INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:
  
  (1) your assigned number  
  (2) the number of the question you are answering  
  (3) the position of the page in the sequence of pages used to answer the questions.

Example:

<table>
<thead>
<tr>
<th>MACRO THEORY</th>
<th>7/30/10</th>
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</thead>
<tbody>
<tr>
<td>ASSIGNED #</td>
<td>_______</td>
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<td>Qu # _______</td>
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- Do not answer more than one question on the same page!
  When you start a new question, start a new page.

- DO NOT write your name anywhere on your answer sheets!
  After the examination, the question sheets and answer sheets will be collected.

- Please DO NOT WRITE on the question sheets.

- Please solve all three problems.

- You are not allowed to use notes, books, calculators, or colleagues.

- Do NOT use colored pens or pencils
Question 1: 100 total points.

A. [40 points] Consider the problem of an agent maximizing the present value of lifetime earnings over $T$ periods by dividing his 1 unit of time between market work and human capital investment. Let $h_t$ denote the human capital stock of the agent at age $t$ and let $1 - l_t$ be his market work. Let $h_t(1 - l_t)w$ be the agent's income at time $t$, where $w$ is the rental rate of human capital. The amount of time the agent does not spend for market work, i.e. $l_t$, is used to produce human capital according to $f(h_t, l_t) = (h_t l_t)^\alpha$ with $\alpha \in (0, 1)$. Human capital depreciates at rate $\delta$. Let $r$ denote the real interest rate. The agent maximizes the present discounted value of lifetime earnings.

(a) Write this problem as a dynamic-programming problem; be clear on the states and controls.

(b) Find the Euler equations for this problem.

(c) Interpret the Euler equations in a couple of sentences.

(d) What is the impact of a change in the rental rate on human capital $(w)$ on human capital accumulation? Explain your result.

(e) What is the impact of a change in the real interest rate $(r)$ on human capital accumulation? Explain your result.

B: [40 points] Consider the following model with consumption $(c)$ and labor supply $(h)$. An infinitely lived consumer chooses the infinite sequences of $c$ and $h$ to maximize $\sum_{t=0}^{\infty} \beta^t [\ln c_t - \gamma h_t]$, $0 < \beta < 1$ subject to $c_t + h_{t+1} = Ak_t^{\alpha} h_t^{1-\alpha}$.

(a) Write this optimization problem as a dynamic programming problem.

(b) Find the policy function and the value function.

(c) Derive the first-order condition and envelope condition (i.e., the Benveniste-Scheinkman formula) associated with the dynamic programming problem. Use these to find the steady state capital stock and hours worked.

C. [20 points] The representative consumer has preferences given by:

$$\sum_{t=0}^{\infty} \beta^t \frac{c_t^{1-\gamma}}{1-\gamma},$$

$\gamma > 0$. The production technology is given by $y_t = \alpha k_t$, where $\alpha > 0$. Capital depreciates at the rate $\delta$, with $0 < \delta < 1$. One unit of capital can be produced from one unit of the consumption goods in period $t$, and becomes productive in period $t + 1$.

(a) Assume that the value function is strictly concave and differentiable, and show that consumption grows at a constant rate. Determine what that rate is, and explain your results.

(b) Under what conditions will consumption increase without bound? Explain why this can happen here.
Question 2: 100 total points.

A. [75 points] Consider the neoclassical growth model with linear taxation as in class, but instead of there being taxation on capital and labor income, there are linear taxes \( \tau^c_t \) on consumption. That is, the household has preferences:

\[
\sum_{t=0}^{\infty} \beta^t u(c_t, 1 - N_t)
\]

and the household budget constraint (with date-0 trading) becomes:

\[
\sum_{t=0}^{\infty} q_t[(1 + \tau^c_t)\bar{c} + k_{t+1} - (1 - \delta)\bar{k}] = \sum_{t=0}^{\infty} q_t[w_tN_t + r_t\bar{k}_t].
\]

As always, firms operate in a competitive market and produce via a constant returns to scale production function \( F(k, N) \), and the aggregate feasibility condition is:

\[
c_t + k_{t+1} - (1 - \delta)k_t + G_t = F(k_t, N_t).
\]

(a) Show a consumption tax scheme is equivalent, in terms of having the same effect on the household’s optimality conditions, to a particular choice of capital and labor income taxes.

(b) For this part only suppose that leisure is not valued, so labor supply is identically equal to one. Suppose that consumption taxes are increasing over time at a constant rate, so \( 1 + \tau_t^c = (1 + g)(1 + \tau^c_t) \), and this finances a constant level of government spending \( G_t = G \). (Assume that \( (g, G) \) are always set so that the government’s budget balances.) What happens to capital and consumption when both \( (g, G) \) are increased? Consider both the steady state and the transitional dynamics.

(c) Reconsider the case of valued leisure as in part (a). Suppose the government optimally chooses the consumption tax to maximize the agent’s utility. Characterize the optimal consumption tax (its level and how it varies over time) in a steady state of such a Ramsey equilibrium.

B. [25 points] Consider an endowment economy with one good and two assets. Asset 1 pays a constant amount \( R \) in each period. Asset 2 pays a stochastic amount \( x_t \) which is \( \text{i.i.d.} \) and so \( x = R/2 \) with probability \( \theta \) and \( x = 2R \) with probability \( 1 - \theta \). In equilibrium consumption of the nonstorable good is therefore \( c = R + x \). The representative agent has preferences:

\[
E \sum_{t=0}^{\infty} \beta^t \log c_t.
\]

Find the equilibrium price/consumption ratios for the two assets: \( p_1(x)/c(x) \) and \( p_2(x)/c(x) \). Which asset has the greater price/consumption ratio? Interpret your result.
Question 3: Monetary Policy (100 points)

In this question, you are asked to consider two models of the economy that relate output \( y_t \) and inflation \( \pi_t \).

\[ y_t = \alpha y_{t-1} + \beta \pi_{t-1} + \epsilon_t \quad \text{(NK)} \]

and

\[ y_t = \gamma y_{t-1} + \delta (\pi_{t-1} - E_{t-2} \pi_{t-1}) + \epsilon_t \quad \text{(NC)} \]

Assume that the joint stochastic process for output and inflation is 0 mean, second-order covariance stationary.

A. (40 points). Provide conditions under which the NK and NC models are observationally equivalent.

B. (40 points). In what sense do these models provide a tradeoff between the variance of output and the variance of inflation that a policymaker can exploit? Hint: think about what has been omitted in terms of the description of the inflation process.

C. (10 points) Does a tradeoff of variance between the output and inflation series imply that there is an associated tradeoff at each frequency of the associated spectral densities of each series?

D. (10 points). How does relaxation of the second order stationarity assumption affect the answer to part A?