,im} = X_{us}p_{(us,j'),t}^{ie}\pi_{(us,j'),t}^{\$,im} + (1 - X_{us})p_{(us,j'),t}^{ie}\pi_{(us,j'),t}^{\$,im}$$

for all US potential trading partners  $j$  and  $j'$ . Similarly,

$$X_{us}p_{(us,j),t}^{ie}\pi_{(us,j),t}^{\$,im} + (1 - X_{us})p_{(us,j),t}^{ie}\pi_{(us,j),t}^{\$,im} = X_{us}p_{(us,j),t}^{ex}\pi_{(us,j),t}^{\$,ex} + (1 - X_{us})p_{(us,j),t}^{ex}\pi_{(us,j),t}^{\$,ex}$$

The above equations are sufficient to pin down the equilibrium probabilities for importing and exporting to and from each country pair.

Given this and all of the above choices, prospective firms then decide whether or not to pay the fixed cost  $\phi > 0$  in order to become operational this period. Firms enter the import-export sector until the zero-profit condition

$$\max\{\Pi_{jt}^{\$,im}, \Pi_{jt}^{\$,ex}\} - \phi = 0,$$

is satisfied. The entry condition thus determine the equilibrium size,  $m_{jt}$ , of the import-export sector in each country.

## Summary

Given processes for  $\{\Delta_{jt}^{\$,im}, \Delta_{jt}^{\$,ex}, \Delta_{j',t}^j\}$ , an equilibrium in the domestic economy is described by the common prices,  $\{Q_t^{\$,im}, Q_t^{\$,ex}\}$ , the set of country-specific prices,  $\{P_{jt}, P_{jt}^{us}, P_{jt}^{eu}, P_{jt}^{rw}\}$ , and the set of country-specific allocations  $\{C_{jt}, C_{jt}^{us}, C_{jt}^{eu}, B_{jt}^{\$,im}, B_{jt}^{\$,ex}\}$  that satisfy equations (11), (12), (15) through (18), and (20) through (23).

## Rest-of-World Asset Supply

Our baseline economy abstracts from the presence of any savings vehicle issued by the rest of the world. This is in part because, absent a liquidity premium term, adding such an asset would create an indeterminacy in long-run wealth levels. The same sort of indeterminacy is pervasive in open economy models with incomplete asset markets.

As simple way to include a rest-of-world asset market is to assume there exists an exogenous liquidity demand  $z_j$  for the rest of world asset. Though we don't model this role explicitly, we assume it is proportional to the measure of firms in the economy, so that the liquidity wedge for the RW asset is given by

$$\Delta_{jt}^{RW} = \frac{M^f (m_{jt}z_j, \nu P_{rw,t}^{RW} B_{jt}^{RW} Q_t^{\$,im})}{\nu P_{rw,t}^{RW} B_{jt}^{RW} Q_t^{\$,im}} r.$$

Moments	USD Coord.			Symmetric			EUR Coord.		
	US	EU	RW	US	EU	RW	US	EU	RW
Dollar Share	0.90	0.10	0.80	0.90	0.10	0.50	0.90	0.10	0.20
$100 \times (i^{\$} - i^{\text{€}})$	1.03	-	-	0.00	0.00	-	-	-1.03	-
100×Implied rev./GDP	0.85	0.22	-	0.54	0.54	-	0.22	0.85	-
100×Trade bal./GDP	-0.25	-0.28	0.14	-0.12	-0.12	0.07	-0.28	-0.25	0.14
Gross debt/GDP	0.60	0.59	0.40	0.60	0.60	0.40	0.59	0.60	0.40
NFA/GDP	-0.14	0.01	0.03	-0.10	-0.10	0.05	0.01	-0.14	0.03
Home Bias	0.12	0.39	0.06	0.23	0.23	0.00	0.39	0.12	0.06
Consumption	11.492	11.492	12.264	11.466	11.466	12.279	11.492	11.492	12.264

Table 6: Steady-state values for baseline model.

The household euler equation for the RW bond is

$$1 = \beta E_t \left[ \left( \frac{C_{jt+1}}{C_{jt}} \right)^{-\sigma} \frac{P_{jt}}{P_{jt+1}} \frac{P_{rw,t+1}^{rw}}{P_{rw,t}^{rw}} \frac{1}{Q_t^{RW} (1 - \Delta_{jt}^{RW} + \tau'(B_{j,t}^{RW}, B_{j,t-1}^{RW}))} \right].$$

A desirable feature of this approach to determining holding of the RW bond is that the steady state portfolio allocations are independent of a scale shift in the  $z_j$ .

Table 6 reproduces the moments for a calibration targeted to the moments in Panel (a) of Table 1, except that the share of RW assets is increased from zero to 40% of rest-of-world GDP and  $z_j = 0.10$ . Consumption for the US and EU remain extremely close to each other, but two other differences stand out. First, the EU NFA position is now roughly zero, while the US NFA position is reduced to -14%, more consistent with its average value over the past 40 years. Second, as a result of the more moderate NFA position, the US runs a permanent trade deficit, despite its negative NFA position.