University of Wisconsin

Microeconomics Prelim Exam

Friday, June 27, 2011: 9AM - 2PM

Questions and Solutions

- There are four parts to the exam. All four parts have equal weight.
- Answer all questions. No questions are optional.
- Hand in 12 pages, written on only one side.
- Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
- Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV).
- Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
- You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
- Please return any unused portions of yellow tablets and question sheets.
- There are six pages on this exam, including this one. Make sure you have all of them.

Part I — Ireland

Consider a pure exchange economy with three consumers (labeled 1, 2 and 3) and three goods (labeled x, y and z). Agent i's consumption vector is (x_i, y_i, z_i). Each agent is endowed with only one type of good: $\omega_1 = (0, 0, 3)$, $\omega_2 = (1, 0, 0)$, and $\omega_3 = (0, 2, 0)$, where ω_i is i's endowment. The consumers' preferences can be represented by utility functions, as follows:

$$
\begin{aligned}
U_1 &= \log(y_1) + \log(z_1) \\
U_2 &= \sqrt{y_2 z_2} \\
U_3 &= x_3 y_3 z_3
\end{aligned}
$$

1. Find a competitive equilibrium. Is it unique?
2. Find the set of Pareto optimal allocations.

Solution: The Pareto set can be characterized as follows. Assign all of the x good to person 3, and forget that good from now on (this ignores the trivial case in which $y_3 z_3 = 0$, so that any division of x is Pareto optimal).

Pick three positive numbers for y_i, summing to 2. Let m be the MRS between y and z. Then efficiency requires

$$
\begin{aligned}
z_1 &= y_1 m \\
z_2 &= y_2 m \\
z_3 &= y_3 m
\end{aligned}
$$

Summing these gives

$$m = \frac{3}{2}$$

So the set of Pareto optima is the set of allocations such that the proportion of the aggregate endowment of the y good consumed by each person is the same as the proportion of the aggregate endowment of the z good consumed by that person.

Let p be the price of y per unit of z, and take z as the numeraire. By the first welfare theorem, $p = \frac{3}{2}$. The budget constraint for 1 is $py_1 + z_1 = 3$, and half of 1's income is spent on each good. This implies $y_1 = 1$ and $z_1 = \frac{3}{2}$.

The budget constraint for 3 is $p_x x_3 + py_3 + z_3 = 2p$, and one third of 3's income is spent on each good. This implies $y_3 = \frac{2}{3}$ and $z_3 = 1$ and $p_x = 1$ (to clear the market for good 3).

The budget constraint for 2 is $py_2 + z_2 = p_x$, and half of 2's income is spent on each good. This implies $y_2 = \frac{1}{3}$ and $z_2 = \frac{1}{2}$.

	x	y	z
p	3	$\frac{3}{2}$	1
1	0	1	$\frac{3}{2}$
2	0	$\frac{1}{3}$	$\frac{1}{2}$
3	1	$\frac{2}{3}$	1
ω	1	2	3

This is the unique competitive equilibrium.

Part II — Canada

1. Mary Zap spends all her budget, and consumes no inferior goods (where consumption falls with income).What is an upper bound on the share of her budget allocated to a good with income elasticity 4?

   Solution: By Engel aggregation, the budget-share-weighted average of income elasticities is one. So if there are no inferior goods, then all budget-shares $\times$ *income elasticities are at most one. And thus, at most* $1/4$ *of her budget can be allocated to a good with income elasticity 4.*

2. Rockafellar (or Rocky, as his friends call him) has strictly convex preferences over leisure and one other composite consumption good. Rocky's only source of income is a flex-hours job, with a constant hourly wage rate, for however many hours he works. If Rocky faces a *regressive* income tax (i.e. the percentage tax rate he pays is absolutely lower at greater incomes), could he ever be indifferent about two levels of leisure? What if he has a *progressive* income tax?

   Solution: Consider the two good world, with leisure time on the horizontal axis, and the composite consumption good on the vertical. Rocky's endowment point is on the horizontal axis, at the 24 hour point (say). With a regressive income tax, the budget set is non-convex, and you can draw strictly convex indifference curves with two tangencies even with strictly convex preferences — working a little or a lot. But a progressive tax produces a convex budget set, and thus a unique optimum.

3. Assume that Leia obeys the axioms of the von Neumann Morgernstern Expected Utility Theorem, and has the increasing and continuous (Bernoulli) utility function u. Assume that Luke's utility of any lottery with prizes x_i having chances p_i is $\sum_i p_i u(x_i) + [\sum_i p_i u(x_i)]^3$. How do their attitudes to risk compare?

   Solution: By assumption, Leia's expected utility for any lottery (p_i) *is* $\sum_i p_i u(x_i)$. *Luke's utility for any lottery is a strictly increasing function of Leia's. Thus, he will choose lotteries in the same way that Leia will, and their attitudes to risk coincide.*

4. An economy has just three firms f_1, f_2, f_3 and just three employees e_1, e_2, e_3, and pairwise production as specified below. When a match of firm f_i and employee e_j occurs, the output is split among the pair,

yielding profit π_i to firm i and wage w_j to employee j. A firm can only employ one employee and an employee can only work for one firm.

	f_1	f_2	f_3
e_1	10	0	0
e_2	15	10	0
e_3	17	14	10

Find the efficient matching. Then find the tightest possible upper and lower bounds on the profit of firm 2 in a competitive equilibrium.

Solution: Checking all possible pairings, we see that assortative matching is optimal, with firm f_i paired with employee e_i, for $i = 1, 2, 3$. In this case, we have $\pi_1 + w_1 = \pi_2 + w_2 = \pi_3 + w_3 = 10$. But since other matches are not mutually desired, we have $\pi_1 + w_2 \geq 15$ and $\pi_1 + w_3 \geq 17$ and $\pi_2 + w_3 \geq 14$. Together, these inequality imply:

- $\pi_2 \geq 14 - w_3 = \pi_3 + 4 \geq 4$
- $w_2 \geq 15 - \pi_1 = w_1 + 5 \geq 5$ *and thus* $\pi_2 = 10 - w_2 \leq 5$.

Part III — U.S.A.

For questions 1 and 2 below, consider the following interaction between a police officer (player 1) and a motorist (player 2). At the start of the interaction, the police officer observes the motorist speeding. The officer chooses between leaving the motorist alone (L), pulling the motorist over to give her a ticket (T), or pulling the motorist over to arrest her (A). If the officer leaves the motorist alone, then the game ends. If not, then the motorist must decide whether to accept her penalty (a) or drive away (d). When the motorist makes this decision, she only knows that the officer has pulled her over, but cannot tell whether the officer intends to ticket her or arrest her. Whenever the motorist drives away, she is equally likely to get caught (C) and to escape (E). Payoffs are as follows:

- If the officer leaves the motorist alone, then both players receive a payoff of 0.
- If the officer pulls the motorist over to give her a ticket and the motorist accepts the penalty, then the officer gets 3 and the motorist gets -5.
- Arresting the motorist means extra paperwork for the police officer. Therefore, if the officer pulls the motorist over to arrest her and the motorist accepts the penalty, then the officer gets a payoff of just 2, while the motorist gets -10.
- If the officer pulls the motorist over and the motorist drives away and is caught, then the officer gets a payoff of 5, while the motorist gets -15.
- If the officer pulls the motorist over and the motorist drives away and escapes, then the motorist gets 0; the officer gets -10 if he was pulling the motorist over to ticket her, and -11 if he was pulling her over to arrest her (since he will still have to do the extra paperwork).

1. Construct two extensive form representations, Γ and Γ', of this interaction such that (a) Γ and Γ' differ only in the presentation of player 1's decision, and (b) the sets of subgame perfect equilibrium outcomes of Γ and Γ' are different. (In doing so, you should compute the sets of subgame perfect equilibria for the two games.)

2. Compute the sequential equilibria of Γ and Γ'. Are the sets of sequential equilibrium outcomes of the two games the same?

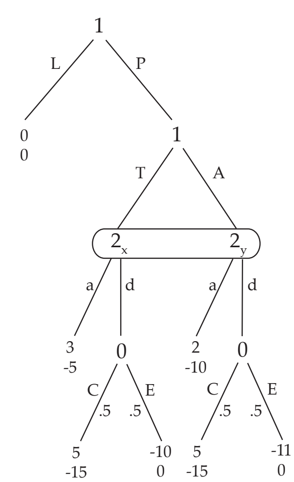
3. Consider the normal form game G below. Assume that $a_i - b_i - c_i + d_i \neq 0$ for $i \in \{1, 2\}$, and that G has a unique completely mixed equilibrium, σ. (The game may have additional equilibria that involve pure strategies.)

		2	
		L	R
1	T	a_1, a_2	b_1, b_2
	B	c_1, c_2	d_1, d_2

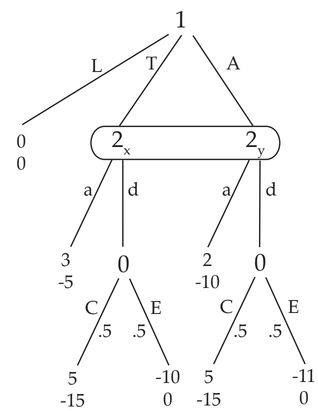
Suppose we increase payoff entry a_1 in such a way that the sign of $a_1 - c_1$ does not change. Call the resulting game G'. Explain why G' must also have a unique completely mixed equilibrium. Denote this equilibrium by σ'. Specify which components of σ' differ from those of σ. For those that differ, specify the directions of the changes in values as functions of the game's payoff parameters.

Solution:

1. *One version of the game is below. In the subgame, after we collapse the moves by Nature, T is dominant for player 1, so player 2 plays a. Thus 1 plays P at his initial node. So $((P, T), a)$ is the unique subgame perfect equilibrium.*



Here is another version of the game. In this version, player 1's choice between T and A does not begin its own subgame.



Once we collapse the moves by Nature, A is dominated by T. If player 1 puts positive probability on T, then player 2 plays a, so player 1 prefers T. Thus (T, a) is a subgame perfect equilibrium. On the other hand, if player 1 plays L, then player 2 is indifferent; if she puts at least probability $\frac{6}{11}$ on d, then L is optimal for 1. Thus $(L, \alpha a + (1 - \alpha)d)$ with $\alpha \leq \frac{5}{11}$ are subgame perfect equilibria.

2. *Since the first game from part (i) has a unique subgame perfect equilibrium, it is also the unique sequential equilibrium (with $\mu(x) = 1$).*

   In the second game from part (i), all of the subgame perfect equilibria are sequential equilibria. The first equilibrium again has $\mu(x) = 1$. For the second component of equilibria, we need $\mu(x) \leq \frac{1}{2}$ for player 2 to play d with probability 1, and we need $\mu(x) = \frac{1}{2}$ for player 2 to randomize.

   Thus, the sequential equilibrium outcomes of the two games are different.

3. *In the completely mixed equilibrium σ of G, player 1 must be indifferent between T and B, implying that*

$$\sigma_2(L) = \frac{d_1 - b_1}{a_1 - b_1 - c_1 + d_1}. \qquad (\dagger)$$

For this number to be in $(0, 1)$*, it must be that* $\text{sgn}(a_1 - c_1) = \text{sgn}(d_1 - b_1)$*. (In other words, player 1 does not have a weakly dominant strategy.) It follows that the numerator and denominator of (†) have the same sign.*

When a_1 *is increased to* a_1'*, there is no effect on player 1's mixed equilibrium strategy:* $\sigma_1' = \sigma_1$*. Player 2's mixed equilibrium strategy changes to*

$$\sigma_2'(L) = \frac{d_1 - b_1}{a_1' - b_1 - c_1 + d_1}. \qquad (*)$$

Since $\text{sgn}(a_1' - c_1) = \text{sgn}(a_1 - c_1) = \text{sgn}(d_1 - b_1)$*, we have that* $\sigma_2'(L) \in (0, 1)$*; since indifference condition (*) uniquely determines* $\sigma_2'(L)$*, the completely mixed equilibrium of* G' *is unique. The numerator and denominator of (*) have the same sign; thus* $\sigma_2'(L)$ *is less than* $\sigma_2(L)$ *if the numerator and denominator are positive, and is greater than* $\sigma_2(L)$ *if they are negative. (Of course* $\sigma_2(R)$ *is affected in the opposite way.)*

Part IV — Poland

Consider the following versions of a signaling model, in which a company wants to hire a worker. Productivity is worker private information and is not observable to the company. The company maximizes expected profits. Worker reservation wage is equal to 0. The labor market is competitive.

1. Suppose that a worker has productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, and $\Pr(\theta = \theta_H) = \mu$. Worker chooses an education level $e \in [0, \infty)$ and is paid $w(e)$. A worker's utility is given by $U(w, e, \theta) = w - c(e, \theta)$, where the cost of education is $c(e, \theta) = e\theta$. Verify whether the single-crossing condition holds. Describe *all* the equilibria of this model (education choices, wages, beliefs).

2. As above, workers have productivity $\theta \in \{\theta_L, \theta_H\}$, $\theta_H > \theta_L$, but there is an equal probability of each type. Assume the cost of education is *the same* for both worker types, $c(e) = e$. Suppose the utility of worker θ who is paid wage w and undertakes education e is $U(w, e, \theta) = \theta w - e$. Is there an equilibrium where different types of workers choose different education levels? If there is, please describe all such equilibria. If not, please explain why. Show your work.

Solutions:

The solutions sketch is only for effort levels and beliefs are omitted.

(1) Let $p(e) \equiv p(\theta = \theta_H | e)$. Note that the single-crossing property does not hold. There are no separating equilibria. To find the pooling-equilibrium effort e^*, consider the IC conditions, given the competitive wage: for all e,

$$\begin{aligned} (H)\ E(\theta) - \theta_H e^* &\geq p(e)\theta_H + (1 - p(e))\,\theta_L - \theta_H e, \\ (L)\ E(\theta) - \theta_L e^* &\geq p(e)\theta_H + (1 - p(e))\,\theta_L - \theta_L e \end{aligned}$$

where $E(\theta) = \mu\theta_H + (1 - \mu)\,\theta_L$. The constraints are easiest to satisfy if $p(e) = 0$ for $e \neq e^*$. This gives effort levels

$$0 \leq e^* \leq \frac{E(\theta) - \theta_L}{\theta_H}.$$

(2) Essentially, worker utility is re-normalized and single-crossing still holds. Separating equilibria exist and can be found from the following conditions: for all e,

$$\begin{aligned} (H)\ \theta_H\theta_H - e_H^* &\geq p(e)\theta_H\theta_H + (1 - p(e))\,\theta_L\theta_H - e, \\ (L)\ \theta_L\theta_L - e_L^* &\geq p(e)\theta_H\theta_L + (1 - p(e))\,\theta_L\theta_L - e. \end{aligned}$$

Setting $p(e) = 0$ *if* $e \neq e_L^*, e_H^*$, *we obtain*

$$\begin{aligned}(H)\ \theta_H\theta_H - e_H^* &\geq \theta_L\theta_H - e, e \neq e_L^*, e_H^*, \\ (L)\ \theta_L\theta_L - e_L^* &\geq \theta_L\theta_L - e, e \neq e_L^*, e_H^*.\end{aligned}$$

Hence, $e_L^* = 0$ *and* $\theta_H(\theta_H - \theta_L) \geq e_H^*$. *For type* θ_L *not to deviate to* e_H , *we need*

$$\begin{aligned}\theta_L\theta_L &\geq \theta_L\theta_H - e_H^* \\ e_H^* &\geq \theta_L(\theta_H - \theta_L).\end{aligned}$$

Thus, $\theta_L(\theta_H - \theta_L) \leq e_H^* \leq \theta_H(\theta_H - \theta_L)$.