UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

January 10, 2005
9:00 am - 12:30 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:
  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions.

Example:
MACRO THEORY 1/10/05
ASSIGNED #
Qu # 1 (Page 2 of 4)

- **Do not answer more than one question on the same page!**
  When you start a new question, start a new page.

- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.

- **Please DO NOT WRITE on the question sheets.**

- Please solve any two of the three problems.

- You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- Please return unused portions of the yellow tablet & question sheets.

- There are 6 pages in the exam – please make sure you have all of them.

- Good luck!
• On the top of EACH yellow sheet, write your **ASSIGNED NUMBER**, date and name of exam and question number. **DO NOT** write your name on the yellow pads. After the examination, the question sheets and yellow pads will be collected. **DO NOT** write on the question sheets.

• You are not allowed to use notes, books, calculators, or colleagues.

• Please solve three (3) and only three (3) out of the four (4) problems.

• The total time allotted is three (3) hours. Each problem has a suggested time of an hour.

• If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

• Read the questions carefully. The questions will not be explained. If you think that a question is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.

• Please return unused portion of the yellow tablets and question sheets.

• There are five (5) pages in this exam (including this cover page). Please make sure that you have all of them.

• Good Luck!
Problem I: Dynamic Programming (100 points)

1. Location choice (45 points): Consider a household which can be located on one of three islands in a given period. Label the islands $i = 0, 1, 2$. The household enjoys utility $u^i$ if it is located on island $i$ in a given period. Assume that $u^0 = 0$ and that $u^i > 0$ for $i = 1, 2$. The household lives forever and discounts at a rate of $\beta$ with $0 < \beta < 1$.

Households are restricted in terms of their movement between islands. If the household is located on island $i$ in the current period, the probability of being on island $j$ in the next period is given by $\pi_{ij}$. Assume that the household is unable to go directly from island $i = 1, 2$ to island $j = 1, 2$, for $i \neq j$ without going through island 0. Thus a household on island $i = 1, 2$ in period $t$ remains on that island or switches to island 0 for period $t + 1$.

a. Write down a transition matrix describing the evolution of the household from one period to the next. What is the probability that a household on island 1 in the current period is on island 2 two periods from now? (5)

b. Write down the system of equations (the Bellman equations) which describe the value of being on each of the three islands. To do so, let $V^i$ be the value of being on island $i = 0, 1, 2$. Compute $V^i$, $i = 0, 1, 2$ in terms of the primitives of the model. (15)

c. Explain why an increase in $u^1$ will increase $V^0$, $V^1$ and $V^2$. (5)

d. Suppose that, in addition to the assumptions made above, $\pi_{10} = \pi_{20} = 0$. Further suppose that $u^2 > u^1$. Prove that $V^2 > V^1$. Provide conditions such that $V^0$ is greater than $V^1$. Explain in these conditions. (20)
2. Stochastic cake eating problem (55 points): Consider a discrete, stochastic cake eating problem. The consumer can either eat the cake or store it until the next period. The utility flow from eating a cake of size $W$ is $\epsilon u(W)$. Assume $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$ and $\epsilon$ is an iid random variable. If the cake is not consumed in the current period then its size in the next period is given by $W' = \rho W$. There is no restriction that $\rho = 1$. The discount factor, $\beta$, satisfies $0 < \beta < 1$.

a. Write down Bellman’s equation for this problem. (5)

b. What are the state and the control variables for this problem? (5)

c. Define a cut-off rule. Prove that the optimal solution to this problem satisfies a cut-off rule. Show that the critical value of $\epsilon$ is independent of $W$. (30)

d. Suppose that $\rho$ falls. How would this influence the solution (the value function and the policy function) to this problem? (15)
1 Problem # 2: Migration and Housing Prices

Consider an economy in which individuals have preferences given by:

\[ U_i = \sum_{t=0}^{\infty} \beta^t u(c_t) \quad 0 < \beta = \frac{1}{1+\rho} < 1, \]

where each \( u \) is twice differentiable strictly increasing and strictly concave. (If necessary, assume that the Inada conditions hold). Individuals differ in terms of where they live. There are two regions, \( A \) and \( B \). An individual located in region \( A \) produces \( x_B \), while an individual located in region \( B \) produces \( x_B \). Each individual consumes one (homogeneous) unit of housing. Let the period rent of a house in region \( j \) be denoted \( q_j \). Let the interest rate in region \( j \) be denoted by \( r_j \). The budget constraint of an individual located in region \( j \) is

\[ c_{jt} + q_{jt} + b_{jt+1} \leq x_j + (1 + r_{jt})b_{jt}. \]

Individuals are free to move between the two regions. Normalize total population to 1, and let the population of region \( j \) be denoted \( N_j \) (it follows that \( N_A + N_B = 1 \)).

1. (25 points) Argue that there are rent prices, \( q_A \) and \( q_B \), such that there is a steady state equilibrium in which no individuals want to migrate and \( b_{jt} = 0 \), \( j = A,B \). Note: For individuals to be indifferent between the two regions the utility levels must be the same in both regions.

2. (20 points) Let \( p_j \) be the price of a house in region \( j \). Show how house prices depend on house rents in each region. Argue that, given that individuals have access to credit markets, it does not matter whether they own or rent their houses.

3. (25 points) Assume that the cost of producing \( N_j \) houses (recall that each person consumes one unit of housing) in region \( j \) is \( Z(N_j) \). Let the marginal cost \( z(N_j) = dZ(N_j)/dN_j \) be increasing in the number of houses, \( N_j \). Assume that competition among builders (which do not live in this region) is such that house prices equal marginal cost. Go as far as you can arguing that the (endogenous) equilibrium size of each region depends on their productivity level. Assume that \( x_A > x_B \). Is this sufficient to imply that region \( A \) is larger, i.e. \( N_A > N_B \).

4. (30 points) Assume that individuals own their houses and that houses last forever. Thus, if there are \( N_A^* \) houses in region \( A \) and if some people migrate out of region \( A \) then competition among those who want to sell their houses will imply that house prices are zero. Assume that there is a permanent, unanticipated shock to labor productivity in region \( A \). Let the new productivity level be \( \tilde{x}_A = x_A(1+\gamma) \), with \( \gamma > 0 \). Assume no borrowing and lending. Go as far as you can analyzing whether it is possible to have a (post-shock) steady state equilibrium in which no
individuals migrate. In this case, describe the implications of the model for the effect of a productivity shock on housing prices.
Problem III: Linear Quadratic Life Cycle Model

Consider a finite horizon, or life cycle, consumption model. Individuals range in age from 0 to $D$, and $D$, the finite age of death, is known and constant. Individuals leave no bequests. In period $t$, an individual who is $N$ years old maximizes

$$-\frac{1}{2}E_t \sum_{j=0}^{D-N} \beta (C_{t+j} - \bar{C})^2$$

s.t.

$$A_{t+j+1} = (1+r)A_{t+j} + Y_{t+j} - C_{t+j} A_{t+(D,N+1)} = 0$$

Symbols are: $E_t$ is mathematical expectations conditional on current and past values of one’s own income and consumption; $\beta$ is the discount rate, $\beta = (1+r)^{-1}$, $r$ = constant interest rate, $r > 0$; $C_t$ is consumption; $\bar{C}$ is the bliss or satiation level of consumption; $Y_t$ is labor income; $A_t$ is nonhuman wealth. An individual who is $N$ years old has $D-N+1$ years to live, so $A_{t+(D,N+1)} = 0$ is the statement that people leave no bequests.

You may assume an interior solution ($C < \bar{C}$) throughout. In answering each part of the question, you may use results of previous parts of the question, even if you were unable to establish those results. For example, in parts (b)-(g), you may assume part (a) holds, whether or not you were able to show part (a).

a. (10 points) Show that

$$-\frac{1}{2}E_t \sum_{j=0}^{D-N} \beta (C_{t+j} - \bar{C})^2 = \frac{1}{\beta} A_t.$$

b. (10 points) Show that for a person alive in $t$ and $t+j$,

$$E_t C_{t+j} = \bar{C}_t.$$

c. (15 points) Let income be i.i.d., $Y_t = \epsilon_t$. Show that for a person of age $N$

$$\frac{\partial C_t}{\partial \epsilon_t} = \theta_N,$$

where $\theta_N = (1-\beta)/(1-\beta^{D-N+1})$. Does the response of consumption to an income shock increase or decrease with age? What is the intuition to this result?

d. (15 points) Let income follow a random walk, $Y_t = Y_{t-1} + \epsilon_t$. Show that

$$\frac{\partial C_t}{\partial \epsilon_t} = 1$$

for any age $N$. Why is the response larger than in the i.i.d. case?

e. (15 points) “Savings in the U.S. have declined because of the aging of the population.” Is such a decline predicted by this model?

f. (15 points) Return to the i.i.d. case (part c). What is the impulse response of $C_t$ to a shock to income, for horizons of 1, 2 and 3 periods, for a person of age $N$ who has at least 3 periods to live ($N \leq D-3$)?

g. (20 points) Return to the random walk case (part d). Suppose that the shocks are heterogeneous, so that for individual $i$, $Y_{it} = Y_{it-1} + \epsilon_{it}$; that the shocks are i.i.d. across both time and individuals, $E_{t+s} \epsilon_{it} = 0$ if $i \neq j$ or $t \neq s$; that population is constant; that every individual starts out with the same asset level, say $A_0 = 0$. Is the cross-sectional variance of consumption increasing or decreasing with age: is there more variability of consumption across individuals who are (say) 40 years old than across those who are (say) 30 years old?