Question #1: “Interest Rates and the Demand for Durables”

Consider an economy populated by a large number of identical households each maximizing the following utility function

\[ U = \sum_{t=0}^{\infty} \beta^t u(c_t, z_t), \quad 0 < \beta < 1, \]

where \( c \) is interpreted as non-durable consumption, and \( z \) is the stock of durables available at time \( t \). The representative household has a budget constraint given by,

\[ c_t + p_{zt}x_{zt} + x_{kt} + b_{t+1} \leq w_t + [(1-t_o)\tau + \delta_p]k_t + R_t b_t, \]

\[ k_{t+1} \geq (1-\delta_p)k_t + x_{kt}, \]

\[ z_t \leq (1-\delta_{zt})z_{t-1} + x_{zt} - (\gamma/2)(z_t - z_{t+1})^2. \]

The last term, \((\gamma/2)(z_t - z_{t+1})^2\), captures cost of adjustment in the stock of durables. Each household supplies inelastically one unit of labor. The economy is open, and \( R_t \) is the interest rate on one-period international bonds between time \( t-1 \) and \( t \). The production side of the economy is fully described by a constant returns to scale production function, \( F(k_t,n) \), which maps total capital and total labor into consumption. Assume (if necessary) that the proceeds from the capital tax, \( \tau_o \), are used to finance usefulness government consumption.

a) Assume that \( R_t = R^* = \beta^t \), \( p_{zt} = p_{zt} \), \( \tau = \tau \) and \( \delta_p = \delta_p \). Describe the steady state. Does the economy converge immediately to the steady state? Explain.

b) Assume, in addition to the assumptions in a), that \( u(c,z) = [\eta c^{\rho} + (1-\eta)z^{\rho}]^{(1-\sigma)\rho} \) where \( 1/(1+\rho) \) is the elasticity of substitution between non-durable and durable consumption. What are the predictions of the model for the ratio \( z/c \) in the steady state for two economies that differ in the (constant) price of durable goods, \( p_{zt} \).

c) Assume that \( u(c,z) = [\eta c^{\rho} + (1-\eta)z^{\rho}]^{(1-\sigma)\rho} \), \( p_{zt} = p_{zt} \), \( \tau = \tau \), \( \delta_p = \delta_p \), and \( \gamma = 0 \) (no adjustment costs). Let \( R_0 = R_t = \beta^t \), for \( t \geq 2 \). Assume that \( R_1 = (1+\varepsilon)\beta^t \), and \( \varepsilon > 0 \). What can you say about the relative demand for durables (the ratio \( z/c \)) at times 0 and 1? Does the qualitative nature of your answer depend on the elasticity of substitution?

d) Assume that \( u(c,z) = [\eta c^{\rho} + (1-\eta)z^{\rho}]^{(1-\sigma)\rho} \), \( R_t = R^* = \beta^t \), \( p_{zt} = p_{zt} \), \( \tau = \tau \) and that \( \gamma = 0 \) (no adjustment costs). Assume now that technological progress increases “durability” (decreases \( \delta_p \)). In particular, assume that \( \delta_{p0} > \delta_{p1} > \delta_{p2} > \delta_{p3} = \delta_{p4} = \ldots = \delta_{px} \). Go as far as you can describing the dynamics of the ratio \( z/c \).
Question #2: “Real Effects of Deficits”

Consider a standard one sector growth model with government spending and infinitely lived households. Assume that each household supplies one unit of labor inelastically and that the aggregate production function is concave, twice differentiable, homogeneous of degree one in capital and labor and that, fixing the level of employment at one, there is a maximum sustainable capital stock.

The government consumes $g > 0$ units of the single good in every period. To finance this consumption it uses a mix of taxes and bonds.

a) Assume that the government uses lump sum taxes and one period bonds and that markets are perfect. Consider a feasible policy and show the effect on the real allocation (consumption, investment and the capital stock) of a decrease in first period taxes, followed by a budget balancing increase in tax revenues in period T. How does your result depend on T (if at all)?

b) Assume that the government uses lump sum taxes and one period bonds as in a). Assume that markets are perfect. The government decides to decrease government consumption in the first period by one unit and increase it in the second period by an amount equal to $R$, where $R$ is the real rate of return at time 1 in the original equilibrium. Will this policy have real effects? Explain briefly why or why not?

c) Assume that the representative agent cannot borrow; capital markets are imperfect. (Assume that the representative agent can lend or save). Does your analysis of the policy change in a) need to be modified? Explain.

Note: In answering a)-c) please present a formal model to make your point. The details are up to you but the model must be consistent with the given assumptions.

Question 3

Suppose that you are an advisor for the World Bank and have been asked to determine whether or not a given set of countries is caught in a development trap. In answering the following the following, be sure to be as explicit as possible in your discussion.

A. Provide a rigorous definition of a development trap.

B. Describe an econometric strategy for uncovering the presence of a development trap for the set of countries. Specifically, construct a test of the development trap hypothesis. What factors will determine the power of your proposed tests?

C. How would you formulate and incorporate model uncertainty in your answer to B.? Include variable selection and parameter heterogeneity as forms of model uncertainty in answering.
Question #4

Suppose that you are asked to assign $N \cdot K$ workers to $K$ firms in order to maximize aggregate output. Suppose that each firm produces output $y_i$ according to:

$$y_k = \phi(p_k)$$

where $p_k$ is a vector which represents the production inputs associated with the $K$ workers assigned to $k$.

A. Suppose that $\phi$ is supermodular and that $p_i$ is exogenously determined for each worker $i$. What does this imply about the output maximizing allocation of workers across firms? Formally justify your answer.

B. Suppose that each worker's production input can be written as

$$p_i = \xi(a_i, a_{-i})$$

Here, $a_i$ is an exogenous measure of ability and $a_{-i,k}$ denotes the abilities of others with whom worker $i$ is working.

How does this affect the optimal sorting problem? Be sure to relate your answer to the properties of $\xi$.

C. Suppose that individual productive inputs have the functional form

$$p_i = \beta_0 + \beta_1 a_i + \beta_2 \bar{a} + \beta_3 \bar{p}$$

where $\bar{a}$ is the average ability of all workers, including $i$, in $i$'s firm and $\bar{p}$ is the average productive input of all workers, including $i$, in $i$'s firm. If one had individual-level data on $a_i$ and $p_i$, are the parameters of this relationship identified? Would knowledge of the firms' outputs be useful?

D. How would your answer to C. change if

$$p_i = \beta_0 + \beta_1 a_i + \beta_2 \bar{a}_{-i} + \beta_3 \bar{p}_{-i}$$

where, as before, the subscript "-$i$" denotes the firm average not including $i$. 