1. Answer both A (70 points) and B (30 points). The two parts are unrelated and can be answered in either order. Points for individual subsections of the two parts are given in parentheses.

Part A. Consider a standard real business cycle model, simplified so that labor supply is inelastic, and with a different capital accumulation equation:

(1) \[ \max E \sum_{t=0}^{\infty} \beta^t U(C_{t+1}) \]

s.t.

(2) \[ U(C_{t+1}) = \ln(C_{t+1}) \]

(3) \[ Y_t = \bar{A}_t \bar{K}_t \bar{N} = \bar{A}_t \bar{K}_t \]

(4) \[ Y_t = C_t + I_t \]

(5) \[ K_{t+1} = K_t (I/K_t)^{1-d} (Z_t)^{(1-d)}, \quad 0 < d < 1 \]

In (3), \( \bar{N} \) is the fixed labor supply, and is absorbed into the definition of the exogenous level of technology \( \bar{A}_t \). Other symbols are also standard: \( C = \) consumption, \( Y = \) consumption, \( K = \) capital stock, \( I = \) investment.

The novel feature is the capital accumulation equation (5), which will lead to an exact closed form solution (see below). Here, \( Z_t \) is a strictly positive exogenous technology process; \( Z_t \) is raised to the power \(-(1-d)\) only to simplify (5).^1.

Recall that the standard accumulation equation is

(6) \[ K_{t+1} = (1-\delta)K_t + I_t \]

a. Compare (5) and (6):

(5)

i. Under what special circumstances are the two identical?

(10) ii. What economic rationalization might there be for (5)?

b. Form a Lagrangian, and derive and interpret first order necessary conditions for \( C_t \) and \( K_{t+1} \). Then eliminate the Lagrange multiplier by combining the two first order conditions.

(20)

c. Let lower case letters denote logarithms: \( k_t = \ln(K_t), a_t = \ln(A_t), z_t = \ln(Z_t) \). Show that the following solution satisfies your answer to part b:

(7) \[ C_t / Y_t \] is constant;

(8) \[ k_{t+1} = \text{another constant} + (1-\delta + d\delta)k_t + (1-d)a_t - (1-d)z_t \]

Note that you are not being asked to motivate the solution (7) and (8), but are merely asked to construct a solution by "guess and verify," with the guess supplied in (7) and (8). Of course, for (7) and (8) to hold in equilibrium, they must also be consistent with (3), (4) and (5). You are not asked to show this. (If it helps in answering, you may assume that \( z_t \) and \( a_t \) follow univariate AR(1) processes, though (7) and (8) do not require this.)
d. Take (8) as given. Let \( \alpha_t \) follow a random walk, \( \alpha_t = \text{constant} + \alpha_t + \text{iid shock} \). State a condition on \( z_t \) that will imply that the log of the capital-output ratio will grow over time. (Some have argued that the ratio does grow, in the U.S. and many other countries.) Does this condition seem reasonable to you?

Part B. Consider a trivariate VAR in the three variables \( x_{1t}, x_{2t}, \) and \( x_{3t} \) whose first difference is stationary: the vector of variables in the VAR is \( X_t = (\Delta x_{1t}, \Delta x_{2t}, \Delta x_{3t})' \).

There are three mutually uncorrelated i.i.d. structural shocks, labeled \( v_{st}, v_{pt}, \) and \( v_{mt}. \) Let \( v_t \) denote the \( 3\times1 \) vector \( v_t = (v_{st}, v_{pt}, v_{mt})' \). Apart from inessential constants, the moving average representation of the VAR is

\[
X_t = \widetilde{\psi}_0 v + \sum_{j=1}^{\infty} \widetilde{\psi}_j v_{t-j} + \ldots
\]

where each moving average weight matrix \( \widetilde{\psi}_j \) is \( 3\times3 \). The authors of a certain paper assume that \( v_{mt} \) has no long run effects on levels, \( v_{pt} \) has a long run effect on the level of one variable, and \( v_{st} \) has a long run effect on the level of two variables:

- (R1) \( v_{mt} \) has no long run effect on \( x_{1t}, x_{2t}, \) or \( x_{3t} \),
- (R2) \( v_{pt} \) has a long run effect on \( x_{1t} \) but not \( x_{2t}, x_{3t} \),
- (R3) \( v_{st} \) has a long run effect on both \( x_{1t} \) and \( x_{2t} \) but not \( x_{3t} \).

(20) 1. State restrictions on the \( \{\widetilde{\psi}_j\} \) that impose (R1)-(R3).

(10) 2. A paper using the restrictions above computed variance decompositions for \( \Delta x_{1t} \) and for \( x_{1t} \), for horizons of 1, 4, 8 and 20 quarters. Results are given below, but without identifying which set of results is for \( \Delta x_{1t} \) and which is for \( x_{1t} \): "variable #1" is either \( \Delta x_{1t} \) or \( x_{1t} \), and "variable #2" is the other. (Totals may not add to 100 due to rounding.) Which decomposition is typical for a differenced variable such as \( \Delta x_{1t} \), which for a level variable such as \( x_{1t} \)? Why?

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2. (100 points) The Setup
Consider a two period endowment economy with a continuum of individuals of two types. The total population size is normalized to 2 and each type has measure 1. Endowments are i.i.d. over time, common across agent types, and take on the values 0 or 1 in each period with equal probability of \(\frac{1}{2}\). Thus the aggregate endowment is constant at 1 in each period. We can think of the relevant state of nature \(s_t \in \{0, 1\}, \ t = 1, 2\) as an i.i.d. random variable which determines which type of agent receives the high endowment (of 1) in period \(t\). We will consider different variations on asset structures and constraints, but in each case the timing is such that endowments are realized prior to consumption decisions being made. Both types \(i = 1, 2\) of agents have the common preferences over consumption \(c_t\) at the two dates:

\[
U(c_1, c_2) = E \left[ \sum_{t=1}^{2} \beta^{t-1} u(c_t) \right] = -E \left[ (c_1 - 2)^2 + \beta (c_2 - 2)^2 \right],
\]

where \(0 < \beta < 1\). We allow consumption to be negative, so don’t worry about any nonnegativity constraints.

1. (50 points total) Suppose that agents can trade in a complete set of state-contingent securities (in zero net supply) at date 0.

(a) (20 points) State explicitly what the relevant securities are, and solve fully for the equilibrium allocation of consumption for the two agent types. Evaluate the ex-ante utility \(U(c_1, c_2)\) of each of them.

(b) (20 points) Show how the equilibrium allocation is implemented — that is, what are the equilibrium holdings of the different securities by each type?

(c) (10 points) What is the equilibrium rate of return \(r\) on a risk-free bond? (The bond is purchased at date 1, yielding a sure payoff of a unit of consumption at date 2.)

2. (50 points total) Now consider an incomplete markets economy, in which agents can only trade in a risk free bond in zero net supply. Let \(a_t^i\) be the holdings of this bond at the start of period \(t\) for each type \(i\), and suppose that there is a non-binding natural debt limit on \(a_t^i\). Each agent is endowed with \(a_t^i = 0\) initial holdings, so the holdings at date 2 are:

\[
a_2^i = (1 + r)(e_1^i - c_1^i)
\]

There is no reason to carry bonds after period 2 so:

\[
c_2^i = a_2^i + e_2^i.
\]

(a) (20 points) Taking the interest rate \(r\) as given, solve for each agent’s optimal (state-contingent) consumption and bond holdings \(\{c_t^i, a_t^i\}\).

(b) (20 points) Using the results of the previous part, solve for the equilibrium interest rate \(r\), and hence the equilibrium consumption allocation.

(c) (10 points) How do the interest rates and the allocations compare to the complete markets case? Interpret your answer. (For this, it may help to consider what happens as \(\beta \to 1\).)
Question 3: Fiscal Policy Advice (100 points)

Suppose that the US government is considering a fiscal stimulus of $500 billion. You are asked to provide an assessment of the effects of this stimulus on the economy. Explain how you would do this using 1) structural macroeconomic models and 2) atheoretical time series models. Be explicit in describing models and methods and indicate how you would address issues of model uncertainty. Finally, address the extent to which 1) and 2) can be integrated your policy assessment.