
January 17, 2002 from 9:00 to 12:30

- On the top of EACH yellow sheet, write your ASSIGNED NUMBER, date, and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the question sheets and yellow pads will be collected. Do not write on the question sheets.

- This is a closed book exam.

- Please solve any two (2) of the four (4) problems. The time assigned to each is one hour and fifteen minutes. Thus, you have an extra hour to read the questions and revise your answers.

- The total time allotted is three and one half (3 and 1/2) hours.

- Each problem receives the same weight (100 points). The points allocated to each subsection are indicated in each problem.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- Read the problems carefully and completely before you begin your answer. The problems will not be explained. If you think that a problem is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.

- Please return unused portion of the yellow tablets.

- There are 7 pages in this exam (including this cover page). Please make sure that you got all of them.

- Good luck!
1 Problem # 1: News, Interest Rates, and Asset Prices (100 points)

Consider an economy populated by a large number of identical households with preferences given by

\[
\max E_0 \left[ \sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\eta}}{1-\eta} \right] \quad 0 < \beta = \frac{1}{1+\rho} < 1, \ \eta > 0. \tag{1}
\]

Each household owns a tree that drops dividends, \( \{x_t\} \), according to the following, stochastic process

\[
\frac{x_{t+1}}{x_t} = \gamma(z_t) \varepsilon_{t+1},
\]

where \( z_t \) and \( \varepsilon_t \) are mutually independent and i.i.d. random variables. Assume that \( \gamma(z_t) \) is increasing in \( z_t \); that is, high values of \( z_t \) signal higher future growth rate. In this sense, \( z_t \) is a form of news about future dividends.

The information in this economy unfolds according to the following sequence. Each period \( t \) is divided between morning (\( m \)) and afternoon (\( a \)). In the morning of period \( t \), current \( x_t \) is observed. In the afternoon, \( z_t \) is observed. Consumption does not change between morning and afternoon and there is no discounting between these two periods.

1. (10 points) Define a competitive equilibrium in which trees and one period risk free bonds are traded. (You may include other assets, but make sure that those two are in the set of feasible assets.)

2. (30 points) Ignore the morning periods (i.e. assume that at time \( t \) the information set includes \( \{x_t, z_t\} \)). Describe the equilibrium stochastic process followed by the short term (one period) interest rate.

3. (30 points) Ignore the morning periods (i.e. assume that at time \( t \) the information set includes \( \{x_t, z_t\} \)). Describe the equilibrium stochastic process followed by the price of trees (i.e. stock prices).

4. (30 points) For any asset price given by \( v(x_t, z_t) \), let the period \( t \) morning price be \( v^m(x_t) = E[v(x_t, z_t)] \), where the expectation is taken over the random variable \( z_t \). Let \( v^a(x_t) \) denote the price "before news," \( v(x_t, z_t) \) the price "after news." Discuss the following claim: "The arrival of good news about the economy (a relatively high value of \( z_t \)) that result in increases in interest rates and a decrease in the value of the stock market is evidence that the Fed reacts to good news by pushing interest rates up. The reason for this is simple. In the absence of a response by the Fed good news about future dividends would have pushed stock prices up."
2 Problem # 2: Financing Unemployment Insurance (100 Points)

Consider an economy in which all workers have preferences given by

\[ E[\sum_{t=0}^{\infty} \beta^t y_t] \]

where \( 0 < \beta < 1 \), and \( y_t \) is income at time \( t \). A worker can be either employed (and not searching for another job) or unemployed (and searching for a job). Every job — regardless of the wage it pays — is destroyed with probability \( 1 - \eta \). If an individual is employed at a job that pays \( w_t \) his/her income is \( y_t = (1 - \tau)w_t \), where \( 0 < \tau < 1 \) is a tax rate. If an individual is unemployed (and searching) he/she collects unemployment compensation equal to \( c \), and draws a wage offer \( w \) from the distribution \( F \). Assume that the support of \( w \) is the bounded interval \([0, B]\), where \( B \) is arbitrarily large. Moreover, assume that \( F \) has a density \( f \) that is everywhere positive on the support of the distribution. Thus,

\[ F(w) = \int_{0}^{w} f(x)dx. \]

The government runs an unemployment insurance system. It collects taxes from those employed and uses to proceeds to finance payments to the unemployed \((c)\). Assume that the economy has reached its steady state.

1. (10 points) Describe the worker’s problem and argue that the optimal strategy is of the reservation wage variety. Denote the reservation wage by \( z \).

2. (10 points) Taking \( c \) as given, show how the reservation wage varies with \( \eta \).

3. (30 points) Describe the distribution of jobs as a function of wages and compute the mean wage of those employed, \( \bar{w}^e \). Argue that as \( z \) increases, so does \( \bar{w}^e \).

4. (30 points) Prove that for any \( \tau \) there is a unique equilibrium that satisfies both individual maximization and the government budget constraint. [Note: This section requires you to use the government budget constraint to describe \( c \) as a function of \((\tau, z)\).]

5. (20 points) Go as far as you can describing the impact on \( \bar{w}^e \) and the long run unemployment rate of

a) Increases in \( \tau \).

b) Increases in the degree of job security, \( \eta \).
3 Problem # 3: Innovation and Equilibrium (100 points)

Consider an economy with a consumption good $Y$ produced using an intermediate input according to the production function:

$$Y = Ax^\Phi$$

where $\Phi \in (0, 1)$, $x$ is an intermediate good, and $A$ is a productivity parameter.

There is one type of labor in this economy, with total supply $N$, that can be used in either the intermediate good production or in research. The intermediate good is produced one-for-one with labor:

$$x = L.$$ 

Research produces innovations at a Poisson arrival rate of $\lambda n$ where $n$ is the flow of labor used in research and $\lambda$ is a constant.

Time is continuous and indexed by $\tau$. The index $t = 0, 1, \ldots$ indicates the interval starting with the $t^{th}$ innovation and ending with the $(t + 1)^{th}$ innovation. Prices and quantities will be assumed constant at each interval.

Each innovation is a new intermediate good that can produce the same level of output $Y$ with a lower level of $x$, all else equal, than the previous innovation. In other words, productivity of the intermediate good in consumption good production increases by $\gamma > 1$. The innovator can obtain a monopoly via an infinitely-lived patent. Other markets are perfectly competitive. Assume the rate of interest equals the rate of time preference and is denoted by $r$.

i) (20 points) Assume innovations are always drastic, in the sense that new intermediate goods make the others obsolete. Solving the profit-maximization problems of the intermediate good monopolist and the firms in the research sector, derive an equilibrium condition for the level of research labor $n_t$ at an interval $t$. Use this condition to describe a stationary equilibrium whereby $n_t = \tilde{n} \forall t$.

ii) (30 points) Using this stationary equilibrium condition derived above, determine whether $\tilde{n}$ increases or decreasing for the following changes:

- an increase in $r$
- a decrease in $\gamma$
- an increase in $N$
- a decrease in $\lambda$
- an increase in $\Phi$

Briefly provide an economic intuition for each of these results.
iii) (30 points) Now consider the problem of a welfare-maximizing social planner. Solve for the welfare-maximizing level of $n_t = n_t^*$. 

iv) (20 points) Compare the expressions for the allocation of labor to research in ii) and iii). Provide an economic interpretation for the differences in these expressions, and describe conditions under which the allocation of labor to research is larger or smaller in the social planner's case than in the decentralized equilibrium.

Derive an expression for the average rate of growth of output in order to consider how differences in $n_t$ translate into differences in the growth rate of output.
4 Problem # 4: Monopoly Power with Production Experience and Other Dynamic Problems (100 points)

1. Consider the problem of a monopolist introducing a new product. The monopolist desires to maximize the present-value of his profits. He faces a constant interest rate $r$. In each period the monopolist faces the downward sloping demand curve described by

$$p = D(q) \text{ with } D(0) = \bar{p}, D(\infty) = p \text{ and } D'(q) < 0.$$ 

That is, according to the demand schedule $D$ the monopolist can sell $q$ units of output at price $p$. Marginal revenue is assumed to be strictly decreasing in $q$. Let per unit cost of production be $c$. This cost declines convexly with production experience, $e$, according to

$$c = C(e), \text{ with } C(0) < \bar{p}, C(\infty) > p, C'(e) < 0, \text{ and } C''(e) > 0.$$ 

Production experience is taken to be the cumulative sum of past production, so the law of motion governing experience is given by

$$e' = e + q.$$ 

(a) Formulate the monopolist’s dynamic programming problem. (5 points)

(b) Compute the first-order condition and the Euler equation associated with this problem. Interpret it. Prove that the decision-rule for $q$ is strictly increasing. (15 points)

(c) Consider the problem of a myopic monopolist who maximizes current period profits. Who produces the most: the myopic or the far-sighted monopolist? (20 points)

2. Consider the set $S = R$ with $\rho(x, y) = f(|x - y|)$, where $f : R_+ \to R_+$ is continuous, strictly increasing and strictly concave with $f(0) = 0$. Is the set $S$ together with the metric $\rho$ a metric space? (20 points)

3. Consider the problem of choosing a consumption sequence to maximize

$$\sum_{t=0}^{\infty} \beta^t \{ \log(c_t) + \gamma \log(c_{t-1}) \}, 0 < \beta < 1, \gamma > 0$$

subject to

$$c_t + k_{t+1} \leq Ak_t^\alpha, A > 0, 0 < \alpha < 1, \text{ and } k_0, c_{-1} \text{ given.}$$

Here $c_t$ denotes consumption at time $t$, and $k_t$ is the capital stock at the beginning of period $t$. Note that consumption purchases generate a stream of utility flow that extend for more than one period.
(a) Formulate Bellman's functional equation. (10 points)

(b) Derive a closed-form solution to the value function and the policy function. (30 points)