UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

July 28, 2005

9:00 am - 12:30 pm

INSTRUCTIONS

• Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:

(1) your assigned number

(2) the number of the question you are answering

(3) the position of the page in the sequence of pages used to answer the questions.

Example:
MACRO THEORY 7/28/05
ASSIGNED # ____________

Qu # ____1____ (Page ____2____ of ____4____):

• Do not answer more than one question on the same page!
  When you start a new question, start a new page.

• DO NOT write your name anywhere on your answer sheets!
  After the examination, the question sheets and answer sheets will be collected.

• Please DO NOT WRITE on the question sheets.

• Please solve any two of the three problems.
You are not allowed to use notes, books, calculators, or colleagues.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

Please return unused portions of the yellow tablet & question sheets.

There are 6 pages in the exam – please make sure you have all of them.

Good luck!

On the top of EACH yellow sheet, write your ASSIGNED NUMBER, date and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the question sheets and yellow pads will be collected. DO NOT write on the question sheets.

You are not allowed to use notes, books, calculators, or colleagues.

Please solve three (3) and only three (3) out of the four (4) problems.

The total time allotted is three (3) hours. Each problem has a suggested time of an hour.

If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

Read the questions carefully. The questions will not be explained. If you think that a question is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.

Please return unused portion of the yellow tablets and question sheets.

There are five (5) pages in this exam (including this cover page). Please make sure that you have all of them.

Good Luck!
1 Problem I: Dynamic Programming (100 points)

1. Asset Pricing (50 points). Consider a representative agent, stochastic growth model in which output $y_t$, is equal to $y_t = z_t h_t$ where $h_t$ is capital at the beginning of the period and $z_t$ is a random shock. This shock is i.i.d. over time and can take $n$ possible values with $\pi_i$ being the probability of $z_t = z_i$, $i = 1, ..., n$. The depreciation rate is $\delta = 1$ so that capital depreciates fully within the period. The social planner chooses sequences of consumption and capital in order to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t c_t^{1-\gamma} - \frac{1}{1-\gamma}$$

with $\beta \in (0,1)$. Given this environment, answer the following questions:
   a. Express the maximization problem as a dynamic program. Be explicit in identifying the state and policy variables. (5 points)
   b. Find the necessary conditions for an optimum, that is, the envelope conditions. Interpret these conditions. (5 points)
   c. Derive a closed form solution for the policy function describing optimal consumption. (15 points)
   d. Suppose the following asset, called equity, is introduced in this economy. If the agent buys the asset at period $t$ he is entitled to the consumption stream $c_{t+j}$ for $j \geq 1$. The price of the asset at period $t$ is $q_t$. Compute this price. [Note: the price should be described using the primitives of the problem] (15 points)
   e. Now suppose an option is introduced in the economy. If an agent buys the option at period $t$ he is entitled to sell one, and only one, equity at period $t+1$ at the price $q$. The price of this option is $p_t$. Find an expression for this price. [Again, the price should be described using the primitives of the problem] (10 points)
2. Hyperbolic Discounting (50 points): Suppose a planner chooses to maximize, by choice of $c_0, c_1, c_2, \ldots$, the following expression:

$$u(c_0) + \delta [\beta u(c_1) + \beta^2 u(c_2) + \ldots]$$  \hspace{1cm} (1)

where $u(c_t) = \log(c_t)$ subject to

$$c_t = k_t^\alpha - k_{t+1}, 0 < \alpha < 1, c_t, k_{t+1} \geq 0, k_0 \text{ given}$$

where $0 < \delta < \beta < 1$. Note that when $\delta = 1$, this reduces to the problem commonly studied.

1. Write down the Bellman equation for the above problem. (5 points)

2. Let $k_{t+1} = g_t(k_t)$ denote the policy rule that solves this problem, $t = 0, 1, \ldots$. Derive an explicit formula relating $g_t$ to the primitives of the model, $\beta, \alpha, \delta$. (10 points)

3. Derive a closed-form solution for the value function. (5 points)

4. How does the saving rate from period $t = 1$ and on compare with the date 0 saving rate? (5 points)

5. Is there a unique $k^*$ with the property $k_t \to k^*$ as $t \to \infty$ for all $k_0$? Derive a formula relating $k^*$ to the parameters of the model. (5 points)

6. Basically, the attitude of the planner is ‘I’m very impatient today (the discount rate from period 0 to period 1 is $\beta\delta$), but I’ll be less impatient tomorrow (the discount rate from period 1 to period 2 is $\beta$), so I’ll consume a lot today and save a lot tomorrow.’ Such an attitude is not time consistent because when tomorrow rolls around the planner says the same thing. In the end, the planner just ends up with a low capital stock. This type of model has been used to explain the behavior of smokers, who resolve that ‘tomorrow I’ll quit smoking, but tonight I’ll just have one or two more’. It also has been used to explain the low US saving rate. The notion is that many people say, ‘today I’ll spend, and tomorrow I’ll save’, day after day.

Would a rational person really make decisions in the time-inconsistent way described there? Here is another idea. The idea is to treat the planner in each period as though they were a different person, not bound by any commitments that may have been made in previous periods. This gives rise to a Nash equilibrium concept in which each date’s planner optimizes, taking as given what planners in other periods do. Thus, suppose a planner optimizes today, subject to the constraint that planners at all future dates save according to the rule, $k_{t+1} = dk_t^\alpha$.

Let $v(k_t; d)$ denote the present value of utility, $u(c_t) + \beta u(c_{t+1}) + \ldots$, that occurs when the saving rate, $d$, is followed forever. (20 points)

(a) Display an explicit formula for $v(k_t; d)$. (5 points)

(b) Let $g(k; d)$ denote the policy rule of a planner with preferences, (1), who expects the saving rate in the future to be $d$. Show that $g(k; d) = D(d)k^\alpha$, and derive an explicit formula for $D(d)$. (5 points)

(c) A natural equilibrium concept in this setting is that it is a $d^*$, such that $d^* = D(d^*)$. Display a formula relating $d^*$ to the parameters of the model. (10 points)
2 Problem # 2 (100 Points)

2.1 Optimal Consumption and Saving

Fix a probability space \((\Omega, \mathcal{F}, \mathbb{P})\) and an information filtration \(\{\mathcal{F}_t\}_{t=0}^{\infty}\), and suppose that a consumer’s income at \(t\) is:

\[ y_t = \phi + \sigma \varepsilon_t, \quad \forall t \geq 1, \]

where \(\sigma > 0\), \(y_0\) is given, and \(\varepsilon_t\)'s are i.i.d. standard normal innovations. The consumer maximizes:

\[ U(c) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \delta} \right)^t u(c_t) \right], \]

where \(\delta > 0\) is his subjective discount rate, and \(u(\cdot)\) is CARA utility. That is,

\[ u(c_t) = -\frac{1}{\theta} \exp\{-\theta c_t\}. \]

The consumer can borrow or save at a constant interest rate \(r > 0\), and his sequential budget constraints are:

\[ A_{t+1} = (1 + r)A_t + y_t - c_t, \quad \forall t \geq 0, \]

where \(A_t\) denotes his asset holdings. \(A_0\) is given. Assume that the natural borrowing constraint is in place, but disregard corner solutions in your analysis.

1. (6 points) Write down the Bellman equation for this consumer’s problem. In particular, let the value function \(V(\cdot)\) depend on \(A\) and \(y\). Denote the next period variables with a prime; e.g. \(A_{t+1}\) becomes \(A'\).

2. (14 points) It is known that the value function looks like:

\[ V(A, y) = -\frac{1}{\theta r} \exp\{-\theta r (A + py + q)\}, \]

where \(p\) and \(q\) are coefficients to be determined. Show that the optimal consumption rule is linear in the state variables. To answer this question, you do not have to uncover what \(p\) and \(q\) actually are. Moreover, invoking the Benveniste-Scheinkman theorem may facilitate your derivation.

3. (20 points) Express the optimal consumption rule as a function of state variables and fundamental parameters only. That is, substitute out \(p\) and \(q\). If you are on the right track, you will run into \(\mathbb{E} \exp\{-\theta r \rho \sigma \epsilon\}\). Do not try to compute this. Simply call it \(m(r, \sigma)\) and move on.

4. (14 points) From your optimal consumption rule, now derive the optimal saving rule, \(A' - A\), using the budget constraint. Show that the saving rule has three components: one with \((y - \phi)\), another with \(\log\left(\frac{1 + \delta}{1 + r}\right)\),
and the other with \( \log m(r, \sigma) \). It is known that \( \frac{\partial m}{\partial r} > 0 \). These components are called, not necessarily in the same order, dis-saving due to impatience, permanent-income hypothesis component, and precautionary saving. Match the names and the components. Briefly explain why the given names are appropriate.

2.2 General Equilibrium

From the last question, let \( \Psi(r) = \log \left( \frac{1+\delta}{1+r} \right) \) and \( \Pi(r) = \log m(r, \sigma) \). Assume (correctly) that \( m(r, \cdot) > 1 \) for \( r > 0 \). Also, \( m(0, \cdot) = 1 \), which is easily verifiable.

Assume that the economy is populated by a continuum of ex ante identical but ex post heterogeneous agents of measure one. Each agent solves the problem in Section 2.1. The risk-free asset is the pure consumption loan, and is in zero net supply. The initial cross-section distribution of income and asset is assumed to be its stationary distribution, \( \Phi(A, y) \). By the law of large numbers, provided that we construct the space of agents and the probability space appropriately and assume pairwise independence of incomes, we will have an invariant cross-section distribution of income and asset, \( \Phi(A, y) \). Under this invariant distribution, all the aggregate quantities remain constant.

1. (6 points) Under the invariant distribution, the aggregate saving \( \int (A' - A) d\Phi \) in the economy must be zero. Why?

2. (6 points) With regard to the invariant distribution \( \Phi \), what is the cross-sectional mean of \( y - \phi \)? Can we ignore this component in the determination of aggregate saving under the invariant distribution?

3. (14 points) Express the aggregate saving function under the invariant distribution in terms of \( \Pi \) and \( \Psi \). Prove that there exists no equilibrium with \( r > \delta \).

4. (20 points) Prove that there is at least one equilibrium with the interest rate \( r^* \) such that \( 0 < r^* < \delta \).
3 Problem 3: Government Policy and Unemployment (100 Points)

Consider a simple search economy. Individuals maximize the expected present discounted value of income. Assume that the interest rate is \( r > 0 \). In this economy, unemployed individuals receive unemployment compensation given by \( b \). The probability of receiving an offer (only the unemployed receive job offers) is \( \lambda \). Conditional on getting an offer, wages are drawn from the distribution \( F(w) \). Assume that all jobs are accepted. Employed workers have to pay a tax \( \tau \). Thus, if they have a job that pays \( w \), their income is \( w - \tau \). Employed workers see their jobs disappear with probability \( \eta \).

1. (20 points) Describe this worker’s optimal decision. Go as far as you can providing explicit expressions for the “value of having a job that pays \( w \),” \( V(w) \), and the “value of being unemployed,” \( U \).

2. (20 points) Define the long run unemployment, \( u \), and show how it varies with the exogenous parameters, \( \lambda, \eta, \tau \) and \( b \).

3. (30 points) Assume that the rates at which jobs are found and lost — although exogenous from the point of view of individual workers — depends on the fiscal policy of the government. Consider the case in which the job finding and losing rates are

\[
\begin{align*}
\lambda &= \lambda^* z, & 0 \leq z \leq 1/\lambda^* \\
\eta &= \eta^* - \nu x, & 0 \leq x \leq \eta^*/\nu,
\end{align*}
\]

where \( \lambda^*, \eta^* \) and \( \nu \) are constants. The variables \( z \) and \( x \) are chosen by the government.

The steady state government budget constraint is,

\[ bu + zu + x(1-u) = \tau(1-u), \]

where \( u \) is the unemployment rate. Note that the terms on the left are (all in per capita terms) expenditures on unemployment compensation, job creation programs, and job saving programs, respectively. The right hand side is just per capita income.

Let the government utility by just the ex-ante expected utility. In this case, it is given by,

\[ W = uU + (1-u)E[V(w)], \]

where, as before, \( u \) is the steady state unemployment rate. Consider the case in which per capita taxes, \( \tau \), are given. Go as far as you can to describe what are the factors that influence the allocation of government spending among \( b, z \) and \( x \). Is it possible to get “corner” solutions? Does this policy minimize unemployment?
4. (30 points) Go as far as you can describing the optimal level of taxation, \( \tau \). (To do this, use \( W \) as the social objective function.)
Problem #4 - A Linear Present Value Model

1. This question concerns linear present value models. For concreteness, we cast it in terms of exchange rates, though no part of the question turns on specifics of exchange rate behavior.

Let $s_t$ denote the log of the exchange rate and let $f_t$ be a “fundamental” variable that determines the exchange rate. A certain exchange rate model posits

$$ s_t = (1-b) \sum_{j=0}^{\infty} b^j E(f_{t+j} | I_t), $$

where $b$ is a parameter that satisfies $0 < b < 1$, and $E(\cdot | I_t)$ denotes expectations conditional on the public’s information set $I_t$ (described below).

(5) a. Suppose that the period $t$ realization of shocks is such that $f_t$ is higher than expected and the public’s forecast of $f_{t+1}$ increases; all other future forecasts unchanged. What will happen to $s_t$ - will it rise, fall, or remain unchanged? (Yes, this is as trivial as it sounds.)

The rest of this question relies on a specific parameterization of information. Suppose that the public’s information set $I_t$ consists of current and lagged values of $f_t$ and of two i.i.d. shocks $e_{1t}$ and $e_{2t}$ for $j=0$,

$$ E(\Delta f_{t+j} | I_t) = E(\Delta f_{t+j} | e_{1t}, e_{2t}, e_{1t-1}, e_{2t-1}, \ldots), $$

$$ E(f_{t+j} | I_t) = E(\Delta f_{t+j} | I_t) + E(\Delta f_{t+j} | I_t) + \ldots + E(\Delta f_{t+j} | I_t) + f_t. $$

As well, let $\Delta f_{t}$ following the process

$$ (2) \quad \Delta f_{t} = e_{1t} + e_{2t-1}. $$

In (3), $e_{1t}$ and $e_{2t}$ are i.i.d. and independent of one another, implying among other things that $Ee_{1t}e_{2s} = 0$ for all $t$ and $s$.

(10) b. Let $H_t$ be the subset of $I_t$ consisting of $f_t$ and current and lagged values of $\Delta f_t$. For $j>0$, define

$$ E(\Delta f_{t+j} | H_t) = E(\Delta f_{t+j} | \Delta f_t, \Delta f_{t-1}, \ldots), $$

$$ E(f_{t+j} | H_t) = E(\Delta f_{t+j} | H_t) + E(\Delta f_{t+j} | H_t) + \ldots + E(\Delta f_{t+j} | H_t) + f_t. $$

Define $x_{tl}$ as

$$ (3) \quad x_{tl} = (1-b) \sum_{j=0}^{\infty} b^j E(f_{t+j} | H_t). $$

Show that $E\Delta f_t \Delta f_{t+j} = 0$ for $j\neq 0$. Use this fact to solve for $x_{tl}$ and $\Delta x_{tl}$.

(20) c(i) Solve for $E(f_{t+j} | H_t)$ in terms of current and lagged $f$’s and $e$’s, for arbitrary $j$.

(20) c(ii) Solve for $s_t$ and $\Delta s_t$.

(20) c(iii) Show that $\Delta s_t$ follows a MA(1) process.

(20) d. Show that the variance of the innovation in $\Delta s_t$ is smaller than the variance of the innovation in $\Delta x_{tl}$. What is the intuition to this result?

(20) e. Show that as $b \to 1$, $\Delta s_t$ will behave increasingly like a serially uncorrelated process (i.e., $s_t$ will behave
like a random walk).

(25) For \( h \) very near 1, which of the following patterns of results of empirical Granger causality tests are consistent with your answer to part (e) and with the general relationship between \( s \) and \( f \) illustrated by your answer to part (a):

1. \( \Delta f_i = \Delta s_i, \Delta s_i - \Delta f_i \)
2. \( \Delta f_i = \Delta s_i, \Delta s_i + \Delta f_i \)
3. \( \Delta f_i + \Delta s_i, \Delta s_i - \Delta f_i \)
4. \( \Delta f_i + \Delta s_i, \Delta s_i + \Delta f_i \)

What is the intuition for your answer. (In answering this question, take results in parts (a) and (e) as given, even if you were unable to show those answers.)