INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions.

  Example:
  MICRO THEORY 1/3/07
  ASSIGNED #
  Qu # __________ (Page __ of ___)

- Do not answer more than one question on the same page! When you start to answer a new question, start a new page.

- DO NOT write your name anywhere on your answer sheets! After the examination, the question sheets and answer sheets will be collected.

- Please DO NOT WRITE on the question sheets.

- Please solve any three of the four problems.

- You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- Please return unused portions of the yellow tablet & question sheets.

- There are 4 pages in the exam – please make sure you have all of them.

- Good luck!
Question 1

1. Assume two goods \( y(j), j = 1, 2 \), are produced using two inputs \((k(j), z(j)), j = 1, 2\). Assume the production functions, \( y(1) = f(k(1), z(1)) \) and \( y(2) = g(k(2), z(2)) \), are strictly concave, constant returns to scale, twice continuously differentiable, and increasing in \((k(1), z(1)), (k(2), z(2))\). Consider a small country which takes world prices \( p = (p(1), p(2)) \) for the two goods as given. Assume this country has endowment vector \( (k, z) \) of the two factors. Competitive firms produce \( y(1) \) and \( y(2) \) using \( f(\cdot) \) and \( g(\cdot) \) as specified above, facing prices \( p \) as given.

Define competitive equilibrium in factor markets for this small country. Recall that \( k(1) + k(2) = k, z(1) + z(2) = z \). Show how to find equilibrium factor prices, \( w^* \) for \( k \), \( v^* \) for \( z \). Show how to obtain comparative statics of equilibrium quantities \( (y^*(1), y^*(2)) \) and equilibrium prices \( w^*, v^* \) with respect to \( p \) and with respect to \( (k, z) \). In particular, show how to find the signs of the partial derivatives \( \partial y^*(1) / \partial p(1), \partial y^*(1) / \partial p(2), \partial w^* / \partial k, \) and \( \partial w^* / \partial z \).

2. Now let \( k \) denote endowment of high skilled labor and let \( z \) denote endowment of low skilled labor. Assume competitive firms pay \( w \) for units of \( k \) and \( v \) for units of \( z \) and maximize profits using the production functions \( f \) and \( g \) to produce \( y(1) \) and \( y(2) \), which are sold for prices \( p(1) \) and \( p(2) \). Show how you would do a comparative statics analysis to find \( dw^* \) and \( dv^* \) for small \( dp(1) > 0, dp(2) = 0 \). In particular, show how you would determine whether \( dw^* / dp(1) > 0 \) and whether \( dw^* / w^* > dp(1) / p(1) \). (Hint: Recall that \( f, g \) are constant returns.)

Suppose now that migration of low skilled labor to the small country increases the endowment of low skilled labor from \( z \) to \( z + dz \) where \( dz > 0 \) is small, but that the endowment of high skilled labor \( k \) remains fixed. Show how you would find \( dy^*(1) \) and \( dy^*(2) \), and show how you would determine whether \( dy^*(2) > 0 \) and whether \( dy^*(2) / y^*(2) > dz / z \).
A risk-neutral principal who owns a firm employs a manager (the agent), whose effort is unobservable. The agent's effort \( e \in \{1, 0\} \) determines the probability distribution of profits \( \pi \in \{\pi_H, \pi_L\} \), \( \pi_H > \pi_L \), as follows: \( P(\pi = \pi_H|e = 1) = p \in (0, 1) \), and \( P(\pi = \pi_H|e = 0) = 0 \). The agent's utility from effort \( e \) and wage \( w \) is \( v(w, e) = u(w) - e \), where \( u \) is continuous and strictly concave with \( u(0) = 0 \). The agent's reservation utility is 0. Wages are nonnegative.

A contract is a tuple \((w(\pi_H), w(\pi_L)) = (w_H, w_L)\) of contingent wages to be paid to the agent after the profit is realized.

a) Consider the standard contracting situation: The principal offers a contract which the agent may accept or reject; upon acceptance, the agent makes an effort choice.

i. Derive a necessary and sufficient condition that ensures that the profit-maximizing principal will always prefer to induce high effort in the agent.

ii. If your condition in (i) holds, how do the agent's wages depend on \( p \)? Interpret.

b) Now consider a contracting situation which is the same as the one in a) until the agent has chosen his effort level; however, after \( e \) has been chosen and before profits realize, the wage contract can be renegotiated. In this renegotiation stage, the principal offers a contract \((\hat{w}_H, \hat{w}_L)\) to the agent (this may be the same as the original contract \((w_H, w_L)\)). If the agent accepts, the new contract \((\hat{w}_H, \hat{w}_L)\) is in force; if the agent rejects, the original contract \((w_H, w_L)\) remains in force.

Let \( q := P(e = 1|(w_H, w_L), (\hat{w}_H, \hat{w}_L)) \) be the probability that the agent chooses the high effort in this game.

i. For any \((w_H, w_L), q\), set up the principal's maximization problem when he chooses the optimal renegotiated contract \((\hat{w}_H, \hat{w}_L)\) such that the agent will weakly prefer to accept this contract, for any previous action choice. Assume the principal's beliefs are correct.

ii. From the optimization problem above, what is the optimal renegotiated contract \((\hat{w}_H, \hat{w}_L)\) when \( q = 1 \)? Give an intuitive explanation.

iii. Now consider the agent's effort choice when he correctly anticipates the renegotiated wages the principal will offer (which you have derived in (ii)). Can you find conditions under which \( q = 1 \) could be a Perfect Bayesian equilibrium?

iv. Assume again the condition you derived in a (i). Compare your results from (iii) with the results you derived in a (i). Give an intuitive explanation.
Agent $F$ (firm) has an opportunity to start a firm, but doing so requires 1 unit of money, which agent $F$ does not have. Agent $I$ (investor) has 1 unit of money. $I$ can invest this money and receive a safe payoff of $R > 1$, or $I$ can give the money to $F$, who can then start the firm. If $F$ starts the firm, the firm is a success (profit of $G$) with probability $p$ and a failure (profit of zero) otherwise.

1 (1 point) What is the efficient outcome, as a function of $R$, $p$ and $G$?

2 (2 points) Suppose that agent $F$ can propose a contract to agent $I$, which $I$ either accepts or rejects. If $I$ accepts, the contract is implemented. If $I$ rejects, $I$ gets the safe payoff of $R$ and $F$ gets nothing. A contract specifies that $I$ gives the unit of money to $F$ who starts the firm, as well as an amount that $F$ pays to $I$ if the firm is a success. ($F$ necessarily pays nothing to $I$ if the firm is a failure.) Identify the subgame perfect equilibrium outcome and payoffs of this interaction.

3 (1 point) Now suppose that only $F$ can observe whether the firm is a success or failure. It is impossible for the firm to make a payment to the investor if the firm fails. Hence, no matter what the contract, there is no way to compel the firm to make a payment to the investor, since the firm can always claim failure. Identify the subgame perfect equilibrium outcome and payoffs of this interaction between the firm and investor. (Hint: no tricks—this part really is easy.) Given your answers to this and the previous part, if the firm had to pay a cost to make its outcome verifiable, how much would it be willing to pay?

4 (4 points) Suppose now that investment is efficient and the game between the investor and firm is played twice. Assume that the outcome of the firm is observable only to the firm, so that contracts cannot be conditioned on that outcome. However, assume that the agents can contract on whether $F$ makes a payment to $I$ in the first period. For example, a contract can specify that $I$ will fund $F$ in period 2 if and only if $F$ makes a payment to $I$ in period 1. Find the subgame perfect equilibrium of this interaction and its payoffs.

5 (2 points) Now suppose the investor has 2 units of money in each period and there are two identical firms, whose outcomes are independent. Continue to assume that there are two periods in the interaction. Each firm can observe its outcome and the outcome of the other firm, no one else can observe these outcomes. How much (if any) could the firms increase their payoffs by merging?
Question 4

a. Compute all rationalizable strategies and all Nash equilibria of the normal form game below.

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b. Let $G$ be a normal form game, and let $G'$ be a normal form game obtained from $G$ by eliminating a strictly dominated strategy. Let $G_\infty(\delta)$ and $G'_\infty(\delta)$ be infinite repetitions of $G$ and $G'$ with discount rate $\delta < 1$. Can the sets of Nash perfect equilibria of $G_\infty(\delta)$ and $G'_\infty(\delta)$ differ? What about the sets of subgame perfect equilibria? Explain.