Optimal Bank Regulation In the Presence of Credit and Run Risk[*]

Anil K Kashyap[†] Dimitrios P. Tsomocos[‡] Alexandros P. Vardoulakis[§]

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Abstract

We modify a Diamond and Dybvig (1983) model so that besides offering liquidity services to depositors, banks also raise equity funding, make loans that are risky and can invest in safe, liquid assets. The bank and its borrowers are subject to limited liability. Banks monitor borrowers to insure that they repay loans and in doing so they expand the supply of credit. Depositors may choose to run based on conjectures about the resources that are available for people withdrawing early. We use a new type of global game to solve for the run decision. We find that banks opt for a more deposit-intensive liability mix and more lending-intensive asset mix than a social planner would choose. The privately chosen scale of intermediation also can be higher or lower than the level that a planner would choose. To correct these three distortions, a package of three regulations is warranted.

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[†]University of Chicago Booth School of Business, United States and Bank of England; email: anil.kashyap@chicagobooth.edu

[‡]Saïd Business School and St. Edmund Hall, University of Oxford, United Kingdom;
email: dimitrios.tsomocos@sbs.ox.ac.uk

[§]Board of Governors of the Federal Reserve System, United States; email: alexandros.vardoulakis@frb.gov

1 Introduction

It is well understood that banks can provide useful services through both the lending that they undertake and the deposits that they offer. Yet, most banking models tend to focus only one service or the other because tractable models that include both have proved to be illusive. In this paper, we propose a new model that features banks that offer both services and use the model to study alternative regulations that have been proposed since the global financial crisis.

Our model starts with the setup that was developed by Diamond and Dybvig (1983) that explains how an intermediary (that we will call a bank) can provide liquidity services to depositors with uncertain consumption/funding needs. We modify their model so that the bank also raises equity funding, makes loans that are risky and can invest in safe, liquid assets. We also assume that banks and borrowers are subject to limited liability so that each of them has incentives to take risks that they do not fully bear the costs of. We suppose, as in Diamond (1984), that banks monitor the loans to guarantee that they are repaid. This monitoring expands the supply of credit (relative to world without banks). However, because the loans are risky and have an uncertain liquidation value, depositors may opt to run on the bank.

We characterize the run decision by introducing a new type of global game that is inspired by Goldstein and Pauzner (2005) approach to analyzing Diamond-Dybvig style models. In our setup, depositors get signals about the value of loans if they are called early to help pay depositors. Depositors will use a threshold rule in deciding to run that depends on the resources available to meet a run (the combined value of liquid assets and recalled loans) relative to promised deposit repayments. The proof that we use to confirm the uniqueness of the threshold is novel and may be of independent interest for other applications of global games.

In our context, capital and liquidity regulations differently alter the ratio that governs the run. This property makes the model well-suited to studying the impact of these types of regulations and makes it easy to understand how the outcomes they deliver compare to the ones that a social planner would prefer.

We have four main findings. First, both borrowers and banks are prone to take risks that raise the probability of a run (relative to the allocations that a social planner would choose). This is largely a consequence of the limited liability protection that allows them to keep the profits that result from these choices without facing the full costs of the losses that sometimes result. Second, the banks' risk-taking happens on both sides of their balance sheets. They opt for a more deposit-intensive liability mix and more lending-intensive asset mix each of which make the run risk higher than is it socially optimal. The aggregate size of the banking sector is also unlikely to be consistent with the scale that a planner would prefer; in our calibrated model, there is less intermediation (and higher profits from intermediation) than a planner would pick. This is not generically true and the private equilibrium could feature more intermediation than a planner would select. The allocations that a planner chooses will depend on how much weight is placed on borrowers versus savers. Third, individual regulations can be used to move the private choices closer to those that a planner would choose. But no single regulation can deliver the planning outcomes. Finally, to mimic the planners preferred allocations a combination of three regulations are needed to correct the distortions associated with the bank asset and liability mixes, and the scale of intermediation. The asset and liability distortions can be corrected using a capital and a liquidity requirement. To raise or lower the overall size of the banking system other regulations, such as deposit subsidies, or funding for lending subsidies would be needed. Capital and liquidity requirement are jointly helpful at reducing run risk, but are less effective at boosting the overall amount of intermediation which the planner may want to do to expand lending to favor borrowers. To manage the scale of intermediation, we show other regulations can be helpful.

Our analysis of potential runs can be contrasted to several other approaches that have been developed in the literature. For instance, a bank-run in our setup can occur because the information about fundamentals is very bad. Thus, our analysis can be compared to the many prominent papers that analyze information-based runs such as Chari and Jagannathan (1988), Jacklin and Bhattacharya (1988), Allen and Gale (1998), Uhlig (2010), Angeloni and Faia (2013) and Boissay, Collard and Smets (2016).

A bank-run can also potentially occur due to a coordination problem among depositors even if the bank is solvent in the long-run. This type of run can be interpreted as being panic-based, and for analyzing this kind of run, it is important to know what determines the panic. In the Diamond-Dybvig model panics are a multiple equilibrium outcome. Cooper and Ross (1998), Peck and Shell (2003) and Keister (2015) suppose instead that the probability of a bank-run is driven by sunspots. In our earlier working paper, Kashyap, Tsomocos and Vardoulakis (2014), in Gertler and Kiyotaki (2015) and in Choi, Eisenbach and Yorulmazer (2016) the probability of a run is determined by an exogenous function of key fundamentals.

Ennis and Keister (2005) take an axiomatic approach to equilibrium selection and link the probability of a particular equilibrium being played to appropriately defined incentives of agents. Instead, we use the global games approach developed by Morris and Shin (1998) and applied to banks runs by Goldstein and Pauzner (2005) to derive a unique probability of run which depends on fundamentals. Rochet and Vives (2004) and Vives (2014) also take a global game approach, but delegate the withdrawal decision to a (deposit) fund manager with a simpler payoff function.

The remainder of the paper is separated into four parts. In section 2, we describe the model and show the privately optimal choices for the bank, the savers and the entrepreneurs. In section 3 we study the efficient allocations chosen by a social planner and derive expressions for the wedges between the private and social decisions. In section 4, we explore how regulation can be used to correct the private inefficiencies. Section 5 concludes by summarizing the main findings, reiterating the intuition for them, and describing a few directions for future research. All proofs as well as additional derivations and model extensions are relegated to an online appendix.

2 Model

The model consists of three periods, $t = \{1,2,3\}$, features a single consumption good and includes three types of (representative) agents; an entrepreneur (E), a saver (S) and a banker (B).

The entrepreneur has access to a productive, but illiquid, risky technology and chooses how much to borrow to invest in it. Funds invested at date 1 yield A per unit of investment at date 3 with probability ω (which we call the "good" state of the world) and zero otherwise (the "bad" state). The project delivers no output at date 2 but it can be liquidated. The liquidation value, ξ, is uncertain and independent of the productivity shock (A).

The banker manages an institution, which we call a bank, that acts as an intermediary between the entrepreneurs and savers. The bank is funded by raising equity from the banker and deposits from the saver.[1] The funds raised at date 1 are invested into either a liquid storage asset or in a loan to the entrepreneur, which the bank can recall in the intermediate period. Moreover, the banker decides whether to monitor the entrepreneur's project at $t = 3$ or not. Monitoring is important because the productivity shock is private information to the entrepreneur. Without monitoring, the entrepreneur would report the bad state of the world and default.

The saver has a large endowment at date 1 that is used to fund initial consumption and savings. The savers have uncertain future consumption needs and, as in Diamond and Dybvig (1983), after date 1, some fraction will need to consume at t=2 and the rest can wait to consume at date 3. The saver invests in bank deposits or holds a liquid storage asset. The deposits are demandable, which is important to provide incentives to the banker to monitor as we describe in detail later.

A loan can be liquidated to pay deposits. Upon being recalled it yields an immediate gross return ξ. This value is uncertain, and ξ, follows a uniform distribution $U \sim \left[\underline{\xi},\overline{\xi}\right]$ with $0 \leq \underline{\xi} < 1 < \overline{\xi}$ and $\Delta_\xi = \overline{\xi} - \underline{\xi}$. The fact that $\overline{\xi}$ can exceed 1 will be important in what follows. When a loan is called the entrepreneur forfeits the portion of the project that is funded by the loan.

Depending on the value of ξ and the rest of its balance sheet, the bank may not have enough resources to serve withdrawals if all depositors decide to withdraw. Typically, aside from extremely high or low realizations of ξ, the bank is exposed to self-fulfilling runs: a patient saver will demand her deposits early if she believes that other patient savers will do the same. In order to address the coordination problem and obtain a unique equilibrium, we assume that savers receive noisy signals about the true realization of ξ at $t = 2$. These signals not only provide information about the fundamental ξ, but also about the beliefs of other savers, and so serve to coordinate the patient savers' decisions. In particular, we show that there is a unique threshold ξ^*, such that all patient savers withdraw their deposits when the true realization is below that threshold and keep their deposits

[1] We assume that savers cannot buy equity in the bank in order to simplify the exposition of our baseline model. In the online appendix, we present a more complicated model where where the bank raise both inside equity from bankers and outside equity from savers. Therein, the bank shares purchased by savers are tradable in a frictionless market in the intermediate period and, thus, also provide liquidity services. Although bank equity can also provide liquidity services, because it can be traded in a secondary market similar to Jacklin (1987), an all-equity funding structure would not be optimal in even in this richer setup due to the disciplinary role of runnable debt. Overall, the main results from the model in the body of the paper continue to hold in the expanded model.

in the bank otherwise. We will refer to ξ^* as the run threshold, while the probability of a run is $q = (\xi^* - \underline{\xi})/\Delta_\xi$.

The liquidation value can be justified in several ways. For instance, the incomplete project could have a secondary use in the interim period because it can be used in conjunction with an alternative short-term technology. Or, we could assume that it can be sold to some outside investors as in Shleifer and Vishny (1992). In other words, ξ does not strictly represent the salvage value of the long-term investment, as for example in Cooper and Ross (1998), but rather the liquidation/resale value of long-term investment. $\overline{\xi}$ has to be high enough that the bank can always withstand a panic for some realizations. Yet, $\underline{\xi}$ has to be low enough that the bank may run out of liquidity even if a panic does not occur. We describe the importance of these bounds in section 2.4.[2]

The run threshold depends on the savers beliefs about what they will receive by being patient as opposed to joining a run. More precisely, they need to form expectations over a variety of possible outcomes that involve their own utility function, the utility function of the entrepreneur and the production function. Essentially, to judge the risk of the period 3 deposits, the saver needs to infer the bank's profitability and its balance sheet, which will depend on the entrepreneurs' loan demand. In turn, the loan demand depends on the curvature of the entrepreneurs' utility function and the shape of the production function. The potential payoffs for period 3 deposits can then be contrasted to the value of withdrawing early. This comparison will depend on the depositors' attitudes towards risks in period 2 versus 3. Moreover, the run risk will enter into the welfare calculation of savers and entrepreneurs, and the planner will account for this dependence in choosing allocations.

We will make three assumptions to make these calculations as simple as possible, essentially by making the key schedules that are relevant for these expectations and comparisons linear. The main cost of assuming linearity is that it leaves savers and entrepreneurs at a point where they break-even from their savings and borrowing choices. This means that intermediation generates no surplus for them. The absence of any surplus does not distort the basic properties of the private equilibrium, but it does have powerful implications for the nature of optimal regulation. So we reintroduce curvature into the savers' and entrepreneurs' problems in other ways that make the results less extreme without complicating either the calculations of the expectations that matter for the run threshold or the derivations of distortions that motivate policy interventions. We will clearly identify and explain each of the three assumptions as we describe the rest of the model, but none of them are responsible for any of our main results.

Figure 1 presents the timeline of the model. Sections 2.1-2.4 describe the agents' optimization problems and the determination of the run threshold. We will see that solving for the indirect utility

[2] Our model can easily be adjusted to make the liquidation value depend on the expected value of the loans, i.e, $\xi \cdot \omega(1 + r_l)$, where r_l is the loan rate and ω is the good state where the loan is repaid. Then, ξ would capture the fraction (between 0 and 1) of the expected value that can be obtained at liquidation. The liquidation value would vary because ξ varies. Given that the expected value of loans is higher than one, the two approaches would yield qualitatively similar results. Alternatively, we could have assumed that ξ does not vary, but the probability distribution $\tilde{\omega}$ varies as in Goldstein and Pauzner (2005). Then, the liquidation value would continuously vary with the realization of the true probability distribution ω. The upper and lower dominance regions in the incomplete information game would still be endogenously determined in these cases. Matta and Perotti (2016) also consider runs resulting from assets liquidity risk and study how secured debt can adversely impact the incentives to run.

$t=1$	$t=2$	$t=3$
E borrows and invests in risky project S invest in demandable bank deposits and potentially in the liquid asset B raises equity and deposits and invests in loans and liquid assets	S learn their type S receive noisy signals about ξ and decide whether to withdraw B recalls loans and pays withdrawals If $\xi < \xi^*$, a run occurs	State of the world is determined E privately learns the realization B decides whether to monitor Loans and deposits are repaid in the good state and default in the bad

Figure 1: Model Timeline

functions of the savers and entrepreneurs facilitates much of the ensuing analysis. So those will be two of the key objects that we derive. Section 2.5 defines and characterizes the private equilibrium.

2.1 Savers

The savers are endowed with e_S at $t=1$ and decide how much of their endowment to invest in bank deposits, D, and how much to hold in the liquid storage technology, LIQ_S. respectively. At $t=2$, a portion of savers, δ, receive a preference shock to consume immediately, while the rest, $1-\delta$, want to consume at $t=3$. The preference shock is private information, i.i.d. and is not contractible ex-ante.

Deposits are demandable, early withdrawals are serviced sequentially, and the $t=2$ and $t=3$ non-contingent interest rates are r_D and $\bar{r}_D$ respectively. This contract structure creates the possibility of a run, since patient savers may choose to demand their deposits early depending on their own information and their expectations about the actions of other patient savers.

We denote all variables that are not (pre-)determined at $t=1$ as functions of the liquidation value, ξ, and the portion of savers who decide to withdraw at $t=2$, $\lambda \in [\delta, 1]$. In equilibrium, either all savers choose to withdraw, $\lambda = 1$, or only the impatient savers withdraw, $\lambda = \delta$. However, the out-of-equilibrium beliefs, which play an important role in the determination of the run probability, depend on the conjectured portion of savers withdrawing. This conjecture can be anywhere between δ and 1.

It is instructive to review each of the different scenarios. If there is no run, i.e., $\xi \in \left[\xi^*, \bar{\xi}\right]$, only impatient depositors withdraw and they receive the full amount of promised payment, $D(1+r_D)$. Patient depositors' repayments will depend on the realization of the technology shock in the next period. In a run, all depositors attempt to withdraw and there is probability $\theta(\xi, 1)$ that any depositor is repaid.[3] Conditional on the bank surviving to $t=3$ patient depositors receive their promised payment, $D(1+\bar{r}_D)$, with probability ω and zero otherwise.

Savers have quasi-linear preferences for consumption in all periods, such that they value consumption linearly at $t=2$ and $t=3$. This is the first of the three aforementioned assumptions that make our numerical calculations easier. This assumption greatly simplifies the patient savers'

[3]This probability of receiving payment when fundamental are ξ and λ savers withdraw is denoted by $\theta(\xi, \lambda)$ and is determined by equation (10), derived in section 2.3. In a run all savers attempt to withdraw, $\lambda = 1$, hence the probability of being repaid is $\theta(\xi, 1)$.

decision about whether to join a run because it means that all that must be computed is the expected payoff from the deposit in period 2 versus 3. In contrast, if the savers were risk-averse, then finding the threshold that determines whether to run is much more complicated. The complication arises because a patient saver needs to compute her expected utility differential between waiting and withdrawing accounting for all possible out-of-equilibrium beliefs about the actions of other savers. Computing the expected deposit payoffs is a lot simpler that computing the expectation of a non-linear function of the deposit payoffs. Moreover, quasi-linearity yields a very tractable deposit supply schedule, which substantially simplifies the normative analysis of alternative regulations.

The disadvantage of the linearity assumption is that it greatly degrades the usefulness of a deposit to a saver. Essentially, the deposit becomes a pure financial instrument whose only value is that it pays more interest than the liquid asset. Put differently, the deposit essentially would be equivalent to a bond.

To make deposits more useful, we assume that for impatient savers there is a transactional advantage to having a deposit instead of a liquid asset in normal situations where there is no run. For now, we suppose that this advantage is described by a concave function V that is increasing in the amount withdrawn, $D(1+r_D)$. This additional benefit of deposits partially offsets the stark implications of the linearity assumption for savers' utility while still making it simple to solve for the run threshold. Modeling things this way leads to no qualitative changes relative to a model with concave utility and no transactions services, but makes it substantially easier and faster to solve the model.

The expected utility of an individual saver is given by

$$\mathbb{U}_S = \overbrace{U\left(e_S - D - LIQ_S\right)}^{\text{Utility at } t=1} + \overbrace{\int_{\underline{\xi}}^{\xi^*} \left[\beta\delta + \beta^2(1-\delta)\right] \cdot \left[\theta(\xi,1)\cdot D(1+r_D) + LIQ_S\right] \frac{d\xi}{\Delta_\xi}}^{\text{run}}$$
$$+ \underbrace{\int_{\xi^*}^{\overline{\xi}} \beta\delta\left[D(1+r_D) + LIQ_S\right] \frac{d\xi}{\Delta_\xi}}_{\text{no run,impatient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} \beta^2(1-\delta)\left[\omega\cdot D(1+\bar{r}_D) + LIQ_S\right] \frac{d\xi}{\Delta_\xi}}_{\text{no run,patient}} + \underbrace{\int_{\xi^*}^{\overline{\xi}} V\left(D(1+r_D)\right) \frac{d\xi}{\Delta_\xi}}_{\text{transaction services}}, \quad (1)$$

where $U(\cdot)$ is the utility function for $t=0$ consumption with $U'(\cdot)>0$ and $U''(\cdot)<0$; $V(\cdot)$ captures the transaction services of deposits with $V(0)=0$, $V'(\cdot)>0$ and $V''(\cdot)<0$; and $\beta \leq 1$ is the time-discount factor.

Savers choose the level of deposits and their holdings of the liquid asset to maximize (1). An individual saver takes the run threshold, ξ^* and the probability of being repaid in a run, $\theta(\xi,1)$ as given. These objects depend on the aggregate bank portfolio and we suppose that the individual saver is sufficiently small so as to not account for her impact on them. In contrast, a social planner would internalize the effect of the choices. Finally, short-selling of deposits and the liquid asset is not allowed, i.e., $D \geq 0$ and $LIQ_S \geq 0$.

The optimal choice of deposits by S yields the following deposit supply (DS) schedule:

$$U'(e_S - D - LIQ_S) \geq \left[\beta\delta + \beta^2(1-\delta)\right](1+r_D)\int_{\underline{\xi}}^{\xi^*} \theta(\xi,1)\frac{d\xi}{\Delta_\xi}$$
$$+\left[\beta\delta(1+r_D) + \beta^2(1-\delta)\omega(1+\bar{r}_D) + V'(D(1+r_D))(1+r_D)\right](1-q), \quad (2)$$

which holds with strict equality if savers choose to hold deposits in equilibrium, i.e., $D > 0$. Condition (2) says that savers equate the marginal utility of forgone consumption at $t = 1$ to the expected marginal utility gain from holding deposits in the future. In a run, all savers withdraw; an individual saver will receive $1 + r_D$ per unit of deposits with probability $\theta(\xi, 1)$ for each realization of ξ below the run threshold ξ^*. Otherwise, a run does not occur, which happens with probability $1-q$, and only impatient savers withdraw in equilibrium. In this case, an individual saver is either impatient with probability δ and receives the period 2 deposit rate $1 + r_D$ or is patient with probability $1-\delta$ and receives the period 3 deposit rate $1+\bar{r}_D$ with probability ω. Absent a run, S also enjoys the marginal benefit of transaction services, $V'(D(1+r_D))(1+r_D)$.

Savers can choose to self-insure and hold the liquid asset. The optimal liquid holdings, LIQ_S, are given by:

$$U'(e_S - D - LIQ_S) \geq \beta\delta + \beta^2(1-\delta), \quad (3)$$

which holds with strict equality if savers choose to self-insure in equilibrium, i.e., $LIQ_S > 0$. Unless stated otherwise, we consider cases that savers do not want to hold the liquid asset, but want to hold deposits, i.e., (2) hold with equality, while (3) is slack. They will choose to do this when their endowments are not excessive, when banks offer high enough deposit rates or when transaction services are sufficiently valuable.

Substituting the deposit schedule (2) in (1) and using the definition of run probability q, we get the following indirect utility function,

$$\mathbb{U}_S^* = U(e_s - D) + U'(e_s - D)D + (1-q)\left[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)\right], \quad (4)$$

for the benchmark case that $LIQ_S = 0$.[4] Given our assumptions about $V(\cdot)$, it is easy to show that the third term in (4) is strictly positive.[5] Moreover, the first two terms in (4) must be higher than $\mathbb{U}_S^\alpha$ (the utility level from saving only using liquid assets), otherwise savers could self-insure using the liquid asset and attain that level of utility. Hence, savers are always better-off using the bank

[4]For completeness, if $LIQ_S > 0$ then the savers' indirect utility is $\mathbb{U}_S^\alpha + (1-q)\left[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)\right]$, where $\mathbb{U}_S^\alpha$ is the utility in autarky, where savers only use the liquid asset to smooth intertemporal consumption. The autarkic utility is $\mathbb{U}_S^\alpha = U(U'^{-1}(\beta\delta + \beta^2(1-\delta))) + [\beta\delta + \beta^2(1-\delta)] \cdot [e_S - U'^{-1}(\beta\delta + \beta^2(1-\delta))]$. Alternatively, savers could attempt to directly lend to entrepreneurs assuming that they could also monitor them. We present this case in Section B.7 the online appendix.

[5]Given that $D > 0$, the term can be written as $(1-q)D[V(D)/D - V'(D)]$. Because $V(0) = 0$, $V(D)/D$ is the slope of the straight line starting at zero and passing through D. Given that V is strictly concave, its image is always above the image of the straight line for any point $x \in (0, D)$. Because V' is strictly decreasing, V will necessarily cross the straight line connecting zero and D from above and, hence, the derivative of V at D is strictly smaller than the slope of the straight line.

compared to autarky.

Even with our simplifying assumptions, this framework produces an equilibrium and a set of decision rules for savers that have very sensible, intuitive properties. In particular, savers use the bank because it offers a better way to save for the future and because it facilitates transactions. The former is captured by the gain/surplus in terms of period 1 consumption (the first two terms in (4)). The latter is captured by the gain/surplus from the transaction services (the last term in (4)).

2.2 Entrepreneurs

Entrepreneurs have the rights to operate real projects that are in perfectly elastic supply, require a unit of funding at $t = 1$, are infinitely divisible when liquidated, and mature at $t = 3$. For simplicity, E does not have an endowment, but borrows I from the bank at interest rate r_I to invest in the risky technology. Moreover, E is risk-neutral and derives utility only from consumption at $t = 3$. Finally, E is protected by limited liability when projects mature and loans are due.

The risk neutrality of entrepreneurs together with the absence of initial endowment is our second simplifying assumption. Risk neutrality means that the entrepreneur cares only about the expected profits from operating the technology. The absence of an endowment means that the decision to default will depend only on the productivity of the technology, and not on entrepreneurs' leverage. More importantly, the savers also know that the entrepreneurs' loan demand and decision to default depend only on the expected value of profits. Otherwise, the savers run decision becomes more intertwined with how much the entrepreneur wants to borrow and the relative risk-aversion of the two parties will matter for how risks are shared.

The linearity of the production function for the technology is the last of our three simplifying technical assumptions. This assumption pushes the entrepreneur to borrow as much as possible and simplifies the derivation of the loan demand. Unfortunately, the linearity also means that the loan rate that the bank will choose will depend only on the technology shock. Were the production function to have any curvature, then that curvature would be an important determinant of the level of loan demand and the interest rate on loans. However, in that case, the relative curvature of the production function would non-trivially complicate the loan demand as well as the determination of banking allocations even with risk-neutral savers and entrepreneurs.

This complication arises because the realized level of total production depends on the decision of the bank to recall loans, which happens after the realization of the liquidation value of loans, ξ. The loan demand trades off the expected revenue from borrowing to produce against the cost of repaying the loan. And it is much easier to compute the expected profits to entrepreneurs over the realization $\xi \geq \xi^*$ when production is linear rather than when it is a non-linear function of the number of projects that are not recalled.

In order, to make loan demand less mechanical while still making it straightforward to solve for expected loan demand, we introduce a cost of effort that the entrepreneur incurs upon investing. We suppose that this cost is convex and is recognized before the payoff from the investment is known; we could also call this cost an adjustment cost. The cost now becomes another factor that matters for loan demand and the interest rate charged will depend on the marginal adjustment cost that must be paid. We assume that this cost is convex and it pertains to total investment.

The expected utility of an individual entrepreneur can be written as:

$$\mathbb{U}_E = \int_{\xi^*}^{\overline{\xi}} \{\omega \cdot [\underbrace{A \cdot (1 - y(\xi,\delta)) \cdot I}_{\text{realized output}} - \underbrace{(1 - y(\xi,\delta)) \cdot I \cdot (1 + r^I)}_{\text{loan obligation}}] - \underbrace{c(I)}_{\text{cost}}\} \frac{d\xi}{\Delta_\xi}, \tag{5}$$

where $y(\xi,\lambda)$ is the portion of loans that the bank recalls to serve early withdrawals, given by equation (11) derived in section 2.3. If a run does not occur, E repays the outstanding loans in the good state, $(1 - y(\xi,\delta))I$, as long as the per unit payoff, A, is higher than the promised gross loan rate $1 + r_I$. Naturally, E defaults in the bad state when the project pays zero. In a run, all projects funded by bank loans are liquidated, i.e., $y(\xi,1) = 1$ for all $\xi < \xi^*$, and no production takes place. Finally, $c(I)$ is the effort cost with $c(0) = 0$, $c'(\cdot) > 0$ and $c''(\cdot) > 0$.[6] E needs to incur this cost in order to produce before she learns the realization of the state of the world and after run uncertainty has been resolved. Hence, E will choose not to exert effort and thereby avoid the cost in a run (when all her loans are recalled). Absent a run, given that the net payoff to E is increasing in ξ, she will choose to produce if the following incentive compatibility constraint holds:

$$\omega\,[A - (1 + r_I)]\,(1 - y(\xi^*,\delta))I - c(I) \geq 0. \tag{6}$$

Entrepreneurs choose the level of investment and, thus, borrowing, I, to maximize (5). An individual entrepreneur takes the run threshold, ξ^* and the aggregate portion of loans recalled, $y(\xi,\delta)$ as given. These objects depend on the aggregate bank portfolio and we suppose that the individual entrepreneur is sufficiently small so as to not account for her impact on them. A social planner would internalize the effect of the choices.

The optimal choice of I by E yields the following loan demand (LD) schedule:

$$\int_{\xi^*}^{\overline{\xi}} \{\omega[A - (1 + r^I)](1 - y(\xi,\delta)) - c'(I)\} \frac{d\xi}{\Delta_\xi} = 0. \tag{7}$$

Condition (7) says that E equates the expected profit margin on the remaining projects–given by the difference between the marginal product of investment and the gross loan rate–to the marginal effort cost over the realizations of ξ where a run does not materialize.

Substituting the loan demand schedule (7) in (5) and using the definition of the run probability, q, we get the following indirect utility function

$$\mathbb{U}_E^* = (1 - q)\left[c'(I)I - c(I)\right], \tag{8}$$

[6]As we show in Corollary 3 later on, the adjustment cost matters for policy analysis, because without it the entrepreneur borrowers to the point of paying out all proceeds to the bank. In this case, the entrepreneur essentially drops out of the problem. We could also assume that the cost depends only on portion of investment not recalled, which would complicate the analysis but would create the same motive for a planner to take account of the entrepreneurs as in the setup we analyze.

which is the surplus accruing to the entrepreneur when a run does not occur. Given our assumptions about $c(\cdot)$, it is easy to show that (8) is always strictly positive, i.e., E is strictly better-off than in autarky where $E's$ utility is zero.[7] If effort is costless, then $\mathbb{U}_E^* = 0$ and the loan rate is equated to the marginal product of the project, i.e., $1 + r_I = A$ from (7).

Despite our simplifying assumptions about entrepreneurs' preferences and shape of the production function, the entrepreneurs make choices that are very conventional. Their surplus from operating is increasing in the amount of investment and decreasing in the probability of a run. Hence, entrepreneurs' welfare in equilibrium depends critically on the risk of a run. Our assumptions allow us to capture these relationships in a very tractable way in (8). They also allow us to cleanly identify the distortions that any policies will aim to correct.

2.3 Bankers and Banks

The banker makes all investment and funding decisions to maximize her own utility. At $t = 1$, she is endowed with e_B and decides how much equity, E, to put into the bank. In addition, she decides how many deposits to raise, D, and decides how much to invest in the liquid assets, LIQ, and illiquid loans, I, subject to the following balance sheet (BS) constraint:

$$I + LIQ = D + E. \tag{9}$$

The balance sheet and profits/dividends after t=2 depend on the realization of ξ and the number of people withdrawing, λ. If a bank-run occurs, i.e., $\xi < \xi^*$, then the bank is liquidated and the proceeds are distributed according to a sequential service constraint. Thus, the probability that any saver is served is equal to

$$\theta(\xi, \lambda) = \frac{LIQ + \xi \cdot I}{\lambda \cdot D \cdot (1 + r_D)}. \tag{10}$$

If there is not a run, i.e., $\xi \geq \xi^*$, the bank will recall and liquidate a portion $y(\xi, \lambda)$ of its loan portfolio to serve the early withdrawals. The amount recalled is given by

$$y(\xi, \lambda) = \frac{\lambda \cdot D \cdot (1 + r_D) - LIQ}{\xi \cdot I}. \tag{11}$$

Our assumptions regarding the distribution of ξ imply that lending is sufficiently attractive that the bank will opt to hold insufficient liquid assets to service all early deposit withdrawals even when only the impatient savers withdraw. So the bank is always planning to call some loans. In principle, the bank might also want to liquidate loans beyond the need to serve early withdrawals and carry the proceeds forward using the storage technology. But this would only occur if the realization of ξ is higher than the expected return from holding the loan to maturity, which we have excluded

[7] To show that (8) is strictly positive it suffices to show that $c'(I) - c(I)/I > 0$ given that $I > 0$; otherwise E is in autarky (short-selling of projects is not allowed). Because $c(0) = 0$, $c(I)/I$ is the slope of the straight line starting at zero and passing through I. Given that c is strictly convex, its image is always below the image of the straight line for any point $x \in (0, I)$. Because c' is strictly increasing, c will necessarily cross the straight line connecting zero and I from below and, hence, the derivative of c at I is strictly higher than the slope of the straight line.

by assumption. As a result, $y(\xi,\lambda)$ will take interior values between zero and one, and it will be decreasing in ξ and increasing in λ.

The dividends the banker receives in the good state of the world at $t = 3$ are equal to the repayment on the remaining loans minus the payment on the remaining deposits. In the bad state, B defaults, so the dividends zero. The banker needs to incur a monitoring cost, X, to learn the true state of the world. Alternatively, the banker can forgo the monitoring, in which case the entrepreneur will report that productivity was zero and default, even if the good state of the world materialized. Hence, there is a moral hazard problem in which the banker will choose to monitor only if the expected dividends are higher than the monitoring cost.[8]

Given a level for the liquidation value ξ and the number of withdrawals λ, the banker will choose to monitor if the following incentive compatibility constraint is satisfied:

$$\omega\left[(1-y(\xi,\lambda))I(1+r_I)-(1-\lambda)D(1+\bar{r}_D)\right]-X\geq 0. \tag{12}$$

The first term in (12) is the expected payoff to the banker if she monitors. It is multiplied by ω because the banker has to decide whether to monitor before she learns the true state of the world and so she takes an expectation. The second term, X, is the monitoring cost.

Using (11) and the fact that (12) is decreasing in λ, we can derive a threshold $\hat{\lambda}(\xi)$, such that the banker chooses to monitor for $\lambda \leq \hat{\lambda}$:

$$\hat{\lambda}(\xi)=\frac{(\xi\cdot I+LIQ)(1+r_I)-\xi(D(1+\bar{r}_D)+X/\omega)}{D[(1+r_D)(1+r_I)-\xi(1+\bar{r}_D)]}. \tag{13}$$

Note that this threshold is consistent with all out-of-equilibrium beliefs of savers that govern their decision to withdraw or keep their deposits in the bank, which we describe in section 2.4. In equilibrium, only impatient depositors will withdraw when $\xi \geq \xi^*$. In turn, consistency requires that the bank always chooses to monitor when a run does not occur. Indeed, from the determination of the run threshold in (20) below, we obtain that $\delta < \hat{\lambda}(\xi^*)$, i.e., banker's monitoring incentives are consistent with the equilibrium behavior of savers.[9]

Overall, the banker's expected utility is given by:

$$\mathbb{U}_B=W\left(e_B-E\right)+\int_{\xi^*}^{\bar{\xi}}\{\omega\cdot[\underbrace{(1-y(\xi,\delta))\cdot I}_{\text{outstanding loans}}\cdot\underbrace{(1+r_I)}_{\text{loan rate}}-\underbrace{(1-\delta)\cdot D}_{\text{patient deposits}}\cdot\underbrace{(1+\bar{r}_D)}_{\text{deposit rate}}]-\underbrace{X}_{\text{monit. cost}}\}\frac{d\xi}{\Delta_\xi}, \tag{14}$$

where $W(\cdot)$ is $B's$ utility function for $t = 0$, with $W'(\cdot) > 0$ and $W''(\cdot)$, which may differ from $U(\cdot)$ of the savers. The banker has also quasi-linear preferences, but unlike the saver, never needs to

[8]The productivity level is common across projects. Therefore, as in Diamond (1984), monitoring costs are conserved by having a bank monitor all borrowers, relative to having individual lenders monitor individual borrowers. Thus, the bank monitoring expands the supply of credit.

[9]In equilibrium, monitoring requires that $\omega\left[(1-y(\xi,\delta))I(1+r_I)-(1-\delta)D(1+\bar{r}_D)\right]-X\geq 0$ for all $\xi \geq \xi^*$. However, it suffices that the constraint only holds for ξ^*, because dividends are increasing in ξ. In turn, this is guaranteed by the fact that $\delta < \hat{\lambda}(\xi^*)$.

consume in the interim period.

2.4 Global Game and Bank-run Threshold

Having described the choices facing all of the agents in the economy, we can now determine when individual patient savers will want to withdraw her deposits at $t = 2$. The decision depends not only on the saver's belief about the bank's financial health, but also on their beliefs about how other savers' will behave. The source of fundamental uncertainty in our model is the liquidation value ξ. Apart from really high or low realization of ξ, multiple equilbria arise when savers have complete information about the realization of ξ. The outcomes for the extremely low and high realizations are known as the lower and upper dominance regions, and their existence is critical for obtaining a run threshold (see Lemma 1 for formal derivations).

The lower dominance region is defined by a threshold ξ^{ld} for fundamentals such that every individual patient saver will withdraw her deposits when $\xi < \xi^{ld}$, irrespective of what other patient savers do. This region occurs naturally if signal about liquidation value is sufficiently bad, because if depositors believe the recalled loans have very little value then they may conclude that even the impatient depositors cannot be fully paid. In that case, the bank is going to fail so attempting to withdraw is the dominant strategy.

The upper dominance region is defined by a threshold ξ^{ud} for fundamentals such that every individual patient saver will not withdraw her deposits when $\xi > \xi^{ud}$, irrespective of what other patient savers do. Intuitively, this occurs when the liquidation value is so high that even if everyone were to run, the bank would be able to pay them. In that case, running makes no sense. Allowing the liquidation value of loan to vary, enables us to obtain endogenously a well-defined upper dominance threshold. In Goldstein-Paunzer, the upper dominance is exogenously assumed because the liquidation value is not allowed to vary. However, our modification complicates the proof of uniqueness for the run threshold, and a new argument is needed, which we present below.

Aside from the two extreme cases, for intermediate values of fundamentals, the bank cannot serve all early withdrawals, and a patient saver may decide to withdraw her deposits if she believes that other patient savers will withdraw as well. In order to resolve this coordination problem, we assume that at $t = 2$, each patient saver i receives a private signal $x_i = \xi + \varepsilon_i$, where ε_i are small error terms that are independently and uniformly distributed over $[-\varepsilon, \varepsilon]$. These signals not only provide information about ξ, but also about other savers' signals, so that inference about their actions is possible. The higher the signal is, the higher is the posterior belief about ξ and the smaller is the likelihood that other savers receive bad enough signals so that they will opt to withdraw. Both effects reduce the incentive to withdraw. As a result, the incomplete information leads patient savers to coordinate their actions so that they withdraw if fundamentals are below a threshold. These incomplete information games are known in the literature as global games (see also Carlsson and van Damme, 1993).

We seek a symmetric equilibrium characterized by two thresholds (x^*, ξ^*) such that: an individual patient saver will withdraw her deposits if her private signal realization x_i is lower than x^* and the bank will experience a run at $t = 2$, and be liquidated, if the realization of ξ is below ξ^*.

Under such a threshold strategy, the number of savers that withdraw at a given level of fundamentals ξ is

$$\lambda(\xi,x^*) = \begin{cases} 1 & \text{if } \xi < x^* - \varepsilon \\ \delta + (1-\delta) Prob\,(x_i \leq x^*) & \text{if } x^* - \varepsilon \leq \xi \leq x^* + \varepsilon \\ \delta & \text{if } \xi > x^* + \varepsilon \end{cases}. \tag{15}$$

If the fundamental value ξ is lower than $x^* - \varepsilon$, then all savers receive signals $x_i < x^*$. Hence, all patient savers withdraw and $\lambda(\xi,x^*) = 1$. The opposite is true for $\xi > x^* + \varepsilon$. In this case, all patient savers receive signals $x_i > x^*$ and keep their deposits in the bank and only the impatient ones withdraw, $\lambda(\xi,x^*) = \delta$. Finally, if fundamentals are close to x^*, i.e., $\xi \in [x^* - \varepsilon, x^* + \varepsilon]$, some patient savers will receive signals that are lower than x^* and, thus, will withdraw their deposits; and others will receive a signal higher than x^* and, thus, will keep their deposits in the bank. Because ε_i, the noise in the private signals, is independently and identically distributed, the law of large numbers holds. This means the number of savers withdrawing for a given level of ξ in the intermediate region is $\lambda(\xi,x^*) = \delta + (1-\delta) Prob\,(x_i \leq x^*) = \delta + (1-\delta)\,(x^* - \xi + \varepsilon)\,/2\varepsilon$.

The signal and fundamentals thresholds are derived in two steps as follows. First, given the threshold strategy x^*, we can derive the threshold for fundamentals, ξ^*, which determines whether the bank is fully liquidated at $t = 2$ or survives to $t = 3$. Because the number of savers withdrawing is decreasing in ξ from (15), the bank is fully liquidated only if $\xi < \xi^*$. That is, ξ^* as a function of x^* is the solution to $\theta\left(\xi^*, \lambda\left(\xi^*, x^*\right)\right) = 1$, which from (10) gives:

$$\xi^* = \frac{\varepsilon\left[(1+\delta)D(1+r_D) - 2 \cdot LIQ\right] + x^*(1-\delta)D(1+r_D)}{2\varepsilon I + (1-\delta)D(1+r_D)}. \tag{16}$$

In other words, for threshold strategy x^*, if ξ is lower than ξ^*, then the numbers of savers withdrawing are owed more than the bank can pay even by liquidating all of its assets. Alternatively, if ξ is higher than ξ^*, fewer savers withdraw, allowing the bank to liquidate fewer assets and survive to $t = 3$.

Next, given the fundamentals threshold ξ^*, an individual saver can compute the signal threshold x^*, below which is it optimal to withdraw conditional on its expectation over the number of savers withdrawing and the private signal she receives. The threshold x^* depends on the utility differential between keeping the deposits in the bank and withdrawing. We denote this differential by $\nu(\xi,\lambda)$ and report in (17) the value it takes for different levels of withdrawals λ when fundamentals are ξ:

$$\nu(\xi,\lambda) = \begin{cases} \omega D(1+\bar{r}_D) - D(1+r_D) & \text{if} \quad \hat{\lambda}(\xi) \geq \lambda \geq \delta \\ -D(1+r_D) & \text{if} \quad \theta(\xi,1) \geq \lambda > \hat{\lambda}(\xi) \\ -(LIQ + \xi \cdot I)/\lambda & \text{if} \quad 1 \geq \lambda > \theta(\xi,1) \end{cases}. \tag{17}$$

Consider the first two cases, where there are fewer withdrawals than the maximum that the bank can repay in full, i.e., $\lambda \leq \theta(\xi, 1)$. If $\lambda \leq \hat{\lambda}(\xi)$, the banker chooses to monitor, and S expects to get $\omega D(1+\bar{r}_D)$ if she waits, or $D(1+r_D)$ if she withdraws her deposits. Hence, the utility differential is $\omega D(1+\bar{r}_D) - D(1+r_D)$. We call this region of $\lambda' s$ a *partial run with monitoring*. On the other hand, if $\lambda > \hat{\lambda}(\xi)$, then the banker will not monitor and S get nothing if she chooses to wait. Thus, the utility differential is $-D(1+r_D)$. We call this region of $\lambda' s$ a *partial run without monitoring*. Finally, if $\lambda > \theta(\xi, 1)$, the bank cannot fully repay all savers that withdraw and it is liquidated. S get zero if she waits and is repaid with probability $\theta(\xi, \lambda)$, yielding, in expectation, a payoff of $\theta(\xi, \lambda) D(1+r_D) = (LIQ + \xi \cdot I)/\lambda$. Hence, the utility differential between waiting and withdrawing is equal to $-(LIQ + \xi \cdot I)/\lambda$. We call this region of $\lambda' s$ a *full run*. The following Lemma examines in which of these three regions ν may lie for different levels of ξ.

Lemma 1. *Consider $X < \overline{X}$, where $\overline{X} \gg 0$ is an upper threshold for the monitoring cost. For $\xi < \hat{\xi}$ only the full run region is non-empty, i.e., $\hat{\lambda}(\xi) < \delta$ and $\theta(\xi, 1) < \delta$; $\hat{\xi}$ is the solution to $\theta(\hat{\xi}, 1) = \delta$. For $\xi \in [\hat{\xi}, \xi^{ld})$, only the partial run without monitoring and full run regions are non-empty, i.e., $\hat{\lambda}(\xi) < \delta < \theta(\xi, 1) < 1$; ξ^{ld} is the solution to $\hat{\lambda}(\xi^{ld}) = \delta$. For $\xi \in [\xi^{ld}, \hat{\hat{\xi}})$, all three regions are non-empty, i.e., $\delta \leq \hat{\lambda}(\xi) < \theta(\xi, 1) < 1$; $\hat{\hat{\xi}}$ is the solution to $\theta(\hat{\hat{\xi}}, 1) = 1$. For $\xi \in [\hat{\hat{\xi}}, \xi^{ud})$, only the partial run with and without monitoring regions are non-empty, i.e., $\delta < \hat{\lambda}(\xi^{ud}) < 1 \leq \theta(\xi, 1)$; ξ^{ud} is the solution to $\hat{\lambda}(\xi^{ud}) = 1$. For $\xi > \xi^{ud}$, only the partial run with monitoring region in non-empty, i.e., $\hat{\lambda}(\xi) > 1$ and $\theta(\xi, 1) > 1$.*

Lemma 1 says that when fundamentals are very bad, an individual saver's beliefs are only consistent with aggregate withdrawal behavior in a full run. As fundamentals improve, her beliefs are consistent with other run regions as well, while when fundamentals are very good, beliefs are only consistent with the partial run with monitoring occurring. In particular, a patient saver will withdraw for $\xi < \xi^{ld}$, and will wait for $\xi > \xi^{ud}$, irrespective of the actions of other savers.

To understand the decision to withdraw, consider an individual patient saver who receives signal x_i. The saver will use the signal to update her beliefs about the realization of ξ. Given that both ξ and ε_i are uniformly distributed, the posterior distribution of ξ given x_i is $\xi | x_i \sim U[x_i - \varepsilon, x_i + \varepsilon]$. This implies that the utility differential between waiting and withdrawing for a patient saver who receives signal x_i as a function of the cutoff value for running is

$$\Delta(x_i, x^*) = \frac{1}{2\varepsilon} \int_{x_i - \varepsilon}^{x_i + \varepsilon} \nu(\xi, \lambda(\xi, x^*)) d\xi. \tag{18}$$

In a threshold equilibrium, a patient saver prefers to withdraw, i.e., $\Delta(x_i, x^*)<0$, for all $x_i < x^*$, and prefers to roll over, i.e., $\Delta(x_i, x^*) > 0$, for all $x_i > x^*$. $\Delta(x_i, x^*)$ is continuous in x_i, because a change in the signal only changes the limits of integration $[x_i - \varepsilon, x_i + \varepsilon]$ and the integrand is bounded. Hence, a patient saver that receives signal $x_i = x^*$ is indifferent between waiting and withdrawing if

$$\Delta(x^*, x^*) = \frac{1}{2\varepsilon} \int_{x^* - \varepsilon}^{x^* + \varepsilon} \nu(\xi, \lambda(\xi, x^*)) d\xi = 0. \tag{19}$$

Equations (16) and (19) jointly determine the fundamentals' threshold, ξ^*, and the threshold strategy, x^*. We need to show that these thresholds exist and that they are unique. In the body of the paper, we provide the intuition underlying the proof and we provide the details in the online appendix.

Solving for $x^*(\xi^*)$ from (16) and substituting it in (19), we obtain a single equilibrium condition in terms of fundamentals' threshold ξ^*, i.e., $\bar{\Delta}(\xi^*, \xi^*) = \Delta(x^*(\xi^*), x^*(\xi^*)) = 0$. We need to show that such a ξ^* is unique and that it is indeed a threshold equilibrium. The typical approach in the global games literature considers cases characterized by global strategic complementarities and state monotonicity (see Morris and Shin, 2003, for details). The first property requires that $v(\xi, \lambda)$ is decreasing in λ, i.e., the relative payoff from withdrawing is higher when there are more withdrawals. The second property requires that $v(\xi, \lambda)$ is non-decreasing in ξ and strictly increasing for some ξ, i.e., the relative payoff from waiting is higher when fundamentals are stronger. We these properties obtain, the proof goes as follows: From the existence of lower and upper dominance regions, $\bar{\Delta}$ is negative for low values and positive for high values of ξ, and, thus, there exists a solution ξ^* due to monotonicity. Moreover, from the strict monotonicity property, $\hat{\Delta}$ is strictly increasing and, thus, the candidate solution ξ^* is unique. This in turn implies a unique strategy threshold $x^*(\xi^*)$. This candidate solution is indeed a threshold equilibrium if (18) is higher than (19) for $x_i > x^*(\xi^*)$ and lower for $x_i < x^*(\xi^*)$, i.e., a patient saver that receives a signal that is higher (lower) than the threshold chooses to wait (withdraw). With global strategic complementarities, these conditions are automatically satisfied because a higher signal means that fundamentals are higher and, thus, from (17) there are fewer withdrawals resulting in higher utility differential from waiting.

In our environment, neither of these typical properties hold. First, the model does not exhibit global strategic complementarities because v is increasing in λ in the full run region. As Goldstein-Pauzner argue, this is a typical property of bank-run models and is due to the fact that the marginal gain from running is lower as more people opt to run because more people are competing for the same liquid resources. Goldstein-Pauzner shows that one-sided strategic complementarities are adequate to recover the threshold equilibrium under the stricter assumption that noise in private signals in uniformly distributed, which we maintain in our model.

On top of this complication, our model also exhibits what we call *perverse state monotonicity* because v is decreasing in the state ξ in the full run region. Intuitively, this property means that savers have higher incentives to run on a stronger than a weaker bank, conditional on a run occurring, because the chances of getting paid are higher. This occurs because the signals provide information about liquidation value of loans. Presuming that the signals pertain to liquidation values allowed us to endogenously derive the upper dominance region. But, it restricts us from using the usual argument to establish uniqueness.

Because of the perverse state monotonicity, we need to need to use a new proof that covers three possible cases that are depicted in Figure 2. A first possibility is similar to what is studied in the literature where ν is always (weakly) increasing in the state irrespective of whether the model exhibits global strategic complementarities (Rochet and Vives, 2004) or one-sided strategic complementarities (Goldstein and Pauzner, 2005). If this happens to be true, the threshold solving $\bar{\Delta}$ will

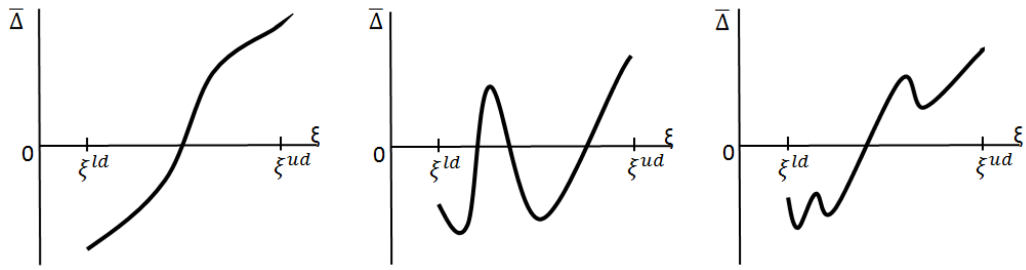

Figure 2: Perverse state monotonicity

be unique as depicted in the left panel in Figure 2.

However, v may not be increasing in the state. Instead, it could be the case that a situation as in the middle panel, where there are multiple solutions, or in a situation in the right panel, where there is a unique solution, despite the fact that $\bar{\Delta}$ is not increasing in ξ *everywhere* in the domain.

To establish uniqueness in the presence of perverse state monotonicity, we observe that to rule out the problematic middle case, $\bar{\Delta}$ does not need to be increasing for all ξ (as is typically required), but instead only needs to be strictly increasing at candidate solutions that solve $\bar{\Delta}(\xi^*, \xi^*) = 0$. We know that such points exist because of continuity and the existence of the extreme regions established in Lemma 1. This weaker requirement is graphically depicted in Figure 3. In the left panel, where there are multiple solutions, $\bar{\Delta}$ will necessarily cross the x-axis both from below and from above because of continuity and the existence of the lower and upper dominance thresholds. This means that the derivative of $\bar{\Delta}$ at the candidate solutions, which are depicted by the dots on the x-axis, can be either positive or negative. On the contrary, in the unique solution in the right panel, the derivative at the candidate solution is *strictly* positive. Hence, the strategy to establish uniqueness comprises of showing that there do not exist solutions such that the derivative of $\bar{\Delta}$ at these solution is strictly negative, i.e., the set to $\{\bar{\Delta}(\xi^*, \xi^*) = 0 \cap \partial\bar{\Delta}/\partial\xi | \xi = \xi^* < 0\}$ is empty.

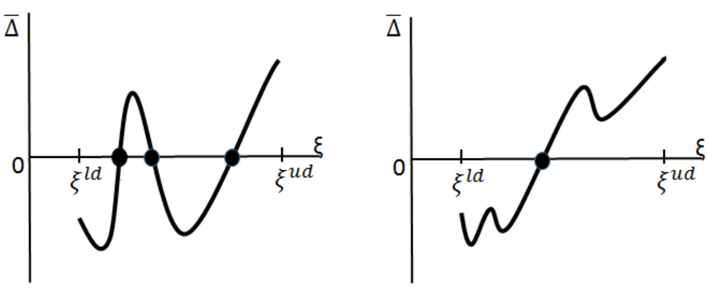

Figure 3: Graphical representation of uniqueness argument

Proposition 1. *Given equilibrium allocations that satisfy the regions in Lemma 1, there exists a unique threshold, x^*, such that patient savers keep their deposits in the bank if $x_i > x^*$, and withdraw if $x_i < x^*$. Moreover, there exists a unique threshold ξ^*, such that the bank does not experience a run if $\xi \geq \xi^*$, and is fully liquidated if $\xi < \xi^*$.*

Proposition 1 establishes the uniqueness of the run threshold provided that there exist equilibrium allocations that satisfy the conditions in Lemma 1. We verify that these conditions hold in the equilibria we examine.

Hereafter, we focus on the case that the noise becomes arbitrarily close to zero. Note that taking the limit $\varepsilon \to 0$ implies that $x^* \to \xi^*$ from (16). The posterior distribution of $\lambda(\xi, x^*)$ for a patient saver who receives signal x^* is uniform over $[\delta, 1]$.[10] As ξ decreases from $x_i + \varepsilon$ to $x_i - \varepsilon$, λ increases from δ to 1. Changing variables in (19) provides the indifference condition, $GG = \int_0^1 v(\xi^*, \lambda) d\lambda = 0$, that determines the unique ξ^*:

$$\int_{\delta}^{\lambda^*} [\omega D(1+\bar{r}_D) - D(1+r_D)]\, d\lambda - \int_{\lambda^*}^{\theta^*} D(1+r_D) d\lambda - \int_{\theta^*}^{1} \frac{LIQ + \xi^* I}{\lambda} d\lambda = 0, \tag{20}$$

where $\lambda^* \equiv \hat{\lambda}(\xi^*)$ and $\theta^* \equiv \theta(\xi^*, 1)$.[11] We will use (20) to help demonstrate that the conditions we need to obtain do in fact hold. Define $GG(\xi)$ the value of GG for general ξ. Then, we obtain the following derivative

$$\frac{\partial GG}{\partial \xi} = \omega D(1+\bar{r}_D) \frac{\partial \hat{\lambda}}{\partial \xi} - \int_{\theta}^{1} \frac{I}{\lambda} d\lambda, \tag{21}$$

which we cannot unambiguously sign for arbitrary values of ξ. The first term strengthens the incentives to wait because the monitoring threshold $\hat{\lambda}$ is increasing in ξ. The second term strengthens the incentives to withdraw because of the perverse state monotonicity. Thus, (21) cannot be used by itself to establish that the solution ξ^* in (20) is unique. Instead, we evaluate (21) only at candidate solutions for ξ^*. Combining (20) and (21) we get

$$\frac{\partial GG^*}{\partial \xi} = \frac{1}{\xi^*} \left[\int_{\theta^*}^{1} \frac{LIQ}{\lambda} d\lambda + \int_{\delta}^{\theta^*} D(1+r_D) d\lambda \right] + \omega D(1+\bar{r}_D) \left[\frac{\partial \lambda^*}{\partial \xi} - \frac{\lambda^* - \delta}{\xi^*} \right] > 0, \tag{22}$$

where $\partial GG^*/\partial \xi \equiv \partial GG/\partial \xi|_{\xi=\xi^*}$ and $\partial \lambda^*/\partial \xi \equiv \partial \hat{\lambda}/\partial \xi|_{\xi=\xi^*}$. The first two terms are necessarily positive. The last term can be written as $\omega D(1+\bar{r}_D)[(\lambda^* - \delta)\xi^* D(1+r_3^D) + (\delta D(1+r_D) - LIQ)(1+r_I)]/[\xi^* D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D))]$ and is greater than zero from Lemma 1. Hence, $\partial GG^*/\partial \xi$ in (B.47) is strictly positive and the ξ^* solving (20) is unique.

Because ξ^* will be a critical factor in the welfare analysis, we prove the following corollary that provides some useful information about its characteristics.

Corollary 1. *The run threshold ξ^* is decreasing in the loan rate and investment, while it is increasing in deposits and the period 2 deposit rate. The effects of the liquid asset holdings and of the period 3 deposit rate on ξ^* are ambiguous.*

The overall effect of the banking variables on the run threshold combines the indirect effect on the incentives of the bank to monitor and the direct effect on the payoff differential in (17). Because

[10] This is true because $Prob\,(\lambda(\xi, x^*) \le N | x_i = x^*) = 1 - Prob\,(\xi \le x^* + \varepsilon - (N-\delta)/(1-\delta)2\varepsilon | x_i = x^*) = 1 - (x^* + \varepsilon - (N-\delta)/(1-\delta)2\varepsilon - x^* + \varepsilon)/(2\varepsilon) = (N-\delta)/(1-\delta)$, hence $\lambda(\xi, x^*) \sim U\,[\delta, 1]$.

[11] Equation (20) is sufficient to guarantee that a patient saver will not withdraw if a run does not occur, i.e., $\omega D(1+\bar{r}_D) - D(1+r_D) > 0$. Thus, only impatient savers withdraw in equilibrium.

of one-sided strategic complementarities, higher I or LIQ increase the payoff from withdrawing in the full run region, which pushes ξ^* up, but also strengthen the incentives to monitor, i.e., increases λ^* which pushes ξ^* down. For investment, the first effect is mitigated because of the discounted liquidation value in a full run, $\xi^* I$, and we are able to show that increasing I decreases ξ^* other things equal. For liquidity, the first effect is relative stronger than for investment, and we cannot unambiguously show which effect dominates. However, Proposition 4 below establishes conditions under which liquidity reduces ξ^* and these conditions are easily satisfied in the examples we consider. More deposits reduce the incentives to monitor, which at the same time increase the payoff from waiting. But the first effect dominates and ξ^* goes up (other things equal). Moreover, higher loan rates increase bank profits and, thus, the incentive to monitor, which pushes ξ^* down.

Finally, the effect of the period 3 deposit rate is ambiguous. On the one hand, it reduces the incentives to monitor as bank profits go down. On the other hand, it increases the payoff from waiting in the partial run region with monitoring. Our ability to analyze this case highlights the benefit from explicitly modeling the actual payoffs in the incomplete information game. In our examples, we find that increasing $\bar{r}_D$ pushes ξ^* down and, thus, reduces the probability of a run other things equal.

The conclusion would be completely reversed if we had considered a reduced-form incomplete information game à la Rochet and Vives (2004) (or in models with *ad hoc* run probability functions). In their framework, depositors delegate the decision to withdraw to fund managers who have a very different payoff function than the one described in (17). In particular, they assume that managers receive an exogenous payoff $\mathcal{A} > 0$ if the run does not occur or the bank does not default, and suffer a negative payoff $\mathcal{B} < 0$ otherwise. In terms of our model, this formulation would imply that the indifference condition (20), which determines ξ^*, becomes $\int_{\delta}^{\lambda^*} \mathcal{A} d\lambda + \int_{\lambda^*}^{1} \mathcal{B} d\lambda = 0$. As a result, $\bar{r}_D$ affects ξ^* only via the monitoring threshold λ^*. In particular, if a planner wants to reduce the run probability, then she would have to implement a lower $\bar{r}_D$ to increase bank profits and, thus, the out-of-equilibrium monitoring incentives. This would not be beneficial for depositors. In contrast, because the run threshold also depends on the actual payoffs in our model, the planner can reduce the run probability by setting a higher $\bar{r}_D$, without hurting the depositors. A higher deposit rate could encourage a higher supply of deposits and more intermediation, which could mitigate any negative effect of more capital and liquidity on credit extension.

2.5 Private Equilibrium

The banker will choose both sides of the balance sheet to maximize her own profits as shown in (14). The choices of loans, liquidity, deposits and equity, need to satisfy the global game constraint (20), which determines the run threshold, the deposit supply schedule (2), which determines the deposit rates, and the loan demand schedule (7), which determines the loan rate. The balance sheet identity also holds so only three of the four variables can be freely chosen. The banker will internalize the effect of her actions on the run threshold, on deposit supply and on loan demand. As a result, conditions (20), (2) and (7) not only need to be satisfied in equilibrium, but will be explicit constraints in the banker's optimization problem, the solution to which defines the *private equilibrium* (PE). Essentially, this means that the banker also chooses directly the level of the run threshold as well as the deposit and loan rate, rather than letting them being determined implicitly in equilibrium. Finally, the banker will never choose allocations that result in lower welfare than her outside option. The bank can either invest in the storage technology or lend to entrepreneurs without accepting savers' deposits. Thus, the banker's outside option is $\overline{\mathbb{U}}_B = \max\,[\mathbb{U}_B^{\alpha}, \mathbb{U}_B^{n}]$, where $\mathbb{U}_B^{\alpha}$ is the utility in autarky (where the bank holds only liquid assets) and $\mathbb{U}_B^{n}$ is the utility when the banker does not take deposits and lends to E using only her own capital.[12]

Definition 1. *The private equilibrium is defined as the set of banking assets, $\{I, LIQ\}$, banking liabilities, $\{D,E\}$, the run threshold, ξ^*, deposit rates, $\{r_D, \bar{r}_D\}$, and the loan rate, r_I, that maximize banker's utility, $\mathbb{U}_B$, defined in* (14) *subject to the balance sheet (BS) constraint* (9)*, the global game (GG) constraint* (20)*, the deposit supply (DS) schedule* (2)*, and the loan demand (LD) schedule* (7).

Each first-order condition in the private equilibrium takes the form

$$\frac{\partial \mathbb{U}_B}{\partial \mathcal{C}} + \sum_{\mathcal{Y}} \psi_{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial \mathcal{C}} = 0, \tag{23}$$

where $\mathcal{C} \in \{I, LIQ, D, E, \xi^*, r_D, \bar{r}_D, r_I\}$, $\psi_{\mathcal{Y}}$ are the shadow values on constraints $\mathcal{Y} \in \{BS, GG, DS, LD\}$ and $\partial \mathcal{Y}/\partial \mathcal{C}$ the partial derivatives capturing the effect of choice $\mathcal{C}$ on these constraints. Take for example the lending choice, i.e., $\mathcal{C} = I$. Then (23) says that the optimal level of lending is determined by having the banker trade off the marginal return accruing to her, $\partial \mathbb{U}_B/\partial I$, against the shadow cost of funds, ψ_{BS}, and the way it affects the run threshold determination, the deposit supply and the loan demand. The optimality condition for the other variables can be similarly interpreted and we report the detailed expressions for the partial derivatives in the online appendix.

The optimality condition with respect to E, which is $\psi_{BS} = W'(e_B - E)$, deserves special attention because it highlights another way in which our model differs from others in the literature. This condition says that injecting more equity requires the banker to give up consumption in the initial period in exchange for increasing the funds of the bank.[13] Thus, the shadow cost of funds, ψ_{BS}, is inversely related to the amount of equity the banker puts in the bank. In banking models without endogenous credit and run risk, the higher funding costs of injecting more equity would feed in higher loan rates and lower investment. In our model, this does not need to be true if equity changes

[12] The utility in autarky is $\mathbb{U}_B^{\alpha} = W(e_B - \alpha) + \alpha$, where $a > 0$ is the storage investment satisfying $W'(e_B - \alpha) = 1$. Note that if $W'(e_B) > 1$, then $\alpha = 0$. If the banker lends to entrepreneurs using only her own capital, her utility is $\mathbb{U}_B^{n} = W(e_B - n) + \omega(1 + r_n)n - X$, where r_n is the interest rate determined by the loan demand $1 + r_n = A - c'(n)/\omega$. Because B internalizes how the choice of n affects the loan rate that entrepreneurs are willing to accept, the optimal $n > 0$ is the solution to $-W'(e_B - n) + \omega A - c'(n) - c''(n)n = 0$.

[13] Note that the condition does not include term for the effect of additional equity on constraints *GG*, *DS* and *LD*. This is true because, E does not appear directly in (20), (2) or (7), but this does not mean that equity is irrelevant for their determination. On the contrary, equity issuance can affect the run probability, the deposit supply and the loan demand through its joint determination with other equilibrium variables.

the overall level of risk: higher equity can improve the safety of deposits, which can be compatible with lower loan rates and more investment.

We restrict deposit rates to be positive, which can be particularly important for the choice of r_D. Hence, (23) will be strictly less than zero when r_D hits the non-negativity constraint. Absent this constraint, the banker may want to offer a period 2 deposit rate that is negative, since this would allow her to reduce the probability of a run. Such run-preventing deposit contracts have been studied for example in Cooper and Ross (1998). In our model, however, runnable deposits are important to discipline the banker and there are limits to how low the early deposit rate can be set both because of the disciplinary role and because savers can stop using the bank if the rates become too low. In the numerical examples we present, r_D hits the non-negativity constraint both in the private and planning equilibria, but we have also solved for cases where it is allowed it to take negative values. The implications of our model for the distortions between the private and planning equilibria as well as the effects and desirability of regulation continue to hold under a negative deposit rate for early withdrawals. We present these results in Section B.9 in the online appendix.[14]

While the banker is free to choose any three of the quantities on her balance sheet, to explain how the model works, it is helpful to summarize the choices in terms three different combinations of the quantities. In particular, we will summarize the private equilibrium in terms of the asset allocation choice, the capital structure choice and the scale of intermediation. The three intermediation margins can be easily interpreted (see Section B.1 in the online appendix for detailed expressions).

The *asset allocation margin* is

$$AAM_{PE} = \left(\frac{d\mathbb{U}_B}{dLIQ} - \frac{d\mathbb{U}_B}{dI}\right) + \frac{d\mathbb{U}_B}{d\xi^*}\left(\frac{d\xi^*}{dLIQ} - \frac{d\xi^*}{dI}\right). \tag{24}$$

Hence, AAM_{PE} captures the decision to shift a unit of risky loans into liquid asset holdings, which consists of the effect on B's utility via bank profitability (the first term) and the effect via the run probability (the second term).

Similarly, the *capital structure margin* is

$$CSM_{PE} = \left(\frac{d\mathbb{U}_B}{dE} - \frac{d\mathbb{U}_B}{dD}\right) - \frac{d\mathbb{U}_B}{d\xi^*}\frac{d\xi^*}{dD}, \tag{25}$$

and captures the decision to replace a unit of deposits with equity.

Lastly, the *scale of intermediation margin* is

$$SIM_{PE} = \left(\frac{d\mathbb{U}_B}{dI} + \frac{d\mathbb{U}_B}{dD}\right) + \frac{d\mathbb{U}_B}{d\xi^*}\left(\frac{d\xi^*}{dI} + \frac{d\xi^*}{dD}\right), \tag{26}$$

and captures the decision to raise a unit of deposits in order to expand credit extension. This mar-

[14] See also Keister (2015) for a model with flexible deposit contracts, i.e., the payment that a depositor receives is determined by the bank as a best response to realized withdrawals in the intermediate period. Runs in his framework are partial in the sense that the bank can alter payments to stop withdrawals by patient depositors and avoid liquidation once the run state is revealed.

gin is a proxy for the intermediation spread between r_I and $\bar{r}_D$ (see equation (B.18) in the online appendix). Intuitively, increasing both I and D pushes r_I down and $\bar{r}_D$ up given that $\partial r_I/\partial I < 0$ and $\partial \bar{r}_D/\partial D > 0$. Instead, if higher I is funded with equity or higher D is used to buy liquid assets, the spread will not necessarily shrink. Thus, the spread can proxy for our notion of intermediation, which is the amount of deposits channeled to loans, or equivalently the amount of loans funded by deposits.[15]

To understand how these margins determine the structure of the bank's balance sheet start in reverse order. Given a level of D and LIQ, the SIM_{PE} determines the level of lending by fixing the intermediation spread; higher I requires a lower spread other things equal and vice versa. Then, AAM_{PE} and CSM_{PE} simultaneously fix LIQ and D, or equivalently, the liquidity ratio $\ell = LIQ/(I + LIQ)$ and the leverage ratio $k = E/(E+D)$, given that $E = I + LIQ - D$ from the balance sheet identity.

Corollary 2. *The liquidity ratio, ℓ, the leverage ratio, k, and the intermediation spread, $r_I - \bar{r}_D$, are sufficient to characterize banking allocations.*[16]

Before we compare the private equilibrium and the social one, we briefly discuss the assumptions about the banker's behavior that give rise to the private equilibrium. First, the possibility of a run disciplines the banker, who internalizes how her lending and funding choices affect the probability of a run, and hence the probability that she will make profits. These considerations are captured by the terms multiplied by the shadow value on the global game constraint, ψ_{GG}, in (23).

Second, the banker internalizes how all of her choices affect the deposit rate demanded by savers. We suppose this is possible because the debt is demandable and the depositors can fully observe the bank's balance sheet. So at the moment of entering into a deposit contract, depositors observe the balance sheet and can rationally compute the underlying risks. Hence, if the banker alters the balance sheet, savers can request new deposit contract terms, so that their optimal supply of deposits reflects the new risks. The banker anticipates this behavior and, thus, internalizes, initially, the impact of all of her choices on the deposit supply. These considerations are captured by the terms multiplied by the shadow value on the deposit supply schedule, ψ_{DS}, in (23).

[15]Our model is not scale invariant, or in other words, the equilibrium conditions cannot all be normalized by the balance sheet size $I + LIQ$. There are four reasons that equilibrium allocations depend on size. First, the concave utilities for savers initial consumption and transaction services of deposits depend on the levels of these variables. Second, the convex effort cost by the entrepreneurs depends on the level of investment. Third, the banker's initial consumption level of consumption determines utility in the initial period and and, lastly, the level of the monitoring cost is a constant. If we relax these assumptions, then the equilibrium will be scale invariant and could be characterized by only the asset allocation and capital structure intermediation margins. We elaborate on this special case later in conjunction with the social efficiency of private allocations.

[16]The statistics in Corollary 2 are not exclusive. For example, one could combine (23) with respect to LIQ and D on the one hand, and with respect to E and I on the other, to obtain alternative intermediation margins to AAM and CSM. These margins would be proxied by a reverse ratio, LIQ/D, and a risk-weighted capital ratio, $CR = E/I$, respectively. We focus on the intermediation margins described above as they provide an intutive way to describe banking allocation. However, in Section 4 we examine the effect of many alternative regulatory tools beyond liquidity and leverage requirements, such as risk-weighted capital requirements, liquidity coverage ratio and net stable ratio requirements. These regulations will impact the three intermediation margins in different ways; some will be complements, while others will be substitutes.

Alternatively, one could assume instead that deposit contracts are incomplete and that not all banking choices are perfectly observed. In that case, after collecting deposits could take more lending risk than the depositors would prefer. This would result in a *commitment problem* for the banker (see, also, Matutes and Vives, 2000; Martinez-Miera and Repullo, 2017). Under these assumptions, the banker realizes that taking more risk increases the cost of raising deposits, and would ideally want to promise depositors that it will behave prudently. But, after the deposit contract has been signed, the bank has an incentive to deviate towards lending more, holding fewer liquid assets and raising less equity – and the depositors could not do anything about that. Consequently, the banker would only internalize the terms that are specified in the deposit contract, which would, at minimum, be the amount of deposits and the deposit rates. As a result, only the first-order conditions in (23) with respect to $\{D, r_D, \bar{r}_D\}$ would include the terms multiplied by ψ_{DS}. As we discuss in Section 3, this lack of commitment generates additional distortions, which would be present even in the absence of run-risk. Thus, abstracting from it, allows us to isolate the impact of regulation on distortions induced only by runs. See Section B.3 in the online appendix for a characterization of equilibria under deposit contract incompleteness and lack of commitment by the banker.

Third, the banker internalizes how all of her choices affect the loan rate that entrepreneurs are willing to accept. These considerations are captured by the terms multiplied by the shadow value on the loan demand schedule, ψ_{LD}, in (23). Alternatively, we could have assumed that the banker is a price-taker with respect to the loan rate, which could be the case in anonymous competitive lending markets. This assumption would imply that $\psi_{LD} = 0$ in (23). Apart from being at odds with the monitoring function of the bank, the price-taking assumption would introduce additional reason why private and social outcomes may diverge as a planner would internalize the impact of banking choice on the loan demand. In order to focus on distortions induced by runs and simplify the analysis, we relegate the characterization of equilibria under price-taking behavior in loan markets in Section B.4 in the online appendix.

3 Efficient Allocations

Our banks offer socially useful services, but face the risk of a run. One of the main points of our analysis is that private banking choice generate *run externalities*, which adversely affect the welfare of savers and borrowers. Run risk is harmful for all agents in the economy; banks, savers and borrowers. But the effects on their respective welfare differ. Banks would benefit from low run risk, but not at the expense of substantially lower profits. Savers would prefer lower run risk and more deposit services, while borrowers would benefit from higher investment accompanied by lower run risk.

The private choices that the agents make work as follows. Bankers fully internalize how their lending and capital decisions change the probability of a run, the deposit rates and the loan rate. However, bankers exploit their limited liability and act to maximize their own utility, disregarding the direct effect of their actions on the welfare of savers and entrepreneurs. In turn, savers and entrepreneurs are too small to internalize how their own decisions matter for aggregate bank allocations (and hence on the probability of a run). In order to examine how these externalities distort outcomes, we consider a social planner who internalizes the effects of lending and capital structure decisions on all agents, but still is constrained by the market structure of the economy. We will show the externalities operate via all three intermediation margins derived in Section 2.5 and create three independent distortions, which the planner would like to address. Section 3.1 sets up the planner's problem and identifies the sources of difference between the private and social optimization margins. Section 3.2 presents a numerical solution to the model and describes how the private and social planner's allocations differ.

3.1 Social Planner

The social planner will choose all endogenous variables to maximize the following social welfare function:

$$\mathbb{U}_{SP} = \mathbb{U}_B + w_S \mathbb{U}_S + w_E \mathbb{U}_E, \tag{27}$$

where $w_S \geq 0$, $w_S \geq 0$ are the weights assigned to S and E—we have normalized the weight on B to one to facilitate the comparison of the planner's and the private optimality conditions (other normalizations are also possible; for example, assign weights to all agents adding up to one). Agents' utilities are given by (1), (5) and (14). The planner is constrained by the market structure of the economy, i.e., she cannot use lump-sum transfers to allocate resources across agents or complete any missing markets.[17] Moreover, the planner respects the run determination given by (20) as well as the deposit supply and loan demand schedules (2) and (7).[18] Give that the latter two hold with equality in the planner's problem, we can substitute the indirect utilities (4) and (8) in (27) to get the following social welfare function:

$$\mathbb{U}^*_{SP} = \mathbb{U}_B + w_S \mathbb{U}^*_S + w_E \mathbb{U}^*_E. \tag{28}$$

Using (28) instead of (27) adds a lot of tractability in the optimizing conditions and, as we show below, allows us to pinpoint the sources of distortions in the private equilibrium that a planner would like to address. In particular, $\mathbb{U}^*_S$ only depends on D and ξ^* with $\partial \mathbb{U}^*_S / \partial D = -U''(e_S - D) - (1 -$

[17]Given the absence of lump-sum transfers, we cannot unambiguously construct a welfare criterium to maximize the total surplus. Thus, we assign weights for different agents in a social welfare function and study different constellations of these weights. Although we remain agnostic about the origin of such weights, we discuss the potential political economy considerations of regulation.

[18]In principle, the planner may want to divert from the privately optimal deposit supply and loan demand schedules, which means that (2) and (7) do not need to hold with equality in the planner solution. For example, the planner could choose deposit or loan rates that do not necessarily satisfy all these conditions with equality and implement the resulting allocations by choosing instruments, such as Pigouvian taxes on interest income/expenses, that distorts (2) or (7). Farhi and Werning, 2016, and Bianchi and Mendoza, forthcoming, consider such taxes to implement the constrained efficient allocations. Note that the taxes can also take negative values, in which case they are interpreted as subsidies. If these conditions do not hold with equality, then the Lagrange multipliers associated with them are zero. Given that our focus is on banking regulation, we abstract in our baseline analysis from such tools and examine their implications in Section B.8 in the online appendix.

$q)V''(D(1+r_D))D(1+r_D)^2 > 0$ and $\partial\mathbb{U}_S^*/\partial\xi^* = -[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)]/\Delta_\xi < 0$. Therefore, a planner that cares about S would like to increase the level of deposits and reduce the probability of a run. Similarly, $\mathbb{U}_E^*$ only depends on I and ξ^* with $\partial\mathbb{U}_E^*/\partial I = (1-q)c''(I)I > 0$ and $\partial\mathbb{U}_E^*/\partial\xi^* = -[c'(I)I - c(I)]/\Delta_\xi < 0$. Hence, a planner that cares about E would like to increase the level of investment and reduce the probability of a run.

Definition 2. *The social planner's equilibrium is defined as the set of banking assets,* $\{I, LIQ\}$*, banking liabilities,* $\{D, E\}$*, the run threshold,* ξ^**, deposit rates,* $\{r_D, \bar{r}_D\}$*, and the loan rate,* r_I*, that maximize social welfare,* $\mathbb{U}_{SP}^*$*, defined in* (28) *subject to the balance sheet (BS) constraint* (9)*, the global game (GG) constraint* (20)*, the deposit supply (DS) schedule* (2)*, and the loan demand (LD) schedule* (7)*.*

Each first-order condition in the planner's equilibrium takes the form

$$\frac{\partial\mathbb{U}_B}{\partial C} + w_S\frac{\partial\mathbb{U}_S^*}{\partial C} + w_E\frac{\partial\mathbb{U}_E^*}{\partial C} + \sum_{\mathcal{Y}}\zeta_{\mathcal{Y}}\frac{\partial\mathcal{Y}}{\partial C} = 0, \tag{29}$$

where $\zeta_{\mathcal{Y}}$ are the shadow values on constraints $\mathcal{Y} \in \{BS, GG, DS, LD\}$ and are different than $\psi_{\mathcal{Y}}$ in the private equilibrium. The planner's optimality conditions differ from the private ones, because the planner explicitly accounts for how her choices affect S and E, and thus will assign different shadow values, $\zeta_{\mathcal{Y}}$, on the same constraints $\mathcal{Y}$ faced by the banker. Following the same steps as for the private equilibrium, we can derive the same three intermediation margins for the planner's problem (see Section B.2 in the online appendix for detailed expressions).

The asset allocation margin for the planner can be written at $AAM_{SP} = AAM_{PE} + AAM_{WD}$, where AAM_{PE} is given by (24) and AAM_{WD} is a wedge, which captures the additional distortions that the planner takes into account and will try to correct, given by:

$$AAM_{WD} = \underbrace{\left(w_S\frac{\partial\mathbb{U}_S^*}{\partial\xi^*} + w_E\frac{\partial\mathbb{U}_E^*}{\partial\xi^*}\right)\cdot\left(\frac{\partial\xi^*}{\partial LIQ} - \frac{\partial\xi^*}{\partial I}\right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E(1-q)c''(I)I}_{\text{Surplus to } E \text{ from additional } I}. \tag{30}$$

The first term in (30) captures the externality from run risk due to the choice of the asset allocation between liquid assets and loans. If shifting a unit of loans to liquid assets reduces the run probability, i.e., $d\xi^*/dLIQ - d\xi^*/dI$ detailed in (B.13) is negative, then a planner that cares about S and E would want a more liquid asset mix than in the private equilibrium. The last term in (30) captures the surplus created for the entrepreneur from an additional unit of investment and this consideration leads the planner to increase the level of investment to favor E. Note that, in principle, a more liquid asset mix and more investment are not incompatible if they are accompanied by a bigger balance sheet.

Similarly, the capital structure margin for the planner is $CSM_{SP} = CSM_{PE} + CSM_{WD}$, where

CSM_{PE} is given by (25) and the wedge CSM_{WD} is given by:

$$CSM_{WD} = -\underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right) \frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}} - \underbrace{w_S [U''(e_S - D)D + (1-q)V''(D(1+r_D))D(1+r_D)^2]}_{\text{Surplus to } S \text{ from additional } D}. \tag{31}$$

Shifting funding from deposits to equity mitigates the run externality and helps S and E, but reduces the surplus to S.

Finally, the scale of intermediation margin for the planner is $SIM_{SP} = SIM_{PE} + SIM_{WD}$, where SIM_{PE} is given by (26) and the wedge SIM_{WD} is given by:

$$\begin{aligned} SIM_{WD} = & \overbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right) \cdot \left(\frac{\partial \xi^*}{\partial I} + \frac{\partial \xi^*}{\partial D}\right)}^{\text{Run externality from intermediation scale}} \\ & + \underbrace{w_S [U''(e_S - D)D + (1-q)V''(D(1+r_D))D(1+r_D)^2] + w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}. \end{aligned} \tag{32}$$

The planner will want less intermediation, if raising an additional unit of deposits to fund investment increases the probability of a run, i.e., $\partial \xi^*/\partial I + \partial \xi^*/\partial D > 0$. Yet, the run externality considerations are countered by the fact that more intermediation increases the surplus to both S and E.

All three wedges feature a component driven by run externalities, which B, S and E do not internalize, and a component that captures the surplus to S and E. The planner trades off reducing run risk in order to tackle the run externalities and improving the surplus accruing to S and/or E when the run does not occur. In doing so, she chooses a different asset allocation, capital structure and scale of intermediation, which have a direct impact on the surplus and an indirect impact on run risk. The following Proposition establishes the inefficiency of the private equilibrium.

Proposition 2. *The private equilibrium is generically inefficient and there are three independent distortions in intermediation margins given in* (30)*,* (31) *and* (32)*.*

Correcting some of these distortions involve tradeoffs because the interventions skew allocations that favor borrowers over savers (or vice versa). For instance, if the bank holds more safe assets and makes fewer loans, that switch marginally helps the savers because it makes their deposits safer. Conversely, the opposite choice of more loans and fewer safe assets creates more opportunities for the borrowers but reduces the buffer that helps mitigate the riskiness of deposits. In the next section we present a numerical example of the private and planning equilibria and discuss how the allocations differ in reference to the aforementioned intermediation margins and associated distortions. Before turning to that, we show in the following Corollary the conditions under which the private equilibrium is constrained efficient, or in other words all wedges are zero.

Corollary 3. *The private and social planner's equilibria coincide if all of the following conditions hold: (i) $c''(\cdot) = 0$; (ii) $V''(\cdot) = 0$; and (iii) $U''(\cdot) = 0$ or $e_S \geq \overline{e}_S$ such that $LIQ_S > 0$.*

The Corollary 3 essentially says that if S and E are not benefiting from bank intermediation, then there is nothing a planner, who respects the market structure, can do to improve outcomes. The reason is that savers' and entrepreneurs' welfare is constant at the autarkic level and the banker already internalizes everything that matters to her. Additional inefficiencies could be introduced to the model to justify a role for policy. For example, the liquidation value ξ could be a function of the amount of loan recalled, y, and determined in a fire-sale, which the banker would not internalize.[19] Alternatively, one could assume that the run induces a deadweight loss, which S and E do not internalize and B neglects because she is protected by limited liability.

Even with these alternatives, the distortions would manifest themselves through the three intermediation margins we have described. As long as the asset allocation, capital structure and scale of intermediation (I, LIQ and D) load on the fire sale price or deadweight loss, then the market failure will operate through all three margins to distort outcomes. Conversely, the number of distortions is determined by the number of intermediation margins that are misaligned and not by the number of market failures in the model. If we added externalities from fire-sales and deadweight losses to the other frictions in our baseline model, we would still have distortions in the asset allocation, capital structure and scale of intermediation margins, but the wedges would include additional terms.

3.2 Numerical example

The full set of parameters we used to solve the model is shown in Table 1. The parameterization should be taken more as an illustrative example to highlight the mechanisms in the model, rather than as a realistic calibration of the economy that would be suitable for making quantitative statements about the absolute optimal level of banking regulations. We have experimented with various other parameter choices and the findings that we emphasize are quite robust.

Our model would require some obvious modifications to use it for quantitative policy analysis. For example, all liabilities in our model are unsecured, while in practice certain types of deposits are insured. Deposit insurance, even partial, would reduce the market discipline exerted by depositors and hence the credit risk premia in deposit rates bringing them closer to what is observed in reality. Moreover, it is not clear whether the various capital regulations in practice (Basel requirements, stress tests, restrictions on dividend payouts) are indeed binding and whether one should be calibrating to match a regulated economy rather than an unregulated private equilibrium. Finally, the quasi-linearity of preferences, which simplifies the computation of the run threshold and derivation of policy significantly, as well as the finite horizon of the model make depositors willing to accept a higher probability of a run than if they were risk-averse or if there was a continuation value for the bank. One could add convex bankruptcy costs to mimic a higher degree of risk-aversion as well as model the continuation value, but we have not done so because it is not important to make our fundamental analytic points.

With these caveats in mind, let us call attention to some of the considerations that we took into

[19] See Carletti, Goldstein and Leonello (2019) for a model where fire-sales externalities interact with run risk yielding inefficient private outcomes.

account while choosing the model parameters. First, the bank is profitable enough, and the initial equity of the banker and her preference for current consumption are such that she voluntarily uses some of her endowment to buy more equity in the bank. So the banker finds intermediation to be profitable. Second, the deposit services provided by the bank lead savers not to self-insure by directly holding the liquid asset. If savers were opting to self-insure, then the banking sector is under-performing as a provider of liquidity and, hence, intermediation, and regulations that make banks more stable would have an additional positive effect. Third, we have chosen the parameters so that the bank makes loans and invests in liquid assets, but also plans to liquidate some loans to serve early withdrawals. The key parameters that are responsible for this outcome are the size of the liquidity preference shock, the distribution of the liquidation values for recalled loans and riskiness of investment opportunity. Fourth, we have chosen logarithmic utility for period 1 consumption for both the savers and the banker, but we have assumed that the banker values future consumption more than the savers do. In particular, $U(x) = log(x)$ and $W(x) = \gamma \cdot log(x)$, where $\gamma < 1$.[20] Finally, we set $V(x) = c_D \cdot log(1+x)$ and $c(x) = c_I \cdot x^{\phi_I}$, with $\phi_I > 1$, which satisfy the general properties required for the transaction services' and effort cost functions.

Tables 2 shows the private equilibrium and planning outcomes for small perturbation of the welfare weights away from the private equilibrium.[21] We focus the analysis around the three intermediation margins derived in section 3.1.

Before turning to the details, it is helpful to recall three things we already know about the nature of the distortions that the planner is trying to correct. First, the banker is already internalizing everything that matters for her own welfare. The problem is that she is ignoring the consequences of her choices on the saver and the borrower. Therefore, anything the planner does to take this into account will make the banker worse off. So the planner will be constrained on this front by the need to make sure that the banker will still find it profitable enough to monitor loans. The saver generally wants safer deposits. This can be accomplished by reducing the riskiness of the asset mix or by raising more equity from the banker. The banker will only contribute more equity if the expected dividend yield is high enough. Finally, the borrower would like to get more loans but has a downward sloping demand curve. So more lending will only occur at lower interest rates.

First, consider the case where the planner favors E and S equally; for example $w_E = w_S = 0.1$ in Table 2. The planner would like to increase liquid holdings in the asset mix to address the run externality because $\partial\xi^*/\partial LIQ - \partial\xi^*/\partial I < 0$ in (30). Similarly, the planner would like to increase the amount of equity in the liabilities mix because $\partial\xi^*/\partial D > 0$ in (31). However, to benefit E and S the planner needs to pay attention to the level of loans and deposits to make sure that they do not drop.

[20] Assigning to the banker the same utility function requires high enough e^B or low enough γ such that she would be willing to invest enough of her own wealth in equity to provide liquidity benefits to savers. We do the second because we want the banker endowment to represent only a small part of the total endowment in the economy, with the vast majority accruing to the savers. For $\gamma = 1/\beta^2$, such that savers and the banker discount the future the same way, and for logarithmic utility, we can obtain the same equilibrium for banker's wealth $\check{e}^B = E + (e^B - E)/(\beta^2\gamma)$, where E is the equilibrium value of contributed equity.

[21] Given period 1 allocations the run threshold is unique, but it may be the case that there are more that one private equilibria each characterized by a unique run threshold. In order to guarantee that the planning equilibria we report correspond to the stated PE we first so the results for a small perturbation of weights and then move to larger perturbations.

The way to achieve these various goals is to grow the overall size of the of the bank's balance sheet while making assets more liquidity intensive and liabilities more equity intensive. The reduction in run risk mitigates the upward pressure on the deposit rate and the volume of lending has to be enough higher so that, even though the loan rate will be lower, expected dividends grow by enough to induce the banker to supply more equity.

One way to think about what is happening in this experiment is to recognize in the private equilibrium, the banker is restricting lending to prop up loan rates and limiting deposits to suppress the cost of deposits. The planning allocations correct these problems. In doing so, the banker is made slightly worse off, but the other two agents are much better off. Overall, social welfare rises.

Second, consider the case that the planner wants to favor S, but cares little about E; for example $w_E = 0$ and $w_S = 0.2$. Similar to the first case, the planner would like to improve the liquidity of the asset mix to address the run externality. But, the planner is now less concerned about the surplus accruing to E and can more easily shift some of the investment towards the liquid asset. At the same time, the planner would like to increase equity in the liabilities mix in order to address the run externality, but without cutting deposits which is what matters for the surplus accruing to S. The planner can increase both liquidity and deposits, and at the same time guarantee that B will inject more equity in the bank, by cutting the level of lending. This results in a sufficiently large increase in the loan rate to compensate for higher deposit rate, so that bank profitability jumps and the banker is willing to to invest in more equity. The overall size of the balance sheet goes up, but the scale of intermediation, measured by the amount of deposits channeled to investment(I-E), goes down.

Finally, consider the third case where that the planner wants to favor E, but cares little about S; for example $w_E = 0.2$ and $w_S = 0$. The planner would like to increase lending and, thus, the surplus accruing to E. As a result, the planner shifts liquidity to loans, which makes deposits more expensive. The planner could then substitute away from deposits to equity to fund the higher investment, but this would only be possible if such a shift did not worsen the intermediation spread and, thus, the incentives of B to provide equity. Given the optimality of private allocations from B's perspective such a shift from deposits to equity—with a shift from liquidity to loans at the same time—would not be profitable for the banker. However, because the planner is not concerned with helping S, it is possible to reduces deposit taking, which lowers the deposit rate and also the necessary amount of liquidity that is needed to be carried for early withdrawals. So the planner shifts allocations so that more deposits are being used to support lending; the scale of intermediation increase despite the fact that the bank operates with a smaller balance sheet.

The private equilibrium may exhibit over- or under-investment compared to the planner's outcomes depending on the weights on E and S. Lending decreases and liquid asset holdings increase as the more weight is on placed S. Both factors contribute to a higher liquidity ratio $\ell \equiv LIQ/(LIQ+I)$. Lower lending is accompanied by a higher loan rate and induce the banker to provide more equity. The leverage ratio $k \equiv E/(E+D)$ improves as a result. Even though this also helps deposits increase somewhat, the amount of deppsits going to support lending is falling and the spread between the loan rate and period 3 deposit rate rises. The opposite is true when the planner places more weight on E. In this case, investment jumps but deposits fall.

Naturally, the run probability is lower when more weight is placed on S. But, lower run risk accompanied by higher ℓ and k can still be consistent with higher investment. This can happen when the bank has a bigger balance sheet and uses more deposit financing to support lending. Notice that this is what happens when the planner cares equally about depositor and savers.

The enhanced stability of both the asset portfolio and the capital structure of the bank is beneficial to S especially because, as discussed above, it can be accompanied by a higher level of deposit services. Lower run risk is also beneficial to E, but may come at the cost of lower investment and, hence, lower surplus from production. Indeed, when investment falls below its level in the private equilibrium the entrepreneur will be worse off. As more weight is placed on E the level of investment increases pushing up the run probability and reducing the surplus from deposit services. S's welfare goes down and after a point she is worse-off compared to the private equilibrium. In this example, both S and E can be made better-off when the planner cares about them equally. B is always worse-off, because she already internalized what mattered for her and any deviation from the private equilibrium reduces her welfare.[22] Note that the planner not only increases social welfare $\mathbb{U}_{sp} = \mathbb{U}_B + w_S \mathbb{U}_S^* + w_E \mathbb{U}_E^*$, which depends on the weights, but also the overall surplus in the economy, $\mathbb{S}_{sp} = \mathbb{U}_B + \mathbb{U}_S^* + \mathbb{U}_E^*$. Thus, the planner could improve the welfare of all agents if she had access to a re-distributive, non-distortionary (lump-sum), tax system to transfer resources across agents.

We should note that neither the banker nor the planner opt to hold excess liquidity. Holding excess liquidity could be desirable in order to eliminate the probability of a run altogether. If the liquidation value of the bank for the lowest possible realization of ξ was higher than the total runnable deposit obligations, i.e., $LIQ + \underline{\xi} \cdot I \geq D(1+r_D)$, then only impatient depositors would withdraw. The excess liquidity carried over to period 3 would then be $LIQ_{ex} = LIQ - \delta \cdot D(1+r_D) \geq (1-\delta)D(1+r_D) - \underline{\xi} \cdot I$. Such run-proof equilibria may not be desirable when the lowest liquidation value of long-term investment is small or when savers are not very risk-averse. A large literature has focused on run-proof equilibria, which naturally restricts credit intermediation (see Cooper and Ross, 1998, Ennis and Keister, 2006, Diamond and Kashyap. 2016). Run-proof contracts require certain assumptions to be optimal and our work has, instead, focused on optimal policy in the presence of both run risk and credit risk.

4 Regulation

We now explore how the planner's solution can be decentralized via various regulatory interventions. We group tools into two categories. The first category includes tools that target the capital structure and intend to inject more equity in the bank. The second category includes tools that aim to make the asset mix more liquid. Sections 4.1 and 4.2 discuss the effects when the tools are used

[22] This is not the case under incomplete deposit contracts discussed in section B.3. Then, the planner can also improve B's welfare by forcing her to internalize how her actions matter for the supply of deposits.

in isolation. Section 4.3 discusses how the regulations can be optimally combined to implement the planner's solution as a private equilibrium.

4.1 Tools targeting capital

We examine two tools that can be used to increase the amount of equity in the bank. The first one is a requirement $\bar{k}$ on the leverage ratio, where here it is helpful to use the balance sheet identity to write it $k = E/(I + LIQ) \geq \bar{k}$. The second one is a requirement $\overline{CR}$ for the risk-weighted capital ratio, i.e, $CR = E/I \geq \overline{CR}$, where we have assumed a risk-weight of one for loans and zero for liquid assets.

The direct effect of both tools would be to increase the level of equity in the bank's liabilities, which should intuitively push the probability of a run down and help with the run externalities. Both k and CR will affect the probability of a run directly by changing allocations in (20), and indirectly by influencing loan and deposit rates.

It is helpful to keep in mind that the banker will only invest in additional equity if the bank profits rise. Profitability depends on the spread between the loan and period 3 deposit rate, so rising profitability requires either higher loan rates or lower deposit rates. Hence, either lending will have to fall, so the loan rate can rise, or deposits have to be made safer so that the deposit rate can fall.

The following Proposition establishes that the direct effect of higher k or CR, i.e., keeping interest rates and other allocations constant, is to reduce run risk.[23]

Proposition 3. *Set $X = 0$. The partial equilibrium effect of higher requirement $\bar{k}$ or $\overline{CR}$ on q is negative.*

The above proposition establishes that if one fixes the liquidity ratio, ℓ, as well interest rates, then tightening $\bar{k}$ or $\overline{CR}$ reduces run risk in exactly the same way, because $k = CR(1-\ell)$. Hence, in a partial equilibrium setting, one would conclude that these two regulations are equivalent.[24]

However, the general equilibrium effects of the two regulations on the incentive of bankers to hold liquidity and on the deposit supply and loan demand will differ. These differences stem from the way that two regulations impact the three intermediation margins. Recall $k = E/(I + LIQ)$ and $CR = E/I$. The intermediation margins under leverage regulation become $AAM_k = AAM_{PE}$, $CSM_k = CSM_{PE} + \psi_k$, and $SIM_k = SIM_{PE} - \psi_k \bar{k}$, where ψ_k is the Lagrange multiplier on the leverage requirement $k \geq \bar{k}$. The intermediation margins under risk-weighted capital regulation become $AAM_k = AAM_{PE} + \psi_{CR}\overline{CR}$, $CSM_k = CSM_{PE} + \psi_{CR}$, and $SIM_k = SIM_{PE} - \psi_{CR}\overline{CR}$, where ψ_k is the Lagrange multiplier on the risk-weighted capital requirement $CR \geq \overline{CR}$.

The leverage regulation allows banks to costlessly switch from liquid assets to loans. So that substitution will necessitate lowering the lending rate. Hence, to boost profitability deposit rates

[23]We set the monitoring cost equal to zero purely because it simplifies the proof, but nothing qualitatively changes if instead we fix the cost to be a small positive value.

[24]In the online appendix, we further characterize how the two regulations change the boundaries between the three run regions and affect the payoffs for depositors in each of them.

must fall. In contrast, the risk-weighted capital requirement lowers lending and hence will be associated with a higher loan rate. The higher loan rate means that the bank can take on more deposits (than if lending had risen).

Table 3 reports the results for individually tightening $\bar{k}$ and $\overline{CR}$. The change in the two requirements from their private equilibrium level satisfies the aforementioned relationship, i.e., $\Delta k = \Delta CR(1-\ell)$, where ℓ is the liquidity ratio in *PE*. We will consider two cases: both requirements increases by a little and both requirements increase by a lot.

The direct beneficial effect on run risk continues to dominate when general equilibrium effects are accounted for and q decreases under both regulations. However, the magnitude of the decrease differs and so do the effects on other components of the bank balance sheets.

First, consider the case where $\bar{k}$ and $\overline{CR}$ are marginally tightened. To compare the effects notice how the *AAM* is differentially impacted. Loans decreases much more under *CR* than *k*, which improves bank's asset liquidity. Thus, run risk goes down even further than what would expect by just increasing capital. The lower run risk makes deposits cheaper and the bank increases its deposit taking. *S* gains, while both *E* and *B* lose.

The situation is similar for *k*, but the effects are much less pronounced because the bank can shift some liquidity to lending and still satisfy the regulation. Because of the additional lending, the bank does not boost liquidity by as much, so the scope to increase deposits is reduced. The gain for *S* is smaller and the losses for *E* are too.

Considering a much larger increase in capital, we obtain the same, but stronger, effects for *CR*. However, for *k*, the substitution towards lending becomes much more pronounced. The bank actually increases lending relative to the private equilibrium and substantially reduces its holding of liquid assets.[25] Given those changes, the cost of deposits must fall in order to make the bank profitable enough to support the higher level of capital. This drop occurs because the total deposits fall and so *S* is made worse off from this change. The higher level of lending makes *E* better off.

4.2 Tools targeting liquidity

We examine three tools that can be used to increase the amount of liquidity in the bank. The first one is a requirement $\bar{\ell}$ on the fraction of assets that are liquid, $\ell = LIQ/(I+LIQ) \geq \bar{\ell}$. We will refer to this regulation as the liquidity ratio. The second one is a requirement $\overline{LCR}$ on the liquidity coverage ratio, which takes the (lowest) liquidation value of the bank's portfolio in a run relative to runnable liability, i.e., $LCR = (LIQ + \underline{\xi} \cdot I)/(D(1+r_D)) \geq \overline{LCR}$. The third one is a requirement $\overline{NSFR}$ on the net stable funding ratio, which is computed as the fraction of illiquid assets funded by relatively stable sources, i.e., $NSFR = (E + (1-\delta)D)/I \geq \overline{NSFR}$.

Proposition 4. *Set X=0. The partial equilibrium effect of higher requirements $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$ on q is negative if $\delta > e^{-1}$ or $\ell > \hat{\ell}$.*

[25] This is in contrast to models where the bank cannot raise additional equity, where stricter capital/leverage requirements (mechanically) result in a drop in credit extension (see, for example, Corbae and D'Erasmo, 2014, Clerc et al. 2015 and the references therein).

This proposition establishes that if one fixes the leverage ratio, k, as well interest rates, then tightening $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$ reduces run risk in exactly the same way, because $LCR = ((1-\underline{\xi})\ell + \underline{\xi})/((1-k)(1+r_D))$ and $NSFR = (k+(1-\delta)(1-k))/(1-\ell)$. Hence, in a partial equilibrium setting, one would conclude that these three regulations are equivalent.

These regulations have the unintended consequence that extra liquidity raises patient savers' incentives to join the full run. This has been noted by others in models with one-sided strategic complementarities (see also Carletti et al. 2019). We show that these perverse incentives do not dominate when the fraction of patient depositors is small enough or when the liquidity ratio in the private equilibrium is above a threshold. In the private equilibrium we examine, $\hat{\ell} < 0$ because the run risk is big enough to limit the strength of this channel, so the partial equilibrium effect of higher ℓ on q is negative.

But, as was the case for capital regulations, the general equilibrium effects of liquidity regulations may differ as the three regulations alter the intermediation margins in different ways. The intermediation margins under regulation on ℓ become $AAM_\ell = AAM_{PE} + \psi_\ell$, $CSM_\ell = CSM_{PE}$, and $SIM_\ell = SIM_{PE} - \psi_\ell \bar{\ell}$, where ψ_ℓ is the Lagrange multiplier on the liquidity requirement $\ell \geq \bar{\ell}$. The intermediation margins under LCR regulation become $AAM_\ell = AAM_{PE} + \psi_{LCR}(1-\underline{\xi})$, $CSM_{LCR} = CSM_{PE} + \psi_{LCR}\overline{LCR}(1+r_D)$, and $SIM_{LCR} = SIM_{PE} - \psi_{LCR}(\overline{LCR}(1+r_D) - \underline{\xi})$, where ψ_{LCR} is the Lagrange multiplier on the liquidity requirement $LCR \geq \overline{LCR}$. Finally, the intermediation margins under $NSFR$ regulation become $AAM_{NSFR} = AAM_{PE} - \psi_{NSFR}\overline{NSFR}$, $CSM_{NSFR} = CSM_{PE} + \psi_{NSFR}\delta$, and $SIM_{NSFR} = SIM_{PE} - \psi_{NSFR}\overline{NSFR}$, where ψ_{NSFR} is the Lagrange multiplier on the liquidity requirement $NSFR \geq \overline{NSFR}$.

Table 4 reports the results for individually tightening $\bar{\ell}$, $\overline{LCR}$, or $\overline{NSFR}$. The change in the requirements from their private equilibrium level satisfy the aforementioned relationships, i.e., $\Delta LCR = \Delta\ell(1-\underline{\xi})/((1-k)(1+r_D))$ and $\Delta NSFR = \Delta\ell \cdot NSFR/(1-\ell)$, where k, $NSFR$ and ℓ are the leverage, net stable funding and liquidity ratios in PE.

First, focus on requirements $\bar{\ell}$ and $\overline{LCR}$, which are very similar. Mandating that the bank holds more liquidity changes the trade-off between investing in risky loans and liquid assets as can be seen by the way these regulation alter the AAM. The higher liquid asset holdings allow the bank to raise more deposits without increasing run risk. Although the amount of deposits raised increases, the portion that is channeled to loans falls. At the same time, the amount of equity loans goes up.

Why does requiring them to hold more liquidity induce the banks to both raise more deposits and more equity? Banks can raise more deposits and invest them in the liquid asset to satisfy the regulation. This is preferable to raising equity in order to invest in the liquid asset, because equity is more expensive. Despite the fact that run risk decreases, the increased demand for deposits pushes up the deposit rate and, thus, makes loans less profitable. Banks will reduce lending to secure higher loan rates to raise profitability. For a large enough fall in lending, the intermediation spread widens so much that it becomes desirable to increase the amount of equity.

The $NSFR$ regulation operates via the same channels, but the effects are less pronounced. The reason is that this regulation also partially resembles a risk-weighted capital requirement. In particular, notice that the $NSFR$ can be re-written as $1 + CR + \ell + 1/(1 - CR)$ so that it operates through affecting both capital and liquidity. Relative to the other liquidity regulations, the $NSFR$ has a more muted effect on deposits. S is better-off, while E and B are worse-off under all liquidity regulations.

4.3 Combined Regulation and Optimal Regulatory Mix

Finally, we examine whether and how regulation can be combined to implement the social planner's solution as a private equilibrium. The social planner solves for allocations without taking into consideration how the optimal behavior of the bank will change, or in other words the first-order conditions of the banker (adjusted for regulatory interventions) are not taken as additional constraints in Definition 2. Hence, the planner's allocations are computed without tying the planner to specific tools. We have seen that the difference between privately and socially optimal choices can be summarized characterized by the wedges in (30), (31) and (32). The rest of the section shows how the regulatory tools studied above can be combined to mimic the allocations preferred by a social planner.

To do this it is instructive to set-up an augmented planner who is endowed with certain tools. Let $\mathbb{T} = \{k, CR, \ell, LCR, NSFR, \tau_D\}$ be this set of potential tools. These options are the capital and liquidity tools discussed in sections 4.1 and 4.2 as well as a tax on deposit interest expenses, τ_D that we describe below. The details of the implementation are spelled out in section B.5. Herein, we discuss the results using our numerical example.

Table 5 reports the outcomes of combining regulatory tools and compares them to the planner's solution for a case where the planner favors the saver more than the borrower. Given that three margins are distorted, we would expect that three tools will be needed to implement the planner's solution. We need one tool that operates on the asset side and fixes the asset allocation margin, another that operates on the liability side and fixes the capital structure margin, and a third tool that targets the scale of intermediation. In other words, we will need to combine one capital regulation and one liquidity regulation with a tool that can push the bank towards the socially optimal level of intermediation. There are several potential candidates for the third type of tool, but we will consider a tax (or subsidy) on deposit interest expenses. This tool can indirectly control the intermediation spread, which, as discussed in Corollary 2, is a sufficient statistic for the SIM (provided that the AAM and CSM have been fixed by a capital and a liquidity tool).[26]

As discussed, tightening the risk-weighted capital requirement to target the CSM increases the amount of equity in the bank. Raising the capital requirement reduces run-risk and results in lower

[26] The planner cannot use two liquidity or capital tools at the same time because they will not be jointly binding. This result is consistent with the analysis in Checchetti and Kashyap (2016), who show that LCR and $NSFR$ regulations almost surely will never bind at the same time. However, the collinearity of the CR and k regulations may be specific to our model and may not even hold for high k (see discussion in section 4.1). If the bank that could choose between more types of assets with different levels of risk, or to hold off-balance sheet assets, this result may not obtain – though this would not likely deliver the planner's allocations. Although we can only speculate at this point, we believe that such modifications are important avenues for future research. Other papers that study the use of capital and liquidity requirements include Walther (2016) and Kara and Ozsoy (2016) in the presence of fire sale externalities, Boissay and Collard (2016) when the interbank market cannot efficiently allocate resources, and Van den Heuvel (2017) who quantifies the welfare costs of capital and liquidity requirements in a neoclassical growth model.

loan extension. If, in addition, we impose a stricter liquidity requirement, we can get closer to the planner in terms of the asset and capital structure margins. But, the scale of intermediation, as proxied by either the loan-deposit rate spread or the amount of deposit funding investment, moves in the opposite direction of what the planner seeks. A subsidy of $\tau_D = -6.46\%$ can be levied to induce the bank to increase the scale of intermediation to the level that the planner prefers.[27] The combination of the three tools can implement the planner's solution as a private equilibrium. More generally, using two tools that are not redundant would also typically improve welfare relative any single regulation. However, to mimic the planner we find that three regulations are required.

One might expect that capital and liquidity requirement are not useful when the planner would like to increase the lending in the economy (as would be the case when the planner puts high weights on E). Table 6 shows partial and full implementation of the planner's solution for this kind of case. To mimic the planner's outcome we need to use deposit and lending subsidies. The complication arises because, as we see from Table 2, the planner wants higher lending but much less liquidity for this set of weights. So raising liquidity requirements will mean that the planner's allocations cannot be achieved. A capital requirement can be helpful, but it will have to be combined with lending or deposit subsidies. The second column in the table shows what happens when a capital requirement and a deposit subsidy τ_D are combined. These two regulations are sufficient to boost lending all the way to where the planner prefers. Hence we consider how a lending subsidy can be part of regulatory mix.

The last column in the table shows that a lending subsidy, $\tau_I = 3.54\%$, can be combined with $\bar{k} = 0.042$ and $\tau_D = -2.74\%$ to replicate the planner's allocation (column 5).[28] In this case, capital regulation is needed because the bank is tempted to maximize its leverage to reap the benefits of the subsidies. This helps a bit with respect to run risk compared to the case that only the two subsidies are in place (column 4), but unlike a liquidity regulation, capital regulation does not restrict lending materially, which is the primary objective of the planner for this set of weights.

To conclude, our findings suggest that capital is very useful as part of the optimal regulatory mix irrespective of which agent the planner favors, while liquidity requirements are more useful for savers. Indeed, we further verify this in section B.6 where we set $c_I = 0$, so that the entrepreneurs drop out of the planner's objective (see Corollary 3).

5 Conclusions

Banks perform important services for the real economy using both sides of their balance sheet. However, the private banking equilibria may not be socially optimal and regulating banking activities can improve social welfare. We have examined how many regulations that are often discussed in

[27]The tax/subsidy τ_D increases/decreases the interest expense on patient deposits, which becomes $(1-\delta)D(1+\bar{r}_D)(1+\tau_D)$—$\tau_D > 0$ implies a tax. We assume that the planner returns/collects the proceeds from the tax/subsidy to/from the bank in a lump-sum fashion in order to eliminate any income effects and focus on the ability of τ_D to target the SIM. Hence, the lump-sum transfer is equal to $(1-\delta)D(1+\bar{r}_D)\tau_D$.

[28]The easiest way to model this subsidy is to reward banks τ_I per unit of I extended and then tax them lump-sum an equal amount $\tau_I I$.

policy discussions perform in a relatively familiar model of banking. We started from the Diamond and Dybvig (1983) benchmark precisely because it is so thoroughly studied. The modifications that we made trade-off tractability to keep the model relatively simple, against our preference for additional realistics forces that the baseline model excludes.

Our modifications generate endogenous credit risk in banks' portfolios as well as the risk of an endogenous funding run. This simple pair of features interact in interesting and unexpected ways. We draw several general lessons from the model that we believe will carry over to many other models.

First, we identify three general intermediation margins that are distorted: the relative amounts of liquid and illiquid assets, the mix of deposits and equity, and the spread between loan and deposit rates. The way that banks privately set these margins diverges from what a social planner would choose, because bankers do not fully internalize the effects of their choices on savers and entrepreneurs. Provided the social planner cares sufficiently about savers, the planner chooses relatively more liquidity and equity than the banker and would reduce bank profits by boosting deposit rates and lowering loan rates. As a result, the planner reduces run risk, improves the provision of liquidity, guarantees a more stable extension of credit and real production, and delivers more overall intermediation compared to the private equilibrium. Capital regulation if still desirable even when the planner cares little about savers, but liquidity regulation needs to be replaced with tools that encourage further credit extension and, hence, actually create more risk.

Second, the wedges between the private and social choices are not collinear. Thus, more that one regulatory tool is needed to implement the socially optimal allocations. Optimal policy in models without all of these distortions can be misleading. For example, if the liability structure is constrained, say because deposit levels are exogenously determined and equity is fixed, studying asset allocations and distortions becomes much easier. But, regulation, if any is needed, will amount to fixing liquidity ratios. Similarly, shuting down the liquidity demand and liquidity risk makes it easier to focus on the optimal capital structure and level of investment. But, regulation, if again any is needed, would amount to fixing capital ratios. Instead, when both sides of the bank's balance sheet are endogenously determined the distortions from each side interact and a combination of both capital and liquidity requirements emerge in the optimal regulatory mix.

Third, the political economy aspects of regulation deserve attention. Our bankers internalize how their decisions matter for run risk, funding structure and the level of intermediation to maximize their own welfare. Their distorted choices, from a social point of view, have real macroeconomic consequences. Regulation improves aggregate welfare, but reduces the rents accruing to bankers. If possible, therefore, banks' incentives to engage in regulatory arbitrage would be strong. The lack of regulatory arbitrage in the model we have studied is one of its main shortcomings.

There are other interesting avenues to extend our model, some of which we have already been mentioned and are analyzed in the online appendix. One further direction would be to allow banks to issue long-term debt together with demandable deposits and equity. Including loss-absorbing debt instruments in the regulatory mix could introduce additional ways to tackle with run risk and credit risk. But it would not constitute a full remedy by itself due to the disciplinary role that demandable liabilities play. Moreover, our model is flexible enough to incorporate fire-sale dynamics by endogenizing the liquidation value of long-term investment. Although this would introduce pecuniary externalities as an additional reason why private allocations are inefficient, it would not qualitatilvely overturn our main conclusions; the asset and liability side distortions would be similar. Finally, one could enrich the set of risky investments from which a banker could choose and, thus, increase the scope for asset substitution. Setting the (relative) risk-weights in capital requirements to capture social risks would be, then, highly important.

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Tables and Figures

$e_S = 2.50$	$e_B = 0.10$	$A = 4.50$	$\omega = 80\%$	$c_D = 0.27$
$c_I = 0.1$	$\phi_I = 3$	$X = 0.10$	$\gamma = 0.15$	$\delta = 0.50$
$\beta = 0.70$	$\bar{\xi} = 2.20$	$\underline{\xi} = 0.01$		

Table 1: Parameterization.

	PE	SP for weights (w_E, w_S)				
		(0.00,0.20)	(0.05,0.15)	(0.10,0.10)	(0.15,0.05)	(0.20,0.00)
I	0.862	0.785	0.841	0.873	0.899	0.906
LIQ	0.052	0.221	0.119	0.060	0.012	0.000
D	0.875	0.962	0.919	0.894	0.873	0.867
E	0.038	0.044	0.041	0.039	0.038	0.038
r_I	3.097	3.198	3.131	3.089	3.051	3.042
$\bar{r}_D$	0.717	0.804	0.778	0.767	0.761	0.758
q	0.407	0.386	0.398	0.403	0.407	0.408
q_f	0.200	0.147	0.180	0.198	0.211	0.214
ℓ	0.057	0.219	0.124	0.065	0.013	0.000
k	0.042	0.044	0.043	0.042	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.796	2.742	2.705	2.670	2.663
$I + LIQ$	0.914	1.006	0.960	0.933	0.911	0.906
$I - E$	0.824	0.741	0.800	0.834	0.861	0.867
$E(Div)$	0.745	0.755	0.750	0.747	0.743	0.743
$\Delta\mathbb{U}_E$	-	-1.66%	-0.44%	0.33%	1.02%	1.19%
$\Delta\mathbb{U}_S$	-	3.63%	1.74%	0.71%	-0.10%	-0.30%
$\Delta\mathbb{U}_B$	-	-0.44%	-0.13%	-0.05%	-0.08%	-0.09%
$\Delta\mathbb{U}_{sp}$	-	0.29%	0.11%	0.05%	0.07%	0.14%
$\Delta\mathbb{S}_{sp}$	-	1.53%	1.18%	0.99%	0.84%	0.79%

Table 2: Privately versus Socially Optimal Solutions. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	Milder increase in CR	Milder increase in k	Bigger increase in CR	Bigger increase in k
I	0.862	0.856	0.862	0.770	0.880
LIQ	0.052	0.064	0.055	0.224	0.019
D	0.875	0.880	0.876	0.940	0.841
E	0.038	0.040	0.040	0.053	0.058
r_I	3.097	3.106	3.099	3.213	3.077
$\bar{r}_D$	0.717	0.713	0.711	0.703	0.605
q	0.407	0.405	0.406	0.387	0.405
q_f	0.200	0.196	0.199	0.142	0.204
ℓ	0.057	0.069	0.060	0.225	0.021
k	0.042	0.044	0.044	0.053	0.065
CR	0.045	0.047	0.047	0.069	0.066
$r_I-(1-\delta)\bar{r}_D$	2.739	2.749	2.743	2.862	2.774
$I+LIQ$	0.914	0.920	0.916	0.993	0.899
$I-E$	0.824	0.816	0.821	0.717	0.821
$E(Div)$	0.745	0.749	0.749	0.779	0.791
$\Delta\mathbb{U}_E$	-	-0.13%	-0.01%	-2.01%	0.51%
$\Delta\mathbb{U}_S$	-	0.17%	0.02%	2.68%	-1.23%
$\Delta\mathbb{U}_B$	-	-0.01%	-0.01%	-0.61%	-1.24%
$\Delta\mathbb{S}_{sp}$	-	0.03%	0.01%	0.05%	-1.96%

Table 3: Single capital regulations. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	ℓ	LCR	$NSFR$
I	0.862	0.746	0.747	0.770
LIQ	0.052	0.258	0.257	0.217
D	0.875	0.959	0.959	0.942
E	0.038	0.045	0.045	0.044
r_I	3.097	3.236	3.236	3.211
$\bar{r}_D$	0.717	0.746	0.745	0.736
q	0.407	0.385	0.385	0.390
q_f	0.200	0.131	0.131	0.146
ℓ	0.057	0.257	0.256	0.220
k	0.042	0.045	0.045	0.045
LCR	0.069	0.277	0.276	0.238
$NSFR$	0.552	0.703	0.702	0.670
$r_I - (1-\delta)\bar{r}_D$	2.739	2.863	2.863	2.843
$I + LIQ$	0.914	1.004	1.004	0.987
$I - E$	0.824	0.701	0.701	0.726
$E(Div)$	0.745	0.756	0.757	0.756
$\Delta\mathbb{U}_E$	-	-2.50%	-2.49%	-2.04%
$\Delta\mathbb{U}_S$	-	3.50%	3.48%	2.75%
$\Delta\mathbb{U}_B$	-	-0.51%	-0.51%	-0.35%
$\Delta\mathbb{S}_{sp}$	-	0.49%	0.49%	0.36%

Table 4: Single liquidity regulations. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	CR	CR & ℓ	CR, ℓ & τ_D
I	0.862	0.852	0.828	0.841
LIQ	0.052	0.073	0.113	0.119
D	0.875	0.883	0.901	0.919
E	0.038	0.042	0.040	0.041
r_I	3.097	3.112	3.142	3.131
$\bar{r}_D$	0.717	0.710	0.723	0.778
q	0.407	0.404	0.401	0.398
q_f	0.200	0.193	0.181	0.180
ℓ	0.057	0.078	0.120	0.124
k	0.042	0.045	0.043	0.043
$r_I-(1-\delta)\bar{r}_D$	2.739	2.757	2.780	2.742
$I+LIQ$	0.914	0.925	0.941	0.960
$I-E$	0.824	0.811	0.788	0.800
$E(Div)$	0.745	0.752	0.749	0.750
$\Delta\mathbb{U}_E$	-	-0.23%	-0.80%	-0.44%
$\Delta\mathbb{U}_S$	-	0.30%	1.00%	1.74%
$\Delta\mathbb{U}_B$	-	-0.02%	-0.06%	-0.13%
$\Delta\mathbb{S}_{sp}$	-	0.04%	0.14%	1.18%

Table 5: Implementation of the planner's solution for $w_E = 0.05$ and $w_S = 0.15$. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

	PE	k & τ_D	τ_D & τ_I	k, τ_D & τ_I
I	0.862	0.859	0.900	0.899
LIQ	0.052	0.070	0.010	0.012
D	0.875	0.890	0.873	0.873
E	0.038	0.039	0.037	0.038
r_I	3.097	3.104	3.050	3.051
$\bar{r}_D$	0.717	0.742	0.764	0.761
q	0.407	0.404	0.408	0.407
q_f	0.200	0.195	0.212	0.211
ℓ	0.057	0.075	0.011	0.013
k	0.042	0.042	0.041	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.733	2.668	2.670
$I + LIQ$	0.914	0.929	0.910	0.911
$I - E$	0.824	0.820	0.862	0.861
$E(Div)$	0.745	0.746	0.742	0.743
$\Delta\mathbb{U}_E$	-	-0.05%	1.02%	1.02%
$\Delta\mathbb{U}_S$	-	0.55%	-0.11%	-0.10%
$\Delta\mathbb{U}_B$	-	-0.02%	-0.08%	-0.08%
$\Delta\mathbb{S}_{sp}$	-	0.49%	0.84%	0.84%

Table 6: Implementation of the planner's solution for $w_E = 0.15$ and $w_S = 0.05$. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

Optimal Bank Regulation In the Presence of Credit and Run Risk

Anil K Kashyap Dimitrios P. Tsomocos Alexandros P. Vardoulakis

Online Appendix

Appendix A reports the proofs to propositions, lemmas and corollaries in the paper, while appendix B reports additional derivations and extensions.

A Proofs

A.1 Proof of Lemma 1

First, consider that $\xi < \hat{\xi} \equiv (\delta D(1+r_D) - LIQ)/I$. Then, $\theta(\xi,1) = (LIQ + \xi \cdot I)/(D(1+r_D)) < (LIQ+\hat{\xi}I)/(D(1+r_D)) < \delta$. Also, $\hat{\lambda}(\xi)$ in (13) can be written as $[\theta(1+r_D)(1+r_I) - \xi(1+\bar{r}_D) - \xi \cdot X/(\omega D)]/((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))$, which is smaller than $\theta(\xi,1)$ as long as $\theta(\xi,1) < 1 + X/[\omega D(1+\bar{r}_D)((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))]$. So, $\hat{\lambda}(\xi) < \delta$ as well, and only the full run region is possible. Next, define ξ^{ld} as the solution to $\hat{\lambda}(\xi^{ld}) = \delta$; $\hat{\hat{\xi}}$ as the solution to $\theta(\hat{\hat{\xi}},1) = 1$, yielding $\hat{\hat{\xi}} = (D(1+r_D) - LIQ)/I$; and ξ^{ud} as the solution to $\hat{\lambda}(\xi^{ud}) = 1$, yielding $\xi^{ud} = \hat{\hat{\xi}}/(1 - X/(\omega I(1+r_I))) > \hat{\hat{\xi}}$. Moreover, $\hat{\lambda}(\hat{\hat{\xi}}) > \delta$ and $\partial\hat{\lambda}(\xi)/\partial\xi > 0$, so $\xi^{ld} < \hat{\hat{\xi}} < \xi^{ud}$. Finally, $\xi^{ud} < \bar{\xi}$, because $X < \bar{X} \equiv \omega I(1+r_I)(1-\hat{\hat{\xi}}/\bar{\xi})$. Using these observations, it is easy to establish the non-empty regions for the remaining $\xi \in [\hat{\xi}, \bar{\xi})$ in the Lemma.

A.2 Proof of Proposition 1

To Be Completed.

A.3 Proof of Corollary 1

Totally differentiating (19) we get that $\partial\xi^*/\partial z = -(\partial GG^*/\partial z)/(\partial GG^*/\partial\xi^*)$, where z can be any of I, LIQ, D, r_I, r_D, or $\bar{r}_D$. Recall that $\partial GG^*/\partial\xi > 0$ from (B.47). Then, $\partial\xi^*/\partial I < 0$, because $\partial GG^*/\partial I = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial I - \int_{\theta^*}^1 \xi^*/\lambda d\lambda = \omega D(1+\bar{r}_D)[\partial\hat{\lambda}(\xi^*)/\partial I - (\lambda^* - \delta)/I] + (\theta^* - \delta)D(1+r_D)/I + \int_{\theta^*}^1 LIQ/(\lambda I)d\lambda > 0$, from $\partial\lambda^*/\partial I - (\lambda^* - \delta)/I = [(\delta D(1+r_D) - LIQ)(1+r_I) + (1-\delta)\xi^* D(1+\bar{r}_D) + \xi^* X/\omega]/[I \cdot D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] > 0$ from Lemma 1.

Moreover, $\partial\xi^*/\partial D > 0$ because $\partial GG^*/\partial D = \omega D(1+\bar{r}_D)[\partial\lambda^*/\partial D + (\lambda^* - \delta)/D] - (\theta^* - \delta)(1+r_D) < 0$, from $\partial\lambda^*/\partial D = -\xi^*(1+\bar{r}_D)/[D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] - \lambda^*/D$.

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The partial effect of the loan rate and the early deposit rate are, respectively, negative and positive, because $\partial GG^*/\partial r_I = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial r_I = \omega D(1+\bar{r}_D)[(1+r_D)(1+\bar{r}_D)\xi^*(1-\theta^*)+\xi^* X/\omega]/[((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)] > 0$ and $\partial GG^*/\partial r_D = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial r_D - (\theta^*-\delta)D = -\omega D(1+\bar{r}_D)\lambda^*(1+r_I)/[((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)] - (\theta^*-\delta)D < 0$.

On the other hand, the sign of $\partial\xi^*/\partial LIQ < 0$ is ambiguous, because $\partial GG^*/\partial LIQ = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial LIQ - \int_{\theta^*}^{1} 1/\lambda d\lambda = \omega D(1+\bar{r}_D)\cdot[\partial\lambda^*/\partial LIQ - (\lambda^*-\delta)/LIQ] + (\theta^*-\delta)D(1+r_D)/LIQ + \int_{\theta^*}^{1}\xi I/(\lambda LIQ)d\lambda$ and we cannot unambiguously sign $\partial\lambda^*/\partial LIQ - (\lambda^*-\delta)/LIQ = [(\delta D(1+r_D) - \xi^* I)(1+r_I) + (1-\delta)\xi^* D(1+\bar{r}_D) + \xi^* X/\omega]/[I\cdot D((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)]$.

Finally, the sign of $\partial\xi^*/\partial\bar{r}_D$ is also ambiguous, because $\partial GG^*/\partial\bar{r}_D = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial\bar{r}_D + \omega D(\lambda^*-\delta) = \omega D[(\lambda^*-\delta)(1+r_D)(1+r_I) - (1-\delta)\xi^*(1+r_3^D)]/[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)]$, which cannot be unambiguously signed.

A.4 Proof of Proposition 2

To Be Completed.

A.5 Proof of Corollary 3

Given that $c(0)=0$ and $c'(\cdot)>0$, $c''(\cdot)=0$ implies that $c(x)=a_c\cdot x$, with $a_c>0$. Then, $\partial\mathbb{U}_E^*/\partial\xi^* = -[c'(I)I - c(I)]/\Delta_\xi = 0$. Moreover, the surplus to E, $(1-q)c''(I)I$, is zero. Similarly, given that $V(0)=0$ and $V'(\cdot)>0$, $V''(\cdot)=0$ implies that $V(x)=a_v\cdot x$, with $a_v>0$. Then, $\partial\mathbb{U}_S^*/\partial\xi^* = -[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)]/\Delta_\xi = 0$. Moreover, the surplus from the transaction services of deposits, $(1-q)V''(D(1+r_D))D(1+r_D)^2$, is zero. Finally, the surplus in terms of period 1 utility, $U''(e_R-D)D$, is zero for $U''(\cdot)=0$ as well as for $LIQ_S>0$, because then $\mathbb{U}_S^* = \mathbb{U}_S^\alpha$ given that $V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D) = 0$ from condition (ii). In turn, $LIQ_S>0$ if savers endowment is higher than some threshold $\bar{e}_S$ at which (3) holds with equality.

A.6 Proof of Proposition 3

First, set $X=0$ to make the determination of the run threshold in (20) scale invariant. Dividing it by the balance sheet size, $E+D$ (or $I+LIQ$), (20) becomes:

$$GG_{BS} = \int_{\delta}^{\hat{\lambda}} \omega(1-k)(1+\bar{r}_D)d\lambda - \int_{\delta}^{\theta^*}(1-k)(1+r_D) - \int_{\theta^*}^{1}\frac{\xi^*(1-\ell)+\ell}{\lambda}d\lambda = 0, \tag{A.1}$$

where

$$\hat{\lambda}_{BS} = \frac{(\xi^*(1-\ell)+\ell)(1+r_I) - \xi^*((1-k)(1+\bar{r}_D))}{(1-k)[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)]}. \tag{A.2}$$

Thus, k affects the payoff differential in a partial run as well as the range that monitoring occurs, $\hat{\lambda}-\delta$, via its effect on bank profitability. Totally differentiating (A.1) with respect to k, while keeping ℓ, r_I, r_D and $\bar{r}_D$ constant we get:

$$\frac{\partial GG_{BS}}{\partial k} = \underbrace{\frac{\partial\hat{\lambda}}{\partial k}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{(\hat{\lambda}-\delta)[\omega(1+\bar{r}_D)-(1+r_D)]}_{\substack{\text{Lower payoff}\\ \text{given monitoring}}} + \underbrace{(\theta^*-\hat{\lambda})(1+r_D)}_{\substack{\text{'Higher' payoff}\\ \text{absent monitoring}}}, \tag{A.3}$$

where $\partial\hat{\lambda}/\partial k > 0$. Hence, the trade-off from setting a higher requirement $k \geq \bar{k}$ is that monitoring becomes more probable, but the payoff to depositors is smaller given monitoring. Combining the two effects we get that

$$\frac{\partial GG_{BS}}{\partial k} = \left[\frac{\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)} + \delta\right]\omega(1+\bar{r}_D) + (\theta^*-\delta)(1+r_D) > 0, \tag{A.4}$$

which implies that $\partial\xi^*/\partial k = -(\partial GG^*_{BS}/\partial k)/(\partial GG^*_{BS}/\partial\xi^*) < 0$, i.e., higher k reduces the run probability q.

Finally, note that $CR = k(1-\ell)$. So tightening leverage is equivalent to setting a higher capital requirement other things equal and, thus, higher CR reduces the run probability q.

A.7 Proof of Proposition 4

The proof use material described in proof A.6 above. Totally differentiating (A.1) with respect to ℓ, while keeping k, r_I, r_D and $\bar{r}_D$ constant we get:

$$\frac{\partial GG_{BS}}{\partial \ell} = \underbrace{\frac{\partial\hat{\lambda}}{\partial \ell}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{\int_{\theta^*}^{1}\frac{1-\xi^*}{\lambda}d\lambda}_{\substack{\text{Higher payoff}\\ \text{in full run}}}, \tag{A.5}$$

where $\partial\hat{\lambda}/\partial ell > 0$. Hence, the trade-off from setting a higher requirement $\ell \geq \bar{\ell}$ is that monitoring becomes more probable, but the incentives to join a full run increase. Combining the two effects we get that

$$\frac{\partial GG_{BS}}{\partial \ell} = (1-\xi^*)\left[\frac{\omega(1+\bar{r}_D)(1+r_I)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)} + \log\theta^*\right], \tag{A.6}$$

which is definitely positive if $\log\theta^* > -1$ given that $\omega(1+\bar{r}_D) > 1+r_D$. In turn, this is satisfied under sufficient conditions $\delta > e^{-1}$ given that $\theta^* > \delta$ or $\ell > \hat{\ell} \equiv \left[e^{-1}\cdot(1-k)(1+r_D)-\xi^*\right]/(1-\xi^*)$, which is true for high enough ξ^* in the private equilibrium.

Finally, note that $LCR = ((1-\underline{\xi})\ell+\underline{\xi})/(k(1+r_D))$ and $NSFR = (k+(1-\delta)(1-k))/(1-\ell)$. So increasing ℓ is equivalent to increasing LCR or $NSFR$ other things equal and, thus, higher LCR or $NSFR$ reduce the run probability q.

B Extensions and Additional Derivations

B.1 Intermediation Margins in Private Equilibrium

The first-order conditions (23) together with the four constraints in $\mathcal{Y}$ can be combined to characterize the private equilibrium as follows. If (23) gives an interior $r_D > 0$, then it is used it to determine r_D as a function of all other variables in $\mathcal{C}$; otherwise, set $r_D = 0$. Then, use (2) (7), (9) and (20) to express (implicitly) $\bar{r}_D$, r_I, E, and ξ^* in terms of I, LIQ and D. The next steps is the express the shadow values on the four constraints $\mathcal{Y}$ in terms of I, LIQ, and D. The shadow value of funds is determined by the first-condition with respect to E,

$$\psi_{BS} = W'(e_B + D - I - LIQ), \tag{B.7}$$

where we have substituted $E = I + LIQ - D$.

The shadow value on the deposit supply schedule can be obtained from (23) with respect to $\bar{r}_D$, which yields

$$\psi_{DS} = -\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \psi_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}. \tag{B.8}$$

The choice of $\bar{r}_D$ matters for the banker via the effect on profits and on the run dynamics. The shadow value determined in (B.8) captures the sum of these effect as the deposit rate moves along the deposit supply schedule. Because the three variables of interest –I, LIQ, and D– affect the loan demand directly as well as indirectly via $\bar{r}_D$, their overall effect on the deposit supply will be scaled by the shadow value ψ_{DS} in their respective first-order conditions.

The shadow value on the loan demand schedule can be obtained from (23) with respect to r_I, which yields

$$\psi_{LD} = -\left(\frac{\partial \mathbb{U}_B}{\partial r_I} + \psi_{GG}\frac{\partial GG}{\partial r_I}\right)\frac{\partial LD}{\partial r_I}^{-1}. \tag{B.9}$$

Similar to (B.8), condition (B.9) says that the shadow value on the loan demand is measured by how a change in loan rate along the loan demand schedule affects banker's utility.

Equivalently, combining (23) for $\mathcal{C} = \xi^*$, (B.8) and (B.8), the shadow value on the global game constraint is given by

$$\psi_{GG} = -\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)\cdot\frac{dGG}{d\xi^*}^{-1}, \tag{B.10}$$

where $dGG/d\xi^*$ is the total effect of the run threshold ξ^* on the utility differential determining the run behavior, which captures the partial direct effect –$\partial GG/\partial \xi^*$ in (B.47)–and the partial indirect effects via the deposit and loan rate:

$$\frac{dGG}{d\xi^*} = \frac{\partial GG}{\partial \xi^*} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}. \tag{B.11}$$

Overall, ψ_{GG} measures the effect of a change in the run threshold, which is consistent with optimal run behavior, i.e., along the global game constraint, on B's welfare.

Combining (23) with respect to LIQ and I, and substituting in (B.8), (B.9) and (B.10), we obtain the asset allocation margin in the private equilibrium (AAM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial LIQ} - \frac{\partial \mathbb{U}_B}{\partial I} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I} \right) - \frac{\partial \mathbb{U}_B}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I} \right)}^{\text{Effect of asset mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dLIQ - d\mathbb{U}_B/dI}$$
$$+ \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \frac{\partial LD}{\partial \xi^*} \right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*: \ d\mathbb{U}_B/d\xi^*} \cdot \underbrace{\left(\frac{\partial \xi^*}{\partial LIQ} - \frac{d\xi^*}{dI} \right)}_{\text{Effect of asset mix on } \xi^*} = 0, \tag{B.12}$$

where $d\xi^*/dLIQ$ and $d\xi^*/dI$ are obtained from total differentiation of (20), hence

$$\frac{d\xi^*}{dLIQ} - \frac{d\xi^*}{dI} = -\left[\frac{\partial GG}{\partial LIQ} - \frac{\partial GG}{\partial I} - \frac{\partial GG}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I} \right) - \frac{\partial GG}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I} \right) \right] \cdot \frac{dGG}{d\xi^*}^{-1}. \tag{B.13}$$

The asset allocation margin in (B.12) captures the decision to substitute a unit of loans with a unit of liquid assets. The banker in the private equilibrium weighs the effect of the change in the asset mix on the bank profitability (the first line) and on the run threshold, which determines run risk, because both affect her welfare (the second line). The asset mix matters for bank profits due to portfolio effects (first two terms in first line), but also due to the way it will influence the profit margin via the loan rate and deposit rates (remaining terms in first line). The latter (general equilibrium) effect via rates captures how the asset mix matters for the loan rate or deposit rate entrepreneurs and depositors are willing to accept. Similarly, the asset mix changes the payoffs governing the run dynamics directly and indirectly via the loan and deposit rates (captured by (B.13)), which in turn affect the run threshold influencing B's welfare directly and indirectly via the loan and deposit rates.

Similarly, combining (23) with respect to E and D, and substituting in (B.8), (B.9) and (B.10), we obtain the capital structure margin in the private equilibrium (CSM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial E} - \frac{\partial \mathbb{U}_B}{\partial D} + \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial D} + \frac{\partial \mathbb{U}_B}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \frac{\partial LD}{\partial D}}^{\text{Effect of liabilities mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dE - d\mathbb{U}_B/dD}$$
$$- \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \frac{\partial LD}{\partial \xi^*} \right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*: \ d\mathbb{U}_B/d\xi^*} \cdot \underbrace{\frac{d\xi^*}{dD}}_{\substack{\text{Effect of liabilities} \\ \text{mix on } \xi^*}} = 0, \tag{B.14}$$

where

$$\frac{d\xi^*}{dD} = -\left[\frac{\partial GG}{\partial D} - \frac{\partial GG}{\partial \bar{r}_D} \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial D} - \frac{\partial GG}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \frac{\partial LD}{\partial D} \right] \cdot \frac{dGG}{d\xi^*}^{-1}. \tag{B.15}$$

Finally, combining (23) with respect to I and D, and substituting in (B.8), (B.9) and (B.10), we obtain the margin for the scale of intermediation in the private equilibrium (SIM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial I}+\frac{\partial \mathbb{U}_B}{\partial D}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I}+\frac{\partial DS}{\partial D}\right)-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I}+\frac{\partial LD}{\partial D}\right)}^{\text{Effect of intermediation scale on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dI+d\mathbb{U}_B/dD}$$

$$+\underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*:\ d\mathbb{U}_B/d\xi^*}\cdot\underbrace{\left(\frac{d\xi^*}{dI}+\frac{d\xi^*}{dD}\right)}_{\substack{\text{Effect of intermediation}\\ \text{scale on } \xi^*}}=0, \tag{B.16}$$

where

$$\frac{d\xi^*}{dI}+\frac{d\xi^*}{dD}=-\left[\frac{\partial GG}{\partial I}+\frac{\partial GG}{\partial D}-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I}+\frac{\partial DS}{\partial D}\right)-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I}+\frac{\partial LD}{\partial D}\right)\right]\cdot\frac{dGG}{d\xi^*}^{-1}. \tag{B.17}$$

Note that expanding the first two terms in (B.16) we get that

$$\frac{\partial \mathbb{U}_B}{\partial I}+\frac{\partial \mathbb{U}_B}{\partial D}=\omega\left\{[(1-q)-\delta(1+r_2^D)log(\overline{\xi}/\xi^*)/\Delta_\xi](1+r^I)-(1-\delta)(1+r_3^D)\right\}, \tag{B.18}$$

where q is the run probability. Hence, the third margin capturing the scale of intermediation can be proxied by the intermediation spread between the loan rate, r_I, and the late deposit rate, $\bar{r}_D$.

The three intermediation margins pin down the three free variables I, LIQ and D. The remaining variables, E, ξ^*, r_I, and $\bar{r}_D$, are implicitly functions of the three free variables via constraints (9), (20), (7) and (2), which are always binding in equilibrium . Hence, there are three degrees of freedom and the private equilibrium is characterized by (B.12), (B.14) and (B.16).

B.2 Intermediation Margins in Social Planner's Equilibrium

To Be Completed.

B.3 Incomplete Deposit Contracts and Lack of Commitment

To Be Completed.

B.4 Loan Market and Price-Taking Behavior

To Be Completed.

B.5 Tools-augmented planner

We consider a tools-augmented planner who is endowed with the set of tools $\mathcal{T} \in \mathbb{T}$ and want to replicated the social planner's allocations $\mathcal{C}_{sp}$, as a private equilibrium. The tools-augmented planner's problem is akin to a Ramsey planner's problem in the public finance literature.

For each $\mathcal{T} \in \mathbb{T}$ there is a regulatory constraint $RC(\mathcal{T},\mathcal{C}) \geq 0$, which ties the tool with the endogenous variables $\mathcal{C}$ or an additional term in B's utility, $\mathbb{U}_B(\mathcal{T},\mathcal{C})$ in the case the tool is a Pigouvian tax imposed directly on the payoffs to B. It is important to note that the regulatory constraints, RC, are defined as inequalities, i.e., the planner can tighten them, but not loosen them, while there are no restrictions on Pigouvian taxes which can be positive or negative. Let $\psi_{\mathcal{T}}$ be the multipliers that the banker in the private equilibrium assigns to constraint $RC(\mathcal{T},\mathcal{C}) \geq 0$.

Under regulation, the optimization margins change to:

$$AAM_{\mathbb{T}} : AAM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}} \left[\frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial LIQ} - \frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial I} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial LIQ} - \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial I} \right\} = 0, \tag{B.19}$$

$$CSM_{\mathbb{T}} : CSM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}} \left[\frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial E} - \frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial E} - \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial D} \right\} = 0, \tag{B.20}$$

$$SIM_{\mathbb{T}} : SIM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}} \left[\frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial I} + \frac{\partial RC(\mathcal{T},\mathcal{C})}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial I} + \frac{\partial \mathbb{U}_B(\mathcal{T},\mathcal{C})}{\partial D} \right\} = 0. \tag{B.21}$$

We will show that in order to implement the equilibrium allocations of the social planner, denoted by $\mathcal{C}_{sp}$, it is not necessary to solve the full problem of the tools-augmented planner. Instead, it suffices that the there are tools, $\mathcal{T}$, that first satisfy the regulatory constraints $RC(\mathcal{T},\mathcal{C}_{sp}) = 0$ at the planner's allocations, and, second, the intermediation margins in the associated equilibrium are the same as the intermediation margins of the planner. Essentially, this means that the additional terms in (B.19), (B.20) and (B.21) need to equal the wedges derived in (30), (31) and (32). In matrix form, this can be written as:

$$\Delta RC(\mathcal{T},\mathcal{C}_{sp}) \cdot \Psi_{\mathcal{T}} + \Delta \mathbb{U}_B(\mathcal{T},\mathcal{C}_{sp}) \cdot 1_{\mathcal{T}} = WD_{sp}, \tag{B.22}$$

where $\Psi_{\mathcal{T}}$ is the Tx1 vector of the multiplier on the $\mathcal{T}$ regulatory constraints, $1_{\mathcal{T}}$ is a Tx1 unit vector, WD_{sp} is the 3x1 vector of the wedges in the three intermediation margins evaluated at the planner's equilibrium values, $\Delta RC(\mathcal{T},\mathcal{C}_{sp})$ is the 3xT matrix of the partial derivatives of the relevant variables for each intermediation margin on the $\mathcal{T}$ regulatory constraints, and $\Delta \mathbb{U}_B(\mathcal{T},\mathcal{C}_{sp})$ is the 3xT matrix of the partial derivatives of the utility terms introduced by Pigouvian taxation with respect to the relevant variables for each intermediation margin.

Hence, it suffices to find (only) three tools such that, first, the matrix $\Delta RC(\mathcal{T},\mathcal{C}_{sp})$ is invertible, and, second, all elements in $\Psi_{\mathcal{T}}$ are positive. Invertibility, is trivially satisfied under three Pigouvian taxation tool that can be chosen independently such that $\Delta \mathbb{U}_B(\mathcal{T},\mathcal{C}_{sp}) \cdot 1_{\mathcal{T}} = WD_{sp}$. But, three regulatory (capital or liquidity) tools may not be linearly independent because choosing two of them may replicate the value of the third. Hence, the matrix may not be invertible if we do not include Pigouvian tools, i.e., $\Delta\mathbb{U}_B(\mathcal{T},\mathcal{C}_{sp})=0$, which is actually the case in Section 4.3. That is the reason we combine a capital and a liquidity tool with a subsidy on deposit interest expenses. Moreover, the regulatory tools need to be jointly binding, because the constraints RC are inequalities. For example, a capital and a liquidity tool can be jointly binding, but two liquidity tools cannot. This is not the case for Pigouvian taxes as we do not impose any restriction on their sign.

We now show that (B.22) is (generically) a necessary and sufficient condition such that the social planner's solution described in section 3.1 can be decentralized as a private equilibrium by using regulatory tools $\mathcal{T}\in\mathbb{T}$. The tools-augmented planner not only chooses optimally allocations $\mathcal{C}$, but also the level of tools $\mathcal{T}\in\mathbb{T}$ and the multipliers $\psi_{\mathcal{T}}$, which are the shadow values that the bank assigns to constraints $RC(\mathcal{T},\mathcal{C})\geq 0$ in the new equilibrium. Her problem is:

$$\max_{\mathcal{C},\mathcal{T},\psi_{\mathcal{T}}} \quad \mathbb{U}^*_{\text{sp}} \quad s.t. \;\; \mathcal{Y}(\mathcal{C})=0,\;\; RC(\mathcal{T},\mathcal{C})\geq 0,\;\; AAM_{\mathcal{T}}(\mathcal{T},\mathcal{C},\Psi_{\mathcal{T}})=0, CSM_{\mathcal{T}}(\mathcal{T},\mathcal{C},\Psi_{\mathcal{T}})=0,$$
$$SIM_{\mathcal{T}}(\mathcal{T},\mathcal{C},\Psi_{\mathcal{T}})=0. \tag{B.23}$$

Note that the additional utility terms $\mathbb{U}_B(\mathcal{T},\mathcal{C}$ due to Pigouvian taxation tools do not appear in the utility the tools-augmented planner maximizes, because she engages to equal in size lump-sum transfers.

The first-order condition with respect to $\mathcal{C}$ (similar to first-order condition (29)) are:

$$\frac{\partial\mathbb{U}_B}{\partial\mathcal{C}}+w_S\frac{\partial\mathbb{U}^*_S}{\partial\mathcal{C}}+w_E\frac{\partial\mathbb{U}^*_E}{\partial\mathcal{C}}+\sum_{\mathcal{Y}}\zeta_{\mathcal{Y}}\frac{\partial\mathcal{Y}}{\partial\mathcal{C}}$$
$$+\sum_{\mathcal{T}}\zeta_{\mathcal{T}}\frac{\partial RC}{\partial\mathcal{C}}+\zeta_{AAM}\frac{\partial AAM_{\mathbb{T}}}{\partial\mathcal{C}}+\zeta_{CSM}\frac{\partial CSM_{\mathbb{T}}}{\partial\mathcal{C}}+\zeta_{SIM}\frac{\partial SIM_{\mathbb{T}}}{\partial\mathcal{C}}=0, \tag{B.24}$$

where $\zeta_{\mathcal{T}}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM} are the multipliers the tool-augmented planner assigns to regulatory constraints and the three regulation-distorted intermediation margins.

The first-order conditions with respect to the level of tools $\mathcal{T}$ are:

$$\zeta_{\mathcal{T}}\frac{\partial RC}{\partial\mathcal{T}}+\zeta_{AAM}\frac{\partial AAM_{\mathbb{T}}}{\partial\mathcal{T}}+\zeta_{CSM}\frac{\partial CSM_{\mathbb{T}}}{\partial\mathcal{T}}+\zeta_{SIM}\frac{\partial SIM_{\mathbb{T}}}{\partial\mathcal{T}}=0, \tag{B.25}$$

and choosing optimally the multipliers $\psi_{\mathcal{T}}$ yields:

$$\zeta_{AAM}\frac{\partial AAM_{\mathbb{T}}}{\partial\psi_{\mathcal{T}}}+\zeta_{CSM}\frac{\partial CSM_{\mathbb{T}}}{\partial\psi_{\mathcal{T}}}+\zeta_{SIM}\frac{\partial SIM_{\mathbb{T}}}{\partial\psi_{\mathcal{T}}}=0. \tag{B.26}$$

To prove sufficiency, (B.22) implies that there need to be three regulatory tools, including Pigouvian taxes, such that a solution to multipliers $\psi_{\mathbb{T}}$ can be obtained. This means that there are three first-order conditions of the form in (B.25) and at most three of the form in (B.26), because Pigouvian taxes are not assigned a Lagrange multiplier as they are free to be positive or negative. Then, conditions (B.26) can be written in matrix form as $transpose(\Delta RC)\cdot transpose([\zeta_{AAM},\zeta_{CSM},\zeta_{SIM}])=$

0. Given that ΔRC is invertible by the conjecture that (B.22) is sufficient, $\zeta_{AAM} = \zeta_{CSM} = \zeta_{SIM} = 0$. Thus, the only solution requires all ζ_T to be zero and the first-order conditions (B.24) coincide with the first-order conditions (29) of the social planner.

To prove necessity, suppose that (B.22) does not hold. Using conditions (B.24) we can derive intermediation margins $AAM_{TAP} = AAM_{SP} + AAM_{TAP,WD}$, $CSM_{TAP} = CSM_{SP} + CSM_{TAP,WD}$ and $SIM_{TAP} = SIM_{SP} + SIM_{TAP,WD}$ for the tool-augmented planner, where the wedges are linear combination of one multiplier $\zeta_{\mathcal{T}}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM}. The social planner's and tools-augmented planner's solutions coincide if both wedges $AAM_{TAP,WD}$, $CSM_{TAP,WD}$, and $SIM_{TAP,WD}$ are all zero, which in principle is possible because there are at least three tools, hence three degrees of freedom. However, equations (B.25) and (B.26) remove as many degrees of freedom as the number of regulatory tools degrees of freedom. Hence, it is not possible to replicate the social planner's solution with fewer that three independent tools, which leads to a contradiction.

B.6 Zero-profit entrepreneurial sector

To Be Completed.

B.7 Direct lending

To Be Completed.

B.8 Additional distortionary tools

To Be Completed.

B.9 Negative interest rates

To Be Completed.

B.10 Outside equity

To Be Completed.

B.11 Derivatives

This section reports the partial derivatives of banker's utility $\mathbb{U}_B$ in (14), the monitoring threshold $\hat{\lambda}$ in (13), the global game constraint GG in (20), the deposit supply schedule DS in (2), and the loan demand schedule LD in (7) with respect to the choice variables in $\mathbb{C}$. When it is unambiguous, we also report the sign of the derivatives.

Partial derivatives $\partial \mathbb{U}_B / \partial \mathcal{C}$.

$$\frac{\partial \mathbb{U}_B}{\partial I} = \omega(1-q)(1+r_I) > 0. \tag{B.27}$$

$$\frac{\partial \mathbb{U}_B}{\partial LIQ} = \omega(1+r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi > 0. \tag{B.28}$$

$$\frac{\partial \mathbb{U}_B}{\partial D} = -\omega\Big[\delta(1+r_D)(1+r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi + (1-\delta)(1-q)(1+\bar{r}_D)\Big] < 0. \tag{B.29}$$

$$\frac{\partial \mathbb{U}_B}{\partial E} = -W'(e_B - E) < 0. \tag{B.30}$$

$$\frac{\partial \mathbb{U}_B}{\partial \xi^*} = -\left[\omega\left((\xi^* I - \delta D(1+r_D) + LIQ)/\xi^*(1+r_I) - (1-\delta)D(1+\bar{r}_D)\right) - X\right]/\Delta_\xi < 0. \tag{B.31}$$

$$\frac{\partial \mathbb{U}_B}{\partial r_I} = \omega\Big[(1-q)I - (\delta D(1+r_D) - LIQ)\log(\bar{\xi}/\xi^*)/\Delta_\xi\Big] > 0. \tag{B.32}$$

$$\frac{\partial \mathbb{U}_B}{\partial r_D} = -\omega\delta D(1+r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi < 0. \tag{B.33}$$

$$\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} = -\omega(1-\delta)(1-q)D < 0. \tag{B.34}$$

Partial derivatives $\partial\hat{\lambda}/\partial\mathcal{C}$.

$$\frac{\partial \hat{\lambda}}{\partial I} = \xi^*(1+r_I)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] > 0. \tag{B.35}$$

$$\frac{\partial \hat{\lambda}}{\partial LIQ} = (1+r_I)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] > 0. \tag{B.36}$$

$$\frac{\partial \hat{\lambda}}{\partial D} = -\xi^*(1+\bar{r}_D)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] - \hat{\lambda}/D < 0. \tag{B.37}$$

$$\frac{\partial \hat{\lambda}}{\partial E} = 0. \tag{B.38}$$

$$\begin{aligned}\frac{\partial \hat{\lambda}}{\partial \xi^*} &= [I(1+r_I) - (D(1+\bar{r}_D) + X/\omega)]/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] \\ &\quad + \hat{\lambda}(1+\bar{r}_D)/\left[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right] > 0.\end{aligned} \tag{B.39}$$

$$\frac{\partial\hat{\lambda}}{\partial r_I} = (\xi^* I + LIQ)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right]$$
$$- \hat{\lambda}(1+r_D)/\left[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right] > 0. \tag{B.40}$$

$$\frac{\partial\hat{\lambda}}{\partial r_D} = -\hat{\lambda}(1+r_I)/\left[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right] < 0. \tag{B.41}$$

$$\frac{\partial\hat{\lambda}}{\partial \bar{r}_D} = -(1-\hat{\lambda})\xi^*/\left[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right] < 0. \tag{B.42}$$

Partial derivatives $\partial GG/\partial\mathcal{C}$ (see proof of Corollary 1 for the sign of $\partial GG^*/\partial\mathcal{C}$).

$$\frac{\partial GG}{\partial I} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial I} - \int_{\xi^*}^{1} \frac{\xi^*}{\lambda} d\lambda. \tag{B.43}$$

$$\frac{\partial GG}{\partial LIQ} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial LIQ} - \int_{\xi^*}^{1} \frac{1}{\lambda} d\lambda. \tag{B.44}$$

$$\frac{\partial GG}{\partial D} = \omega D(1+\bar{r}_D)\left[\frac{\partial\hat{\lambda}}{\partial D} + (\hat{\lambda}-\delta)/D\right] - (\theta^*-\delta)(1+r_D). \tag{B.45}$$

$$\frac{\partial GG}{\partial E} = 0. \tag{B.46}$$

$$\frac{\partial GG}{\partial \xi^*} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial \xi^*} - \int_{\xi^*}^{1} \frac{I}{\lambda} d\lambda. \tag{B.47}$$

$$\frac{\partial GG}{\partial r_I} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial r_I}. \tag{B.48}$$

$$\frac{\partial GG}{\partial r_D} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial r_D} - D(\theta^*-\delta). \tag{B.49}$$

$$\frac{\partial GG}{\partial \bar{r}_D} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial \bar{r}_D} + \omega D(\hat{\lambda}-\delta). \tag{B.50}$$

Partial derivatives $\partial DS/\partial \mathcal{C}$.

$$\frac{\partial DS}{\partial I} = [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \frac{\xi^* + \underline{\xi}}{2} \cdot \frac{1}{D} > 0. \tag{B.51}$$

$$\frac{\partial DS}{\partial LIQ} = [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \frac{1}{D} > 0. \tag{B.52}$$

$$\begin{aligned}\frac{\partial DS}{\partial D} &= U''(e_S - D) - [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \left(LIQ + I \cdot \frac{\xi^* + \underline{\xi}}{2}\right) \frac{1}{D^2} \\ &+ (1-q)V''(D(1+r_D))(1+r_D)^2 < 0.\end{aligned} \tag{B.53}$$

$$\frac{\partial DS}{\partial E} = 0. \tag{B.54}$$

$$\begin{aligned}\frac{\partial DS}{\partial \xi^*} &= \Big\{[\beta\delta + \beta^2(1-\beta)]\frac{LIQ + \xi^* I}{D} - \delta\beta(1+r_D) \\ &-(1-\delta)\beta^2\omega(1+r_D) - V'(D(1+r_D))(1+r_D)\Big\}\Delta_\xi^{-1} < 0.\end{aligned} \tag{B.55}$$

$$\frac{\partial DS}{\partial r_I} = 0. \tag{B.56}$$

$$\frac{\partial DS}{\partial r_D} = (1-q)\left[\beta\delta + V'(D(1+r_D)) + V''(D(1+r_D))D(1+r_D)\right] > 0. \tag{B.57}$$

$$\frac{\partial DS}{\partial \bar{r}_D} = \omega \cdot \beta^2(1-\delta)(1-q) > 0. \tag{B.58}$$

Partial derivatives $\partial LD/\partial \mathcal{C}$.

$$\frac{\partial LD}{\partial I} = \omega(A - (1+r_I))\frac{\delta D(1+r_D) - LIQ}{I^2}\frac{\log(\overline{\xi}/\xi^*)}{\Delta_\xi} - (1-q)c''(I) \lesseqgtr 0. \tag{B.59}$$

$$\frac{\partial LD}{\partial LIQ} = \omega(A - (1+r_I))\frac{1}{I}\frac{\log(\overline{\xi}/\xi^*)}{\Delta_\xi} > 0. \tag{B.60}$$

$$\frac{\partial LD}{\partial D} = -\omega(A-(1+r))\frac{\delta(1+r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.61}$$

$$\frac{\partial LD}{\partial E} = 0. \tag{B.62}$$

$$\frac{\partial LD}{\partial \xi^*} = -\left[\omega(A-(1+r_I))\frac{\xi^* I - \delta D(1+r_D) + LIQ}{\xi^* I} - c'(I)\right]\Delta_\xi^{-1} \gtrless 0. \tag{B.63}$$

$$\frac{\partial LD}{\partial r_I} = -\omega\left[(1-q) - \frac{\delta D(1+r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi}\right] < 0. \tag{B.64}$$

$$\frac{\partial LD}{\partial r_D} = -\omega(A-(1+r))\frac{\delta D}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.65}$$

$$\frac{\partial LD}{\partial \bar{r}_D} = 0. \tag{B.66}$$