

## Question 1

Suppose that there are 50 million people in the labor force in Mexico and 150 million in the U.S. All workers prefer to work in their own country, but the extent of this preference varies across people. Assume that Mexican and U.S. workers are perfect substitutes (that is, they are equally productive when working in the same country), and assume that the same product is produced in both countries, and that the product price is 1.

The technology in each country is described by a Cobb-Douglas production function with constant returns:

$$Q_i = A_i K_i^\alpha L_i^{1-\alpha}$$

for  $i \in \{1, 2\}$ , with  $\alpha \in (0, 1)$ .

The total supply of capital in the two countries is a fixed amount  $K^0$ , and capital can be moved from one country to the other at no cost. The owners of the capital act so as to maximize income – the capital is rented to the highest bidder.

All markets are competitive, except that there may be restrictions on migration of labor from one country to the other.

- A Suppose that immigration is not allowed, and it is observed that the equilibrium wage is \$30 per hour in the U.S., and \$10 per hour in Mexico. Does this imply that U.S. firms are more productive (i.e. that  $A_1 > A_2$ )? If  $\alpha = \frac{1}{3}$ , and  $A_2 = 1$ , do you have enough information to determine  $A_1$ ?
- B Now suppose that workers can freely migrate from one country to the other. The home country preference depends on the relative wage,  $\omega = \frac{w_1}{w_2}$ , and preferences are uniformly distributed. If  $\omega = 5$ , all Mexican workers would prefer to work in the U.S.; and if  $\omega = 2$ , 40% of Mexican workers would prefer to work in the U.S., and so on.
- How many people will migrate, in the new equilibrium?
  - What happens to wages in each country? Explain why.
  - Does the relative wage rise or fall?
  - What happens to output in each country? What happens to total output (the sum of the outputs in the two countries)?
  - Discuss the welfare implications of your results.

## Question 2

Consider a pure exchange economy with two goods,  $h = 1, 2$ , and two consumers,  $i = 1, 2$ , with utility functions  $u_1$  and  $u_2$  respectively, and aggregate endowment,  $w = (w_1, w_2) \gg 0$ . For each of the following cases, determine which of the Pareto-efficient allocations can be decentralized as competitive equilibrium with lump sum transfers of wealth. Briefly describe the equilibrium prices and transfers for each Pareto-efficient allocation. [Hint: An equilibrium with transfers involves finding (non-negative) prices  $(p_1, p_2)$  and transfers  $(t_1, t_2)$  satisfying  $t_1 + t_2 = 0$ , such that consumers maximize utility at the chosen allocation given budget constraints  $p_1 x_{i1} + p_2 x_{i2} \leq p_1 w_{i1} + p_2 w_{i2} + t_i$ ,  $i = 1, 2$ .]

- (a) (4 points)  $u_1(x_{11}, x_{12}) = \alpha \ln x_{11} + (1 - \alpha) \ln x_{12}$  and  $u_2(x_{21}, x_{22}) = \beta \ln x_{21} + (1 - \beta) \ln x_{22}$ , where  $0 < \alpha < \beta < 1$ .
- (b) (3 points)  $u_1(x_{11}, x_{12}) = \min\{x_{11}, 2x_{12}\}$ ,  $u_2(x_{21}, x_{22}) = x_{22}$  and  $w_1 = w_2$ .
- (c) (3 points)  $u_1(x_{11}, x_{12}) = \max\{x_{11}, 2x_{12}\}$ ,  $u_2(x_{21}, x_{22}) = \max\{2x_{21}, x_{22}\}$  and  $w_1 = w_2$ .

### Question 3

Players 1 and 2 are involved in a dispute about which of them owns a treasure chest. Initially, the treasure chest is in the possession of player 1, but in certain circumstances player 2 may be able to sue player 1 in an attempt to take possession of the chest.

Player 1's legal claim to the chest may be strong ( $S$ ) or weak ( $W$ ), and player 2 may be informed ( $i$ ) or uninformed ( $u$ ) about the strength of player 1's legal claim. Each type of player 1 assigns probability  $\frac{4}{5}$  to player 2 being uninformed. An uninformed player 2 assigns probability  $\frac{1}{4}$  to player 1 having a weak legal claim, and an informed player 2 can distinguish between the two types of player 1.

After each player's type is determined, the interaction between the players proceeds as follows: First, player 1 decides whether to trade the chest to a third party ( $T$ ) or to keep the chest ( $K$ ). If he trades the chest, the game ends, and the payoffs are  $(3, 0)$  (= (payoff to player 1, payoff to player 2)).

If player 1 keeps the chest, then player 2 has three options: give up ( $g$ ), sue cautiously ( $c$ ), or sue boldly ( $b$ ). If player 2 gives up, player 1 will be able to spend more time searching for a buyer, and so payoffs are  $(5, 0)$ . If player 2 sues cautiously, much of the surplus generated by the chest is used up in court fees; in this event, payoffs are  $(-1, 2)$  if player 1 has a weak legal claim, or are  $(2, -1)$  if player 1 has a strong legal claim. Finally, "suing boldly" means that player 2 attempts to obtain the chest by quasi-legal means. This attempt succeeds if player 1 has a weak legal claim (leading to payoffs  $(0, 5)$ ), but leads to player 2's imprisonment if player 1 has a strong legal claim (leading to payoffs  $(5, -10)$ ).

- (i) Represent the situation described above as an extensive form game. (Hint: The game tree should reflect the facts that each player learns his own type, and that a player 2 of type  $i$  learns player 1's type. It may be helpful to think of the information about own and other's type being provided in two stages. In constructing the tree, be sure to account correctly for players' knowledge at each point of play.)
- (ii) Find all of the game's sequential equilibria.
- (iii) Can forward induction be used to eliminate any of the equilibria you found? Explain.

## Question 4

A firm's profit may take one of two values,  $\pi_1$  and  $\pi_2 > \pi_1$ . The firm is run by a manager, who chooses between two levels of effort: high and low. The manager has utility  $U = u(w - c)$  when he exerts high effort and  $U = u(w)$  when he chooses low effort, where  $w$  is the manager's wage,  $u$  is an increasing and concave function ( $\lim_{w \rightarrow \infty} u'(w) = +\infty$ ), and  $c > 0$ . The manager is maximizing the expectation of  $u$ , while his reservation wage is  $U_0 = u(w_0)$ . The shareholders' objective function is the expectation of the net profit  $\pi - w$ .

If the manager exerts high effort, the profit is equal to  $\pi_2$  with probability  $x$  and equal to  $\pi_1$  with probability  $1 - x$ . If he exerts low effort, the profit is  $\pi_2$  with probability  $y$  and  $\pi_1$  with probability  $1 - y$ ,  $0 < y < x < 1$ . The shareholders choose the manager's contract.

First, assume that the manager's effort is observed by the shareholders, who can impose the desired effort level on the manager.

- (a) Suppose that the optimal wage contract induces a low effort level. Describe the optimal contract.
- (b) Suppose that the optimal contract induces a high effort level. Describe the optimal contract.

Now, assume that the manager's effort is not observable by the shareholders.

- (c) Suppose that the optimal contract induces a high effort level. Could the contract you found in part (b) be implemented? If yes, show that it is still optimal. If not, demonstrate why and find the optimal wage structure. Compare the shareholders' profit under observable and unobservable effort.
- (d) Suppose that the optimal contract induces a low effort level. How should the shareholders induce low effort? Compare the shareholders' profit under observable and unobservable effort, and provide economic intuition for any differences between this comparison and the corresponding one from part (c).

Suppose that in the previous model, the optimal contract induces high effort when profit is verifiable. For part (e), assume that the manager effort is observable by the principal (the shareholders) but it is not verifiable, i.e., the principal cannot submit sufficient evidence to a third party (e.g., a court). Thus, contracts that depend directly on the effort cannot be made.

- (e) When profit is observable, but not verifiable, would the principal have an incentive to claim that the profit is low even if it is high? Now suppose that there is a large number of managers, so that the law of large numbers holds (in particular, if all managers exert effort, they know that  $x$  percent of them will yield high profit). As before, the probability that the individual performance is  $\pi_2$  rather than  $\pi_1$  is  $x$  or  $y$ , depending on whether the manager exerts high or low effort; the outcomes are independent across managers. Consider the following contract proposed by the principal: "I will pay a wage  $w_2$  to  $x$  percent of my managers (the ones that I announce to be the most productive ones), and a wage  $w_1$  to the rest," where  $w_1$  and  $w_2$  are the optimal wages under verifiability you found. Can the contract be implemented? Provide an economic interpretation. Assume that the principal does not collude with any of the managers.