

Macro Prelim Q1 - Answers (by K. Fukushima)

1. A competitive equilibrium is a sequence $(p_t^*, c_t^*, s_{t+1}^*)_{t=0}^T$ such that (i) $(c_t^*, s_{t+1}^*)_{t=0}^T$ solves

$$\begin{aligned} \max_{(c_t, s_{t+1})_{t=0}^T} \sum_{t=0}^T \beta^t u(c_t), \quad \text{subject to} \quad (1) \\ c_t + p_t^* s_{t+1} \leq (p_t^* + y_t) s_t, \quad \forall t \\ c_t, s_t \geq 0, \quad \forall t \\ s_0 = 1 \end{aligned}$$

and (ii) markets clear:

$$\begin{aligned} c_t^* &= y_t, \quad \forall t \\ s_t^* &= 1, \quad \forall t \end{aligned}$$

2. The equation is:

$$p_t^* = \beta \frac{u'(y_{t+1})}{u'(y_t)} (p_{t+1}^* + y_{t+1})$$

Proof: Let $(p_t^*, c_t^*, s_{t+1}^*)_{t=0}^T$ be a competitive equilibrium. Then $(c_t^*, s_{t+1}^*)_{t=0}^T$ is an interior solution to the agent's problem (1). (Interiority follows from $c_t^* = y_t > 0$ and $s_{t+1}^* = 1 > 0$.) Thus there are Lagrange multipliers $(\lambda_t^*)_{t=0}^T$ such that the Kuhn-Tucker conditions hold:

$$\beta^t u'(c_t^*) = \lambda_t^*, \quad \forall t = 0, 1, \dots, T \quad (2)$$

$$\lambda_t^* p_t^* = \lambda_{t+1}^* (p_{t+1}^* + y_{t+1}), \quad \forall t = 0, 1, \dots, T-1 \quad (3)$$

$$\lambda_T^* p_T^* = 0. \quad (4)$$

The result follows from (2), (3), and the market clearing condition $c_t^* = y_t$.

3. The final period security price is $p_T^* = 0$. To see this, combine (2) for $t = T$ and (4) to get

$$\beta^T u'(c_T^*) p_T^* = 0.$$

Since $\beta^T u'(c_T^*) > 0$ we have $p_T^* = 0$.

4. The equilibrium security price is

$$p_t^* = \sum_{j=1}^{T-t} \beta^j \frac{u'(y_{t+j})}{u'(y_t)} y_{t+j}.$$

Proof: By recursive application of the Euler equation derived in 2:

$$\begin{aligned} p_t^* &= \beta \frac{u'(y_{t+1})}{u'(y_t)} (p_{t+1}^* + y_{t+1}) \\ &= \beta \frac{u'(y_{t+1})}{u'(y_t)} y_{t+1} + \beta \frac{u'(y_{t+1})}{u'(y_t)} p_{t+1}^* \\ &= \beta \frac{u'(y_{t+1})}{u'(y_t)} y_{t+1} + \beta \frac{u'(y_{t+1})}{u'(y_t)} \beta \frac{u'(y_{t+2})}{u'(y_{t+1})} (p_{t+2}^* + y_{t+2}) \\ &= \beta \frac{u'(y_{t+1})}{u'(y_t)} y_{t+1} + \beta^2 \frac{u'(y_{t+2})}{u'(y_t)} y_{t+2} + \beta^2 \frac{u'(y_{t+2})}{u'(y_t)} p_{t+2}^* \\ &= \dots \\ &= \beta \frac{u'(y_{t+1})}{u'(y_t)} y_{t+1} + \dots + \beta^{T-t} \frac{u'(y_T)}{u'(y_t)} y_T + \beta^{T-t} \frac{u'(y_T)}{u'(y_t)} p_T^*. \end{aligned}$$

We proved in 3 that $p_T^* = 0$, so the result follows.

Answer Key to Question 2

- Let us use “money” as a brief term encompassing “money supply and monetary policy” (meaning, the interest rate when the interest rate is the instrument of monetary policy).
- It is generally agreed (Walsh; Friedman and Schwartz) that (a) money affects prices and inflation in the short and long run, and (b) money has little to no long run real effects. So focus the discussion on the more contentious issue of whether and how money affects real variables in the short run.
- Some relatively unstructured evidence has been used to argue that exogenous movements in money have resulted in movements in real variables (Friedman and Schwartz; Romer and Romer).
- Some fairly structured evidence has been used to argue that certain basic characteristics of business cycles (fluctuations, co-movements, a certain pattern of relative volatility) can be explained in purely real models (Prescott, Hansen). Critics have argued that these models reproduce some of those characteristics mechanically, and do not match a certain hump shaped response (Cogley and Nason). Also, such models do not have any hope of explaining the Phillips curve relationship (positive relationship between inflation and real activity).
- Structured evidence consistent with the money having real effects can be found in New Keynesian models. Such models require price and/or wage stickiness. Monetary movements, which are modeled as adjustments in interest rates, have real effects because they change real interest rates: expected inflation does not move one to one with the movement in nominal interest rates. "Taylor rules" describe one possible way for interest rates to be set.
- Critics have argued that some NK assumptions (such as indexation) are implausible, and that plausible parameterizations of price stickiness do not produce plausible persistence. On the positive side, NK models match some basic characteristics of macro data and business cycles. Evidence for this comes in three forms. One is direct survey evidence on stickiness of wages or prices (Klenow and Kryvtsov). A second is VAR evidence that is consistent with some very simple NK models (Blanchard and Quah, Gali). A third is formally estimated NK models (Christiano, Eichenbaum and Evans, Justiniano and Primiceri), which, according to the authors, fit the data well.

3. Properties of Equilibrium Asset Prices: Answer

1.a.

From the AR (1) assumption, $proj(D_{t+j} | H_t(D)) = \rho^j D_t$. Substitution of this formula into the stock price formulation implies that $proj(p_t | H_t(D)) = \frac{1}{1 - \rho\beta} D_t$.

1.b.

It is immediate from the answer to part a. and the assumption on the dividend process that the joint bivariate relationship is

$$\begin{aligned} p_t &= \frac{\rho}{1 - \beta\rho} D_{t-1} + \frac{1}{1 - \beta\rho} \varepsilon_t \\ D_t &= \rho D_{t-1} + \varepsilon_t \end{aligned}$$

In this formulation, dividends Granger cause prices, but prices do not Granger cause dividends.

2.

Given the MA(1) structure for the dividend process, it is immediate that

$$proj(p_t | H_t(\eta)) = (1 + \beta\xi)\eta_t$$

There are two cases for $\text{proj}(p_t | H_t(\eta))$ which need to be considered. If $|\xi| \leq 1$, then η_t is fundamental, i.e. $\eta_t = \varepsilon_t$. In this case, $\text{proj}(p_t | H_t(\eta)) = \text{proj}(p_t | H_t(D))$. If $|\xi| > 1$, then η_t is not fundamental. However, we can still determine something about the projection of prices onto dividends. By the root inversion approach we developed in the course lectures and notes, $D_t = \varepsilon_t + \xi^{-1}\varepsilon_{t-1}$ and $\text{var}(\varepsilon) = \xi^2 \text{var}(\eta)$. The relevant projection with respect to the Hilbert space $H_t(D)$ is therefore

$$\text{proj}(p_t | H_t(D)) = (1 + \beta\xi^{-1})\varepsilon_t$$

It is immediate that $\text{proj}(p_t | H_t(\eta)) > \text{proj}(p_t | H_t(D))$. This case is economically plausible. Suppose that firm dividends are largely driven by profit shocks in the previous period; this would produce the second case. The point is that forecast errors do not necessarily equal structural shocks. Further, it is obvious that $H_{t-1}(P)$ contains information about ε_t . Put differently, the 1-step ahead forecast error for dividends is smaller when price information is available, since prices contain information about η_t . In this case prices will Granger cause dividends.

3.

Under the assumptions of the problem, agents know the entire history of η_t . Therefore $E(P_t | F_t) = P_t$, so the excess holding return is always 0.

4.

The equilibrium asset price for the utility function given in the problem must fulfill

$$u'(C_t)p_t = E\left(\sum_{j=0}^{\infty} \beta^j (u'(C_{t+j})) D_{t+j} | F_t\right)$$

or

$$p_t = E\left(\sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} D_{t+j} | F_t\right)$$

Hence the original model is a special case of this model in which u' is constant, as occurs when utility is linear in consumption. No credit was given for an answer that said the models are equivalent when consumption is constant; constant consumption is an uninteresting special case.

Comment: the specification of the question ruled out bubbles by assuming second order stationarity, hence any discussion of bubbles was incorrect. Points were subtracted from answers that raised bubbles as part of the answers to the various sections.

4)

$$1) \quad J(s, h_0', h', a_k, a') = \underset{c_0', b'}{\text{MAX}} \left\{ u(c_0') + \theta v(b', h', a_k, a') \right\}$$

$$\text{s.t.} \quad c_0' + b' = s + wh_0'$$

$$V(b, h, a, a_k) = \underset{e, n, e_k, c_y, s}{\text{MAX}} \left\{ u(c_y) + \beta E_{a'/a_k} J(s, h_0', h', a_k, a') \right\}$$

$$\text{s.t.} \quad c_y + \frac{s}{1+r} + e + e_k = b + wh(1-n)$$

2) $\forall (b_0, h_0)$, let $c_y^*, s^*, e^*, e_k^*, n^*, h_0'^*, h_0''^*$ be solution to young parent's problem when $b = b_0, h = h_0$.

Within a neighborhood of (b_0, h_0) define

$$W(b, h, a, a_k) = u(c_y^*) + \beta E J(s^*, h_0'^*, h_0''^*, a_k, a').$$

Clearly, $W(b, h, a, a_k) \leq V(b, h, a, a_k)$, with " $=$ " if $b = b_0$ & $h = h_0$.

$W(\cdot)$ is concave in (b, h)

By the Beneveniste - Scheinkman theorem, $V(\cdot)$ is differentiable w.r.to b & h at $(b, h) = (b_0, h_0)$.

$$3) \quad s: \quad u'(c_y) = \beta(1+r) E u'(c_{y'})$$

$$e: \quad 1 = \frac{w}{1+r} a (nh)^{\delta_1} \delta_2 e^{\delta_2 - 1}$$

$$n: \quad wh = \frac{w}{1+r} a h^{\delta_1} e^{\delta_2} \delta_1 n^{\delta_1 - 1}$$

$$b': \quad u'(c_y) = \beta\theta(1+r) E V_1(\cdot')$$

$$e_k: \quad u'(c_y) = \beta\theta E [V_2(\cdot') a_k h^{\delta_1} \delta_2 e_k^{\delta_2 - 1}]$$

$$b: \quad V_1(\cdot) = u'(c_y)$$

$$h: \quad V_2(\cdot) = u'(c_y) \left[w(1-n) + \frac{w}{1+r} (1-\delta + a n e^{\delta_1} \delta_2 h^{\delta_1 - 1}) \right] \\ + \beta\theta E [V_2(\cdot') [1-\delta + a_k e_k^{\delta_2} \delta_1 h^{\delta_1 - 1}]]$$

$$b': \quad u'(c_y) = \beta\theta(1+r) E u'(c_{y'})$$

$$e_k: \quad u'(c_y) = \beta\theta E \left\{ \frac{u'(c_{y'}) \left[w(1-n') + \frac{w}{1+r} (1-\delta + a_k n' e'^{\delta_1} \delta_2 h'^{\delta_1 - 1}) \right]}{1 + \beta\theta (1-\delta + a' e'^{\delta_2} \delta_1 h'^{\delta_1 - 1})} \times \right. \\ \left. \times a_k h^{\delta_1} \delta_2 e_k^{\delta_2 - 1} \right\}$$

FULLIERS: (i) Reallocating one unit of consumption, at the optimum, from this period to the next, does not make the individual better off.

(ii) The marginal cost of 1 unit of investment in education equals the extra human capital produced through that investment, appropriately converted into utility terms & discounted.

- 4) With complete markets, h' depends only on a_k and h and is independent of parental ability to pay. With incomplete markets, investments in human capital will typically depend on parents' income.
- 5) The Aiyagari-Bewley result will not hold generally. In Aiyagari, in the face of incomplete markets, the individual over-accumulated assets resulting in higher K and lower r relative to complete markets. These parents may invest more in both H & K . Since $r \propto \left(\frac{H}{K}\right)^2$, depending on the degree of overaccumulation of H vis-a-vis K , we can have r greater or less than rate of time preference.
- 6) With iid ability & incomplete markets, there are two sources of intergenerational persistence
- (i) h directly affects h'
 - (ii) Borrowing constraints cause e_k to depend on h .
- With complete markets only effect (i) is present, likely decreasing persistence.