INSTRUCTIONS

• Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

(1) your assigned number
(2) the number of the question you are answering
(3) the position of the page in the sequence of pages used to answer the questions.

Example:

MICRO THEORY 7/23/08
ASSIGNED # ____________
Qu # 1 (Page 2 of 4):

• **Do not answer more than one question on the same page!**
  When you start a new question, start a new page.

• **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.

• **Please DO NOT WRITE on the question sheets.**

• Please solve any three of the four problems.

• You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

• Please return unused portions of the yellow tablet & question sheets.

• There are 4 pages in the exam – please make sure you have all of them.

• Good luck!
George consumes two goods, food, f, and whiskey, x. His utility function is \( u(f,x) = x + \log(f) \). Let \( p \) denote the price of food and set the price of whiskey to 1 for numeraire.

1. (2) What is the slope \( \frac{dx}{df} \) of George’s indifference curves for general values of \( u, x, \) and \( f \)? Make a rough sketch of George’s indifference curves with \( x \) plotted on the vertical axis and \( f \) plotted on the horizontal axis. Label your axes. Do George’s indifference curves intersect the axes? Explain why or why not.

2. (3) Find George’s demand for food \( f(p,y) \) and for whiskey, \( x(p,y) \) where \( y \) denotes income. Recall that whiskey is numeraire with a price set equal to one. George’s wife, Lorena, has said she’d leave him if he doesn’t lay off that whiskey. Find the set \( \{(p,y)\} \) where George demands zero whiskey on his own tastes.

3. (3) Find George’s indirect utility function, his expenditure function, and his Hicksian compensated demand functions.

4. (2) Suppose \( y=1 \) and the price changes from \( p=1 \) to \( p=2 \). Find the changes in the quantities of food and whiskey that George demands. How much income do you have to give George to get him back to his original utility level? Will he consume whiskey after he is compensated? If he does how much will he consume?
Consider a production economy with two goods, labor \((l)\) and a consumption good \((x)\). There is one firm that produces \(f(l)\) units of \(x\) with \(l\) units of labor. Assume that \(f(0) = 0\), \(f' > 0\) and \(f'' < 0\). There are \(N + 1\) agents with endowments \(w_i = (w^l_i, w^x_i)\). Agent 0 owns the firm and has an endowment \(w_0 = (0, 0)\). The remaining agents have endowments \(w_i = (1,0)\) for \(i = 1,...,N\). All agents have the same utility function \(u(l, x) = x\). An allocation is given by \(((l_0, x_0),..., (l_N, x_N))\).

(a) (1 point) Can an allocation be in the core with \(l_i > 0\) for some \(i\)? Explain your answer.

(b) (2 points) Let \(N = 3\). Characterize the core. In a core allocation, what is the highest amount of good \(x\) any agent \(i > 0\) can get? And the smallest amount of good \(x\) agent 0 should get?

(c) (2 points) Describe the core for general \(N > 1\).

(d) (3 points) characterize the set of Walrasian equilibria for general \(N\). [Hint: Pick \(x\) as the numeraire.] Are Walrasian equilibrium allocations in the core? Do agents \(i > 0\) always prefer Walrasian allocations to other core allocations? Explain.

(e) (2 points) What happens to the set of Walrasian equilibria and to the core as \(N \to \infty\)? Do they "converge"? Does your answer contradict the core convergence theorem? Justify your answers.
Consider the classic Cournot model of quantity competition, with asymmetric marginal costs. Two firms \(i \in \{1, 2\}\) simultaneously choose production levels \(q_i \in \mathbb{R}^+\); each firm \(i\) produces those units at a constant marginal cost of \(c_i\) per unit, and sells them at the market price, which is determined by the inverse demand function

\[
P = \max \{0, 100 - q_1 - q_2\}
\]

1. Calculate firm \(i\)'s best-response given a production level \(q_j\) of his opponent. (Be sure to account for the cases where it is optimal to set \(q_i = 0\).)

2. Let \(c_1 = 25\) and \(c_2 = 55\). Find the Nash equilibrium in which both players produce, and calculate both firms' profits.

3. If firm 1 produced 45 units, firm 2's best-response would be not to produce at all. Calculate firm 1's profits in this event. Is it higher or lower than your answer to part 2? Is \((q_1, q_2) = (45, 0)\) an equilibrium? Why or why not?

Now let \(G\) be any two-player simultaneous-move game, with strategy spaces \(A_i = \mathbb{R}^+\) and payoff functions \(u_i : A_i \times A_j \to \mathbb{R}\) which are continuous and differentiable. Let \(G_1\) be a variation on the game \(G\) where player 1 moves first, then player 2 observes 1's action and moves second. (When \(G\) is the Cournot game, \(G_1\) is known as the Stackelberg game.)

4. Let \(BR_i : A_j \Rightarrow A_i\) be player \(i\)'s best-response correspondence for the game \(G\), that is, \(BR_i(a_j) = \arg\max_{a_i \in A_i} u_i(a_i, a_j)\). Show that if \(BR_2\) is single-valued (\(BR_2(a_1)\) is a singleton for every \(a_1\)), then player 1's payoff in any subgame-perfect equilibrium of \(G_1\) is at least as high as his payoff in any pure-strategy Nash equilibrium of \(G\).

5. Now suppose \(BR_1\) and \(BR_2\) are both single-valued. Show that if \(BR_2\) is weakly decreasing and \(u_1\) is weakly decreasing in \(a_2\), then player 1's strategy in any subgame-perfect equilibrium of \(G_1\) is at least as high as his strategy in any pure-strategy Nash equilibrium of \(G\).

6. Let \(G\) be the Cournot game described above, with marginal costs \(c_1\) and \(c_2\), and suppose \(G\) has a unique equilibrium in which both firms produce strictly positive quantities. Show that the statements in parts 4 and 5 hold strictly: firm 1 produces strictly more in the Stackelberg game than in the simultaneous-move Cournot game, and earns strictly higher profits.
Consider the game $\Gamma$ below:

$$
\begin{array}{cccc}
1 & & & \\
\downarrow & & & \\
A & B & & \\
& 25,0 & & 2 \\
\downarrow & & & \\
W & X & & \\
& 30,7 & & 1 \\
\downarrow & & & \\
C & D & & \\
& & 2 & 2 \\
\downarrow & & & \\
Y & Z & Y & Z \\
& 24,8 & 0,0 & 0,0 & 8,24 \\
\end{array}
$$

(i) Compute all subgame perfect equilibria of $\Gamma$.

(ii) Compute all Nash equilibria of $\Gamma$.

Let $G$ be the reduced normal form of $\Gamma$, and let $G^\infty(\delta)$ be the infinite repetition of $G$ with discount rate $\delta \in (0, 1)$.

(iii) Construct a pure strategy profile of $G^\infty(\delta)$ that yields payoffs $(30, 7)$ and that is a subgame perfect equilibrium of $G^\infty(\delta)$ for all large enough values of $\delta$. Describe the set of $\delta$ for which your strategy profile is subgame perfect.

(iv) For $\delta$ large enough, are there subgame perfect equilibria of $G^\infty(\delta)$ in which player 2 obtains a payoff higher than 10? If so, construct a pure strategy profile that satisfies these requirements when $\delta$ is large enough.