This asks you to analyze the effects of indexation of wages on output volatility. Consider an economy governed by the following set of equations:

1) \( y_t = \theta n_t + a_t \)  
(production function)

2) \( m_t = y_t + p_t \)  
(aggregate Demand)

3) \( w_t - p_t = y_t - n_t \)  
(labor market)

4) \( w_t = \bar{w}_t + \lambda (p_t - E_{t-1} p_t) \)  
(wage setting)

\( \bar{w}_t \) = nominal wage set at beginning of period \( t \).

All variables are in logs: \( y_t \) = output, \( n_t \) = labor, \( a_t \) = productivity shock, \( m_t \) = money supply, \( p_t \) = price level, \( w_t \) = nominal wage. Constants have been omitted for simplicity. All variables are stationary (no unit roots).

In equation (1), \( \theta \) is labor's share in the production function, \( 0 < \theta < 1 \). Equation (3) can be interpreted as firms setting \( n_t \) so that the marginal product of labor equals the real wage. In equation (4), \( 0 \leq \lambda \leq 1 \), with \( \lambda \) indicating the degree of indexation. \( \lambda = 0 \) says that nominal wages are fixed to the value set at the beginning of the period (no indexation), \( \lambda = 1 \) says wages respond 1 for 1 with unexpected movements in the price level (full indexation).

For simplicity, we will assume that there are no dynamics in the model:

5) \( \bar{w}_t = 0, m_t = \nu_t, \nu_t \) and \( a_t \) iid and mutually independent, \( E_{t-1} n_t = 0 \).

2)(a) Show that it follows from (5) that \( E_{t-1} y_t = E_{t-1} w_t = E_{t-1} p_t = 0 \).

Thus we can simplify notation by writing

1) \( y_t = \theta n_t + a_t \)

2) \( \nu_t = y_t + p_t \)

3) \( w_t - p_t = y_t - n_t \)

4) \( w_t = \lambda p_t \)

2)(b) Use (1), (2), (3) and (4) to solve for \( y_t \) in terms of the exogenous shocks \( a_t \) and \( \nu_t \), obtaining say

\[ y_t = c \nu_t + d a_t \]

What are \( c \) and \( d \) in terms of the parameters \( \theta \) and \( \lambda \)?

2)(c) Show that indexation lowers the variance of output in the face of nominal demand shocks, but increases the variance of output in the face of real productivity shocks. (If you had trouble solving for \( c \) and \( d \), explain how you would go about showing this, given \( c \) and \( d \).)

2)(d) What is the intuition to the result in part c?

2)(e) Suppose that the result in part c holds for a realistic model of the U.S. economy. In light of your knowledge about sources of business cycles in the U.S., do you think indexation would increase or decrease output volatility in the U.S.?

2)(f) Should it be a major aim of policy to lower output volatility? (One paragraph maximum.)
2. **Answer all 8 parts of this question.**

Consider a world with complete asset markets and households with identical CRRA preferences. Denote by \( R_{t+1} \) the risk-free gross rate of return on bonds and by the \( Z_{t+1} \) stochastic gross return on stocks.

1. Suppose you observed (only and exactly) the consumption of a large number of households in period \( \tau \) and period \( \tau + 1 \), could you test this model of the world? How?

2. Derive the additional expected return required on stocks relative to bonds in terms of the covariance of the return on stocks and the stochastic discount factor, and the risk-free rate.

3. Comment on the validity of the following: given the paths of aggregate consumption, stock returns and bond returns in the US over the past 40 years, the model described above can be rejected.

   Now suppose that the households in the economy are exogenously split into two separate groups, \( A \) and \( B \). Group \( A \) is a fraction \( 1 - \lambda \) of the original population and is forever barred from holding stocks, while both groups are allowed to hold the risk-free asset, bonds. (Asset markets are no longer complete.)

4. Does the equation derived in part 2 still hold? What is the stochastic discount factor for each group of households? Write down the new aggregate stochastic discount factor for each asset.

5. If the consumption of the two groups were perfectly correlated, does the fact that one group is excluded from the stock market affect the equity premium? Which group is likely to have consumption that is more correlated with stock returns and why?

6. Suppose that households in group \( A \) have consumption growth that is uncorrelated with the return on stocks. Derive the additional expected return required on stocks relative to the risk free asset in terms of the covariance of the risky asset return and the new stochastic discount factors, and the risk free rate.

7. Can a model such as this explain the equity premium puzzle?

8. How would you test this model as an explanation of the equity premium puzzle?
Question 3 (45 minutes)

This question requires you to set up a formal model to analyze a policy question. Be as explicit as possible in your analysis and introduce mathematical assumptions as you see fit. Credit will be given according to how well you are able to do the formal modelling and analysis.

Consider an educational subsidy policy which attempts to increase the rate of college graduation among high school seniors. The policy consists of a promise that any student who completes college will receive a lump sum payment of $K$ as a reward. The index means that the subsidy can differ across students. Suppose that the government has partial information on each high school senior, i.e. partial knowledge of the payoffs to completing or not completing college.

A. Describe a formal model which for studying this question.

B. Characterize the relationship between $K$ and the percentage of students who graduate.

C. Is it efficient to give each student who completes college the same subsidy? If not, suggest an alternative.

D. Suppose the population is made of three types of students. Type A always graduates from college and type C never does. Construct an example in which the percentage of type B's who graduate exhibits either multiple or unique equilibrium values depending on the percentage of A and B types in the population.
Problem # 4 "Financial Wealth, Heterogeneity and Unemployment"

Consider the behavior of a risk neutral worker that seeks to maximize the expected present discounted value of wage income. Assume that the discount factor is fixed and equal to $\beta$, with $0 < \beta < 1$. The interest rate is also constant and satisfies $1+r = \beta^1$. In this economy, jobs last forever. Once the worker has accepted a job, he/she never quits and the job is never destroyed. Even though preferences are linear, a worker needs to consume a minimum of a units of consumption per period. Wages are drawn from a distribution with support on $[a, b]$. Thus, any employed individual can have a feasible consumption level. There is no unemployment compensation.

Individuals of type $i$ are born with wealth $a_i$, $i=0, 1, 2$, where $a^0 = 0$, $a^1 = a$, $a^2 = a(1+\beta)$. Moreover, in the period that they are born, all individuals are unemployed. Population, $N_t$, grows at the constant rate $1+n$. Thus, $N_{t+1} = N_t + nN_t$. It follows that, at the beginning of period $t$, at least $nN_{t-1}$ individuals --those born in that period-- will be unemployed. Of the $nN_{t-1}$ individuals born at time $t$, $\phi^0$ are of type 0, $\phi^1$ of type 1, and the rest, $1 - \phi^0 - \phi^1$, are of type 2.

a) Consider the situation of an unemployed worker who has $a^0 = 0$. Argue that this worker will have a reservation wage $w^*(0) = a$. Explain.

b) Let $w^*(i)$ be the reservation wage of an individual with wealth $i$. Argue that $w^*(2) > w^*(1) > w^*(0)$. What does this say about the cross sectional relationship between financial wealth and employment probability. Discuss the economic reasons underlying this result.

c) Let the unemployment rate be the number of unemployed individuals at $t$, $U_t$, relative to the population at $t$, $N_t$. Thus, $u_t = U_t/N_t$. Argue that in this economy the unemployment rate is constant.

d) Consider a policy that redistributes wealth in the form of changes in the fraction of the population that is born with wealth $a^i$. Describe as completely as you can the effect upon the unemployment rate of changes in $\phi^i$. Explain your results.

Extra Credit: Go as far as you can describing the distribution of the random variable "number of periods unemployed" for an individual of type 2.