UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY Prelim Exam

July 27, 2010
9:00 am - 1:00 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions.

  Example:
  MICRO THEORY
  7/27/10
  ASSIGNED #: ____________

  Qu # 1 (Page 2 of 4):

- Do not answer more than one question on the same page!
  When you start a new question, start a new page.

- DO NOT write your name anywhere on your answer sheets!
  After the examination, the question sheets and answer sheets will be collected.

- Please DO NOT WRITE on the question sheets.

- Please solve any four of the five problems.

- You are not allowed to use notes, books, calculators, or colleagues
  Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- Please return unused portions of the yellow tablet & question sheets.

- There are 6 pages in the exam, including this sheet—please make sure you have all of them.

- Good luck!
Meetings Between Buyers and Sellers

Consider the meeting process between 2 identical sellers ($s_1$ and $s_2$) and 2 identical buyers ($b_1$ and $b_2$). Each seller has one unit of a good for sale. The seller has no use for the good herself and if she fails to sell the good it perishes. Each buyer has a valuation of the good of one. A buyer can visit one seller only. A buyer who obtains one unit of the good at price $p$ receives payoff of $1 - p$. If the buyer fails to make a purchase, she receives zero payoff. A seller who sells her good at price $p$ receives payoff $p$, and if she fails to sell her good, she gets zero payoff. The meeting process between buyers and sellers takes place as a two stage game. In the first stage, the sellers simultaneously post a price for their good. In the second stage, the buyers simultaneously decide which seller to go to (could be a mixed strategy) having observed the posted prices. If two buyers show up at the same seller, a fair coin is tossed to determine which buyer gets to buy the good. The other buyer is rationed. If no buyers show up at a seller, the seller fails to sell her good.

1. Determine the symmetric subgame perfect equilibrium (symmetric in the sense that identical individuals adopt identical strategies).

2. In the symmetric subgame perfect equilibrium, what is the average number of meetings between sellers and buyers?
Question 2

Prove or disprove each of the following statements on exchange economies with \( N \) consumers and \( L \) goods, defined by consumer preferences (\( \succ_i \)) consumption sets (\( X_i \)) and individual endowments (\( w_i \)).

(a) (2.5 points) Consider an economy with two consumers, who have locally nonsatiated, convex preferences. If both \((x_1, x_2)\) and \((x_1', x_2')\) are Pareto-efficient allocations, then \((\alpha x_1 + (1 - \alpha) x_1', \alpha x_2 + (1 - \alpha) x_2')\) is Pareto-efficient for any \(\alpha \in [0, 1]\).

(b) (2.5 points) Consider a two-period economy with uncertainty and \( S = 3 \) (i.e. three possible states at date 1). If there are five financial assets in this economy, at least two of them are redundant (an asset is redundant if its returns across states of the world can be replicated by a portfolio consisting of the remaining assets).

(c) (2.5 points) There exists a competitive equilibrium if \( X_i = \mathbb{R}_+^L \), \( \succ_i \) is continuous and locally nonsatiated, and \( w_i \gg 0 \) for \( i = 1, 2, \cdots, N \).

(e) (2.5 points) Walras’ law (\( \sum_{i=1}^N p \cdot (w_i - x_i(p, w_i)) = 0 \)) is satisfied if \( X_i = \mathbb{R}_+^L \), \( w_i \gg 0 \), and \( \succ_i \) is represented by a strictly concave utility function \( u_i \) for \( i = 1, \cdots, N \).
A monopoly airline sells tickets to business travelers (B) and to leisure travelers (L). The proportion of B types is \( \lambda \). There are two periods. At the beginning of period one, the traveler privately learns his type, which determines the probability distribution \( F_B \) or \( F_L \) that will determine his valuation for the ticket (where for example \( F_B(v) \) is the probability that the B type will draw a valuation of \( v \) or less).

The seller and the traveler contract at the end of period one. At the beginning of period two, the traveler privately learns his actual valuation for the ticket, and then decides whether to travel. Each ticket costs the seller \( c \). The seller and the traveler are risk-neutral, and there is no discounting. The reservation utility of each type of traveller is normalized to zero.

A partially refundable ticket contract consists of a pair \( (a, r) \), where \( a \) is an advance payment at the end of period one and \( r \) is a refund that can be claimed at the end of period two if the ticket is not used. The traveler’s payoff under this contract is \( v - a \) if the ticket is used, and \( r - a \) if it is not. The seller offers two contracts \( (a_1, r_1) \) and \( (a_2, r_2) \), and the four parameters describing these two contracts are chosen so as to maximize expected profit. Since the seller does not know the traveler’s type, each traveler can choose either contract.

Suppose \( \lambda = \frac{3}{4} \), \( c = 50 \), \( F_B \) is a uniform distribution on the set \([0, 50] \cup [100, 150]\), and \( F_L \) is a uniform distribution on the set \([50, 100]\).

1. (a) A simple strategy for the seller is to just charge a single ticket price \( p \), that is fully refundable. This can be implemented by setting \( a_1 = a_2 = r_1 = r_2 = p \). What is the optimal ticket price in this case, and how much profit does the seller make?

(b) Can you find two contracts that yield more expected profit than the optimal simple strategy?

2. (a) What are the expected profit maximizing choices of \( (a_1, r_1) \) and \( (a_2, r_2) \)?

(b) If the seller chooses \( (a_1, r_1) \) and \( (a_2, r_2) \) so as to maximize expected profit, is the outcome efficient?
Consider the following normal form game $G(r)$:

\[
\begin{array}{c|cc}
 & I & N \\
\hline 
I & r, r & r - 1, 0 \\
N & 0, r - 1 & 0, 0 \\
\end{array}
\]

In this game, strategy $I$ represents investing, and strategy $N$ represents not investing. Investing yields a payoff of $r$ or $r - 1$ according to whether the player's opponent invests or not. Not investing yields a certain payoff of 0.

i. (2 points) Describe the set of Nash equilibria of $G(r)$ for each $r \in [-2, 3]$.

Now consider a Bayesian game $BG$ in which payoffs are given by the above payoff matrix, but in which the value of $r$ is the realization of a random variable that is uniformly distributed on $[-2, 3]$. In addition, each player $i$ only observes a noisy signal $t_i$ about the value of $r$. Specifically, $t_i$ is defined by $t_i = r + \epsilon_i$, where $\epsilon_i$ is uniformly distributed on $[-\frac{1}{10}, \frac{1}{10}]$, and $r$, $\epsilon_1$, and $\epsilon_2$ are independent of one another.

It follows that if $t_i \in [-\frac{19}{10}, \frac{29}{10}]$, then player $i$'s posterior belief about $r$ conditional on $t_i$ is uniform on $[t_i - \frac{1}{10}, t_i + \frac{1}{10}]$, and that his posterior belief about $t_j$ conditional on $t_i$ is the triangular distribution with support $[t_i - \frac{1}{5}, t_i + \frac{1}{5}]$ (that is, the conditional density equals 0 at $t_j = t_i - \frac{1}{5}$, 5 at $t_j = t_i$, and 0 at $t_j = t_i + \frac{1}{5}$, and is linear on $[t_i - \frac{1}{5}, t_i]$ and on $[t_i, t_i + \frac{1}{5}]$).

ii. (1 point) Describe each player's set of pure Bayesian strategies in game $BG$, and state the definition of pure Bayesian equilibrium for this game.

iii. (1 point) Show that in any Bayesian equilibrium, if the value of player $i$'s signal $t_i$ is less than 0, then player $i$ strictly prefers not to invest.

iv. (3 points) Building on your answer to part (iii), show that in any Bayesian equilibrium, if the value of player $i$'s signal is less than $\frac{1}{20}$, then player $i$ strictly prefers not to invest.

v. (2 points) Building on your answer to part (iv), show that in any Bayesian equilibrium, if the value of player $i$'s signal is less than $\frac{1}{10}$, then player $i$ strictly prefers not to invest.

vi. (1 point) Argue that in any Bayesian equilibrium, player $i$ strictly prefers not to invest when his signal is less than $\frac{1}{2}$. (A symmetric argument shows that in any Bayesian equilibrium, player $i$ strictly prefers to invest when his signal is above $\frac{1}{2}$.)
A company hires a manager whose productivity is \( \theta \). The manager knows his productivity, but the company does not. The company knows that, in the pool of job candidates, there are high- and low-quality managers, \( \theta_H \) and \( \theta_L < \theta_H \), and that \( \alpha \) is the probability of \( \theta = \theta_H \). The manager’s outside opportunity is worth 0. The utility of a manager who has productivity \( \theta \) and who receives wage \( w \) and puts in effort \( e \geq 0 \) is \( w - (e/\theta) \). The company’s utility is then \( 2(e)^{1/2} - w \).

(a) [0.5 pt] Suppose first that the company can observe \( \theta \). Find the optimal contract.

(b) [1 pt] Find the optimal contract (wages, effort levels) based on a separating equilibrium set by a company that cannot observe \( \theta \):

(i) [1 pt] Define a separating equilibrium. Explain why a separating equilibrium is possible in this market.

(ii) [2.5 pt] Write down the optimization problem of the company.

(iii) [3.5 pt] Determine which constraints are binding (explain why) and characterize the optimal contract.

(c) [1.5 pt] Assume \( \theta_H = 2 \) and \( \theta_L = 1 \). Then it also holds that \( w_H = 2 \) and \( w_L = 1 \). Define the Cho-Kreps Intuitive Criterion and determine which among the equilibria you found in part (b) survive the Criterion.

(d) [1 pt] Go back to part (b). Now suppose that with probability \( \beta \) the company observes the manager’s type after the manager chooses a contract. The timing is now as follows: First, the company offers a contract. Then the manager accepts or rejects. If he accepts, the manager chooses an effort level and payoffs are realized. With probability \( \beta \), \( 0 < \beta < 1 \), independently of what has happened in the past, the company observes the manager’s type. Find the optimal contract according to which the manager should be paid.