Question 1

The Magic Mountain

The restaurant in Waldhotel in Davos, Switzerland is popular among businessmen as well as tourists visiting Davos. The Waldhotel is located on top of the Magic Mountain, featured in the famous novel by Thomas Mann ("Der Zauberberg"), who visited Davos in the spring of 1912. To dine in Waldhotel, one needs to make a reservation and pay for getting a table (the fee is essentially a fixed cost, which is independent of dinner expenses). The restaurant does not take reservations in advance. Rather, one can reserve a table only by calling around dinnertime on the day one wishes to dine. The restaurant cannot tell directly whether the group making a reservation consists of tourists or businessmen. The two groups differ, however, in how much they are willing to pay for getting a table without having to wait. One can get to the restaurant only using funicular railway. The restaurant covers the cost of the railway ticket for all visitors; the ticket costs \( c \) per passenger. Among all visitors, tourists represent fraction \( \alpha \) and businessmen represent fraction \( 1 - \alpha \).

For any given amount of waiting time \( t \), the utility function of each type of visitor is given by \( v = \Theta_i p - t \), where \( i \in \{\text{Business}, \text{Tourist}\} \), \( 0 < \Theta_B < \Theta_T \), and \( p \) is the price a traveler is willing to pay for getting a table early. Thus, businessmen are willing to pay more for any reduction in \( t \); and for any given \( t \), the businessmen are willing to pay more for getting a table. The utility from not dining at Waldhotel is 0 for either type. The restaurant wants to maximize profits (i.e., the expected price) by tailoring \( (p,t) \). Assume throughout that prices are nonnegative.

Assume first that the restaurant wants to serve both types of visitors.

(a) (2.5 points) Write down the optimization problem of the restaurant that price discriminates. In \((p,t)\) space, draw the indifference curves for businessmen and tourists, and the restaurant’s isoprofit curves.

(b) (1 point) Show that in the optimal solution, tourists are indifferent between getting a table and not dining at Waldhotel.

(c) (5.5 points) Describe the optimal price discrimination. Characterize how it depends on cost \( c \), proportion \( \alpha \) and parameters \( \Theta_B \) and \( \Theta_T \).

(d) (1 point) Under what conditions will the restaurant choose to serve only businessmen?

(Disclaimer: The scheme practiced by the restaurant is hypothetical. The hotel and its connection with Mann and the novel are not.)
Question 2

In the two-player Bayesian game $BG$, player $i$'s type $t_i$, representing his level of productivity, takes values in the finite set $T_i \subset \{1, 2, \ldots \}$. Types are drawn according to the prior distribution $p$ on $T = T_1 \times T_2$. After types are drawn, each player chooses to be In or Out of a certain project. If player $i$ chooses Out, his payoff is 0. If player $i$ chooses In and player $j$ chooses Out, player $i$'s payoff is $-c$, where $c > 0$ is the cost of participating in the project. Finally, if both players choose In, then player $i$'s payoff is $t_i t_j - c$. Thus, a player who chooses In must pay a cost, but the project only succeeds if both players choose In; in the latter case, the per-player benefit of the project is the product of the players' productivity levels.

i. State conditions defining (pure and mixed) Bayesian equilibria of $BG$.

Now suppose that the type sets are $T_1 = \{3, 4, 5\}$ and $T_2 = \{4, 5, 6\}$, and that the prior distribution $p$ is given by the table below.

<table>
<thead>
<tr>
<th>$t_1$</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>.2</td>
<td>.1</td>
<td>.0</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>.1</td>
<td>.1</td>
<td>.1</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>.1</td>
<td>.3</td>
<td></td>
</tr>
</tbody>
</table>

In answering the following questions, use $a_i^t$ to denote the probability with which a player $i$ of type $t_i$ chooses In.

ii. Find all (pure and mixed) Bayesian equilibria of $BG$ when $c = 15$.

iii. Find all (pure and mixed) Bayesian equilibria of $BG$ when $c = 14$. 
Suppose an Edgeworth Box economy has a strictly positive total endowment \((\bar{w}_1, \bar{w}_2)\) and the agents' preferences are represented by the utility functions \(u_1(x_1) = \min\{x_{11}, x_{21}\}\) and \(u_2(x_2) = \min\{x_{12}, x_{22}\}\).

(a) (2 points) Characterize the set of Pareto optimal allocations when \(\bar{w}_1 = \bar{w}_2 = \epsilon > 0\). Justify your answer.

(b) (4 points) Characterize the set of competitive equilibria when agents' endowments are given by \((3, 5)\) and \((5, 3)\), respectively. Are equilibrium prices unique? Justify your answer.

(c) (4 points) Characterize the set of Pareto optimal allocations when \(\bar{w}_1 = \epsilon > 0\) and \(\bar{w}_2 = 2\epsilon\). Justify your answer.
Question 4

Show each line of your work in this problem. Be precise in stating assumptions that you need.

1. (2 points) Consider a labor supply problem for the utility function $u(C, R) = C^{a_1} R^{a_2}$, $0 < a_1, 0 < a_2$, where C is consumption and R is leisure (time not working). Labor income is taxed with an income tax $t$, $0 < t < 1$, so the budget constraint is $C = w(1 - t) L + m$ and the time constraint is $L + R = T$ where $T = 1$ is the total amount of time (normalized) per day, C-goods are numeraire, $w$ is the real wage rate, m is real balances, $L$ is labor market time, and $R$ is non market labor time (called “leisure”).

Find the supply function for labor, call it $S(w(1 - t); m)$. How does it depend upon $w(1 - t)$ and $m$? Explain in a sentence or two the impact of the income tax on labor supply in this example. Sketch the supply function in a diagram with $w(1 - t)$ plotted on the vertical axis and $L$ plotted on the horizontal axis. Locate a sufficient condition for labor supply to be positive.

2. (3 points) Fix $w$ where $w$ is the before-tax real wage. Suppose an income tax takes $tw$ out of a paycheck of before-tax real wage $w$. Assume that labor supply is positive for tax rate $t = 0$ for this particular fixed value of $w$. For fixed $w$ such that labor supply is positive for tax rate $t = 0$ discuss how government should set the tax rate to maximize revenue, $R(t)$, for the government. Find a formula for the optimal tax rate. Locate sufficient conditions for it to be positive.

3. Suppose now that labor demand is the weakly decreasing differentiable function $d(w)$. Note that labor demand is a function of the before tax real wage $w$ because that is what the firm has to pay. Assume now that the labor supply function $S(w(1 - t))$ is increasing and differentiable in $w(1 - t)$. Assume $S(z)$ is a constant elasticity function with elasticity of supply $\epsilon_s > 0$.

3.1 (2 points) Case 1: Assume $d(w)$ is perfectly elastic at wage $w$. Find the optimal tax rate $t^*$ for Case 1. If $\epsilon_s = 1$ what can you say about the size of $t^*$? Explain the economics.

3.2. (3 points) Case 2: Let $d(w)$ be strictly decreasing and assume a unique $w(t)$ exists such that $d(w(t)) = S(w(t)(1 - t))$ for each $t$ such that $0 < t < 1$. What can you say now about the value of $t$ that maximizes revenue for the government when $w(t)$ is no longer fixed? Is it larger or smaller than the revenue maximizing value of $t$ when $w(t)$ does not change with $t$ as in Case 1? Explain the economics.