UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MICROECONOMIC THEORY Prelim Exam

July 24, 2007

9:00 am - 1:00 pm

INSTRUCTIONS

• Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

(1) your assigned number
(2) the number of the question you are answering
(3) the position of the page in the sequence of pages used to answer the questions.

Example:

MICRO THEORY 7/24/07
ASSIGNED #
Qu # 1 (Page 2 of 4):

• Do not answer more than one question on the same page! When you start a new question, start a new page.

• DO NOT write your name anywhere on your answer sheets! After the examination, the question sheets and answer sheets will be collected.

• Please DO NOT WRITE on the question sheets.

• Please solve any three of the four problems.

• You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well-organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

• Please return unused portions of the yellow tablet & question sheets.

• There are 4 pages in the exam – please make sure you have all of them.

• Good luck!
QUESTION 1

A social planner is regulating a monopolist whose type $\theta \in \Theta = [0, 1]$ he cannot observe. The monopolist produces two different goods, 1 and 2. A monopolist of type $\theta$ has marginal cost $\theta$ of producing good 1; his marginal cost of producing good 2 is $(1 - \theta)$. A monopolist who produces quantity $q = (q_1, q_2)$ and receives transfer $t$ from the planner has profits $\pi(t, q; \theta) = t - \theta q_1 - (1 - \theta)q_2$. The monopolist’s reservation utility is zero. The social planner maximizes consumer surplus $S(q_i)$ in both markets, net of the transfer; his objective function is $V(t, q) = S(q_1) + S(q_2) - t$, where $S(q_i) = 1 - (1 - q_i)^2$.

$\theta$ is distributed uniformly on $\Theta$.

A direct revelation contract is a tuple of functions $(t, q_1, q_2)$, $t : \Theta \rightarrow \mathbb{R}$, $q_i : \Theta \rightarrow \mathbb{R}$, $i = 1, 2$. Rewrite the monopolist’s profit from such a contract as $\pi(\tilde{\theta}; \theta) = \pi(t(\tilde{\theta}), q(\tilde{\theta}); \theta)$, where $\tilde{\theta}$ is the monopolist’s announcement of his type, $\theta$ is his true type. The optimal direct revelation contract maximizes the social planner’s expected utility, subject to the monopolist’s participation and revelation constraints:

$$
\max_{t, q_i} \int_{\Theta} V(t(\theta), q(\theta)) f(\theta) d\theta \text{ s.t. } \pi(\theta; \theta) \geq 0, \forall \theta, \quad (P)
$$

$$
\theta \in \arg \max_{\theta} \pi(\tilde{\theta}; \theta), \forall \theta, \quad (R)
$$

You may use the facts that for any $\tilde{\theta} \in \Theta$, $g : \Theta \rightarrow \mathbb{R}$, $\int_{\Theta} \int_{\Theta} g(\theta) dy f(\theta) d\theta = \int_{\Theta} g(\theta) F(\theta) d\theta$ and $\int_{\Theta} \int_{\Theta} g(y) dy f(\theta) d\theta = \int_{\Theta} g(\theta)(1 - F(\theta)) d\theta$.

a) (4 points) First suppose that production of good 2 is prohibited for some reason. Derive $q_1^\star$, the function determining quantities set by the optimal direct revelation contract $(t^\star, q_1^\star)$. Point out the monopolist’s information rent along the way. Is the contract for the best type distorted away from the first-best?

b) (6 points) Now suppose the monopolist is producing both goods, 1 and 2. The optimal direct revelation contract is a tuple of functions $(t^\star, q_1^\star, q_2^\star)$.

i. (1 point) In the optimal contract, there exists $\tilde{\theta} \in (0, 1)$ such that $q_1^\star(\tilde{\theta}) = q_2^\star(\tilde{\theta})$.

Show that the revelation constraint of any $\theta$, together with the participation constraint of type $\tilde{\theta}$, implies the participation constraint for type $\theta$.

ii. (2 points) Rewrite the revelation constraint in the usual manner, taking care to note whether you have to impose additional conditions on the optimal contract. Then use your result from (i) to derive expressions for $t(\theta)$ for all $\theta$ in an incentive compatible contract.

iii. (3 points) Derive the optimal quantity-setting functions $q_1^\star, q_2^\star$. Which types receive informational rents in the optimal contract? Why?
Question 2

i. (3) Given a utility function of two goods $x_1$, $x_2$, denoted by $u(x_1, x_2)$, the marginal rate of substitution function $\text{MRS}(x_1, x_2)$ between commodities $x_1$ and $x_2$ is defined by

$$\text{MRS}(x_1, x_2) = \frac{\frac{\partial u}{\partial x_1}(x_1, x_2)}{\frac{\partial u}{\partial x_2}(x_1, x_2)}.$$ 

Assume that $u$ is a positive, monotonic, differentiable transformation of a twice differentiable, differentiably strictly concave, homogeneous of degree one function $g(x_1, x_2)$. Prove that $\text{MRS}(x_1, x_2)$ is decreasing in $x_1$ and increasing in $x_2$.

ii. (2) Consider an Edgeworth box economy with two consumers, A and B, and two goods, 1 and 2. Assume the endowment of consumer A is $(1, 0)$ and the endowment of consumer B is $(0, 1)$. Assume the utility functions of the two consumers are identical. Assume the marginal rate of substitution function $\text{MRS}(x, y)$ is strictly increasing in $y/x$. What can you say about the set of Pareto optimal allocations? Be precise. Illustrate your answer with an Edgeworth box.

iii. (2) Assume the utility function in part (ii) above is Cobb-Douglas, i.e. $\log[u(x, y)] = a \log(x) + b \log(y)$, where $a, b > 0$. Assume the endowments are the same as in part (ii). Compute offer curves and competitive equilibria for this Cobb-Douglas economy. Illustrate your findings in an Edgeworth box diagram.

iv. (2) Consider a two-consumer Edgeworth box exchange economy where each consumer has a strongly monotone, strictly convex, twice differentiable, homothetic utility function. Suppose that the marginal rate of substitution decreases along an indifference curve as $x_1$ increases and $x_2$ decreases, so that the curve gets progressively flatter as you move to the right. Illustrate graphically how you can use this information to find points in the Pareto optimal set.

v. (1) Assume the utility function of the two consumers A, B is the same for both consumers: namely, the linear function $u(x, y) = x + y$. Assume the endowments are the same as in part (ii). Compute the set of Pareto optimal allocations. Illustrate your findings in an Edgeworth box diagram.
Question 3

It is the final audition for “American Idol”. All the contestants have interesting voices this year, but their personalities differ. There are three types of contestants: cool, self-controlled and nerdy, with an equal proportion of each type. Regardless of one’s true type, being thought of as cool is worth 90; being thought of as self-controlled is worth 60; and being thought of as nerdy is worth 0. (The singing ability of a contestant does not affect the judges’ decision at this stage. Rather, the judges will make the final selection based on each contestant’s potential of being a good “idol”.)

One way in which a person can try to indicate their personality to the judges is by choosing to signal. This signal is a yes-or-no choice; that is, each contestant can only choose to signal or not to signal. The cost of signaling is 150 for nerdy types; 10 for self-controlled types and 0 for cool types. After he or she has made the decision about whether or not to signal, he or she must take a test (a one-on-one conversation with Simon Cowell, one of the judges). The outcome of the test is independent of whether or not a person has signaled. Cool types always pass the test; nerdy types never pass; self-controlled types pass with probability one half.

i. (1 pt.) Define the payoffs, types, beliefs and strategies in this signaling game.

ii. (1 pt.) Depict the extensive-form game, carefully marking information sets, probabilities, etc. Make sure to include the testing stage in the event tree as well.

iii. (1 pt.) Define a perfect Bayesian equilibrium for this game.

iv. (2 pt.) Is there a perfect Bayesian equilibrium in which only self-controlled types signal? Explain carefully.

v. (2 pt.) Is there a perfect Bayesian equilibrium in which both cool and self-controlled types signal? Assume that the judges believe that anyone who does not signal and who passes the test is self-controlled.

vi. (3 pt.) Do any of the equilibria you found satisfy the Intuitive Criterion?
Question 4

A population game is defined by a set of strategies $S = \{1, \ldots, n\}$ and a continuously differentiable payoff function $F: \mathbb{R}^n_+ \to \mathbb{R}^n$. The function $F_i: \mathbb{R}^n_+ \to \mathbb{R}$, the $i$th component of the function $F$, is the payoff function for strategy $i$. In what follows, we take the set of strategies $S$ as fixed and identify a population game with its payoff function $F$.

The population game $F$ is played by a unit-mass population of infinitesimal agents. Each agent chooses a strategy from the set $S$. (That is, agents always choose pure strategies.) The distribution of strategies in the population is therefore described by a population state, which is an element of the simplex $X = \{x \in \mathbb{R}^n_+: \sum_{i \in S} x_i = 1\}$. When the population state is $x$, agents playing strategy $i$ receive payoff $F_i(x)$.

(i) (1 pt.) Provide a definition of Nash equilibrium for population games.

A population game $F$ is a potential game if it admits a potential function: a twice continuously differentiable function $f: \mathbb{R}^n_+ \to \mathbb{R}$ that satisfies

$$\frac{\partial f}{\partial x_i}(x) = F_i(x) \text{ for all } i \in S \text{ and } x \in X.$$  

(ii) (2 pts.) State the Kuhn-Tucker first-order necessary conditions for maximizing the potential function $f$ on the simplex $X$.

(iii) (2 pts.) Show that population state $x^* \in X$ satisfies the conditions you stated in part (ii) (for some appropriate choices of the Lagrange multipliers) if and only if $x^*$ is a Nash equilibrium of the population game $F$. Also, provide a game-theoretic interpretation of the Lagrange multipliers.

Now let $F$ be a population game (not necessarily a potential game), and introduce a new population game $\hat{F}: \mathbb{R}^n_+ \to \mathbb{R}^n$ defined by

$$\hat{F}_i(x) = F_i(x) + \sum_{j \in S} x_j \frac{\partial F_j}{\partial x_i}(x).$$

(iv) (2 pts.) Provide an economic interpretation of the population game $\hat{F}$. (You may find it helpful to imagine that the payoffs $F_i$ and partial derivatives $\frac{\partial F_j}{\partial x_i}$ are nonpositive, as would be the case if $F$ were a model of network congestion.)

(v) (1 pt.) Prove that $\hat{F}$ is a potential game with potential function

$$\hat{f}(x) = \sum_{j \in S} x_j F_j(x).$$

(vi) (2 pts.) Taking into account your answer to part (iv) and the result stated in part (iii), provide an economic interpretation of the result stated in part (v).