
January 5, 2007, from 9:00 AM to 12:30 PM.

• On the top of EACH yellow sheet, write your ASSIGNED NUMBER, date, and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the questions sheets and yellow pads will be collected. Do not write on the question sheets.
• This is a closed book exam.
• Please solve any two (2) of the three (3) problems. The time assigned to each is one hour and fifteen minutes. Thus, you have an extra hour to read the questions and revise your answers.
• The total time allotted is three and one half (3 and 1/2) hours.
• Each problem receives the same weight (100 points). The points allocated to each subsection are indicated in each problem.
• If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
• Read the problems carefully and completely before you begin your answer. The problems will not be explained. If you think that a problem is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.
• Please return unused portion of the yellow tablets.
• There are 5 pages in this exam (including this cover page). Please make sure that you got all of them.
• Good luck!
1 Problem # 1: Credit Markets and Economic Growth (100 Points)

1.1 Perfectly Functioning Credit Market

Think of an economy populated by two types of infinitely-lived individuals: entrepreneurs and workers. There are measure 1 of entrepreneurs and measure $\mu$ of workers, and all individuals behave competitively. There is no population growth. A worker supplies 1 unit of labor and gets paid the market wage in any given period. An entrepreneur, in any given period, operates a technology that produces output by using labor ($l$) and capital ($k$):

$$f(k, l) = k^{\alpha} l^{\gamma}.$$

It is assumed that $\alpha > 0$, $\gamma > 0$, and $\alpha + \gamma < 1$. Capital depreciates at the rate of $\delta$. The output can be either consumed or invested. Individuals' preference is given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_{i,t}^{1-\sigma}}{1-\sigma},$$

where $i$ indicates the type: an entrepreneur ($e$) or a worker ($w$).

Assume that the initial capital stock in the economy is equally distributed among all the individuals—all workers are alike, and all entrepreneurs are alike. Further assume that the economy has a perfectly functioning capital market, subject only to the natural borrowing limit. Productive capital is the only asset in the economy.

The timeline within a period is as follows. An entrepreneur rents capital (from herself as well as from others) and hires labor. After output is produced, she pays compensation for all the factors of production. Finally, she makes her consumption/savings decision. Likewise, a worker supplies capital and labor, and then gets paid, before she makes her consumption/savings decision.

1. (6 points) Take a period $t$. Given the wage, $w_t$, and the net interest rate, $\tau_t$, compute the quantity of capital and labor demanded by a profit-maximizing entrepreneur. Then, compute the indirect profit function, and denote it by $\pi(w_t, \tau_t)$.

2. (6 points) Write down an entrepreneur's problem with sequential budget constraints. In the budget constraint, use the indirect profit function for notational convenience. Let $a_{e,t}$ denote the wealth (capital holding) of the individual entrepreneur at the beginning of $t$.

3. (3 points) Write down a worker's problem with sequential budget constraints. Let $a_{w,t}$ denote the wealth (capital holding) of the individual worker at the beginning of $t$. 
4. (20 points) Using your answers to the questions above, define the competitive equilibrium for this economy. Carefully list all the prices and allocations that constitute the equilibrium. Be explicit about all the market clearing conditions.

5. (5 points) Argue that the solution to the planner's problem will yield the competitive equilibrium allocations.

6. (15 points) Let's assume that we are only interested in characterizing the time paths of the aggregate variables. Can you formulate the planner's problem recursively? If so, write down the Bellman equation. If not, explain why.

7. (5 points) If we re-distribute the initial capital among the individuals in a way that implements a mean-preserving spread, will the time path of the aggregate output be affected? Why or why not?

1.2 Shutting Down the Credit Market

Now we shut down the credit market so that capital cannot be traded at all. All other things remain as they were. Any individual can still accumulate capital. Note that capital depreciates at the rate of $\delta$.

1. (6 points) Take a period $t$. Given the wage, $w_t$, compute the quantity of labor demanded by a profit-maximizing entrepreneur who has $k_t$ units of capital. Then, compute the indirect profit function, and denote it by $\pi(k_t, w_t)$.

2. (5 points) Write down an entrepreneur's problem with sequential budget constraints. In the budget constraint, use the indirect profit function for notational convenience. Let $k_t$ denote the capital holding of the individual entrepreneur at the beginning of $t$.

3. (2 points) Write down a worker's problem with sequential budget constraints. Let $a_{w,t}$ denote the capital holding of the individual worker at the beginning of $t$.

4. (15 points) Using your answers to the questions above, define the competitive equilibrium for this economy. Carefully list all the prices and allocations that constitute the equilibrium. Be explicit about all the market clearing conditions.

5. (6 points) Can we solve the planner's problem to characterize the competitive equilibrium allocations? Explain.

6. (6 points) If we re-distribute the initial capital among the individuals in a way that implements a mean-preserving spread, will the time path of the aggregate output be affected? Why or why not?
2 Problem # 2: Cagan’s Model of Inflation (100 Points)

Consider the Cagan hyperinflation model, which relates \( m_t \), the log of the money supply, to \( p_t \), the log of the price level, via the relationship

\[
m_t - p_t = \gamma (p_{t+1}^e - p_t)
\]

where \( p_{t+1}^e \) denotes the expected value of \( p_{t+1} \) at time \( t \).

A Characterize all equilibrium price level sequences when expectations are rational and the money supply process obeys

\[
m_{t+1} = \rho m_t + \xi_{t+1}
\]

where \( \xi_{t+1} \) is white noise. When will the price level at \( t \) be finite? Explain.

B Suppose the realized \( p_t \) sequence is nonstationary in levels. Assume that the money supply process is the AR(1) process above and that the parameter \( \rho \) is known.

1. What inferences can be drawn about the Cagan specification and about the presence or absence of a bubble in prices? Make sure to relate your answer to the value of \( \rho \).

2. Repeat 1 assuming that while \( p_t \) is nonstationary in levels, it is stationary in first differences.

3 Problem # 3: Heterogeneity and Asset Prices (100 Points)

Consider an economy populated by two types of individuals. Each agent of type I initially owns one type I tree. These trees drop dividends according to the stochastic process:

\[
y_t^I = s_t y.
\]

Each type II individual initially owns a type II tree, whose dividends satisfy

\[
y_t^{II} = (1 - s_t) y,
\]

where the stochastic process \( \{s_t\} \) is such that \( 0 \leq s_t \leq 1 \). Assume that, at time 0 (before \( s_0 \) is realized) all agents believe that the distribution of \( s_0 \) is given by

\[
Pr[s_0 \leq s] = F(s).
\]

All individuals of type \( j \) (\( j \in \{I, II\} \)) have preferences given by

\[
E \left[ \sum_{t=0}^{\infty} \beta^t u_j(c_t) \right],
\]
where \(0 < \beta < 1\), and the functions \(u_j(c)\) are “nice” (i.e. strictly increasing, strictly concave, twice differentiable and whatever else you need short of a functional form).

Assume that the economy has complete markets (i.e. a full set of Arrow securities) which open just before the realization of \(s_0\).

1. Consider the special case in which \(F(s)\) is uniform between 0 and 1, and that the distribution of future values of \(s_t\) is i.i.d. (not necessarily equal to \(F(s)\)). Go as far as you can characterizing the equilibrium allocation of consumption, the one period risk-free interest and the prices of the two types of trees. Show your work.

2. How would your answers change if the complete set of markets opens after the realization of \(s_0\) but before the realization of \(s_1\).

3. Assume now that the economy is as in 1, except that future (i.e. for \(t \geq 1\)) values of \(s_t\) are given by a Markov process with transition probability density given by \(\pi(s_{t+1}, s_t)\). Go as far as you can describing how the equilibrium consumption allocations and the one period risk free interest rate in this economy differ from the equivalent concepts in the economy of part 1. Show your work.

4. For the economy of section 2 discuss the following claim: “Independently of the coefficient of risk aversion in individual utility functions, the prices of both types of trees are given by the present expected discounted value —using the risk free interest rate— of their future dividends.

5. Consider the basic economy, except that \(F(s)\) is not uniform, and the distribution of \(s_t\), for \(t \geq 1\), is arbitrary. Go as far as you can describing properties of individual consumption allocations and asset prices (bonds and stocks in the two types of trees).