Macroeconomics Preliminary Exam - August 1999

Problem 1 - 60 points

This question asks you to consider the role of a fixed factor, say land, in a simplified version of a standard RBC model, using the approach of working through the decentralized equilibrium. The answers to the three parts of this question are self-contained. You may answer part b taking the results of part a as given (even if you couldn't show those results), and similarly answer part c taking the results of parts a and b as given.

Notation:

(1) \( C_t \) = consumption, \( K_t \) = capital stock, \( L_t \) = total fixed supply of land, \( R_{Kt} \) = return (or rental price) to capital, \( R_{Lt} \) = return (or rental price) to land, \( P_{Lt} \) = price of a unit of land; \( W_t \) = wage; \( Y_t = F(K_t, L_t) \) is the production function;
\( \beta \) = discount factor, \( 0 < \beta < 1 \), \( \delta \) = depreciation rate of physical capital, \( 0 < \delta < 1 \)

For simplicity, labor is ignored throughout. The representative consumer solves

(2) \[
\underset{\{C_{t+1}\}}{\max} \sum_{j=0}^{\infty} \beta^j U(C_{t+j}) \quad \text{s.t.} \quad C_t + K_{t+1} - (1-\delta)K_t + P_{Lt}(L_t - L_{t-1}) = R_{Kt}K_{t+1} + R_{Lt}L_t.
\]

The budget constraint has a time subscript on land \( L_t \) because the representative consumer is free to buy or sell land at the prevailing price. In equilibrium, the price of land must adjust so that \( L_t = L_{t-1} \) for all \( t \). There is no "\( E_t \)" in the problem because for simplicity a nonstochastic environment is assumed.

Perfectly competitive firms rent capital and land from workers in perfectly competitive factor markets. Recall that this means that the first order conditions to the representative firm's static maximization problem include \( F_{Kt} = \frac{\partial F}{\partial K_t} = R_{Kt}, \ F_{Lt} = \frac{\partial F}{\partial L_t} = R_{Lt} \). As a final simplification, let us assume that consumers have access to an asset with a constant, exogenous rate of return \( 1+R \). Such an exogeneity assumption is sometimes justified by assuming that the model is to be applied to a small open economy, with perfect mobility of capital. Constancy of the exogenous return is for simplicity. Recall that

(3) \[
U(C_t) = \beta(1+R)U(C_{t+1}).
\]

Throughout your answer, you may assume the usual terminal or transversality conditions on the growth rate of the variables in the model.

a. (18 points)

Let \( \lambda_t \) be the Lagrange multiplier on the resource constraint in (2).

(i) Derive the consumer's first order condition for \( C_t, L_t \) and \( K_{t+1} \).
(ii) Show that \( R_{Kt} = \frac{(R+\delta)}{(1+R)} \).
(iii) Show that \( P_{Lt} = \sum_{j=0}^{\infty} (1+R)^{-j} F_{Lt+j} \).

Now assume that the production function takes the CES form

(4) \[
Y_t = F(K_t, L_t) = A_t^{1-\delta}[K_t^{1-(1-\delta)} + \gamma L_t^{1-(1-\delta)}]^{\delta}, \ 0 < \delta < 1, \ \delta > 0, \ A_{t+1}/A_t = 1+g, \ g > 0, \ g < R.
\]
Here, $A_t^{1-\theta}$ is total factor productivity and $\theta>1$ is the elasticity of substitution between capital and land.

Let $\tilde{K}_t=K_t/A_t$ and $\tilde{L}_t=L/A_t$ be the capital and land scaled by $A_t$. The marginal products of capital and land evaluated at the equilibrium value of $L_t=L$ are

\begin{align*}
F_{Kt} &= \alpha A_t^{1-\theta} \tilde{K}_t^{-1-\theta} [K_t^{1-(1/\theta)} + \gamma L_t^{1-(1/\theta)}]^{-1-\theta/[(1-\theta)/(1-\theta)\theta]} = \alpha \tilde{K}_t^{-1-\theta} [\tilde{K}_t^{1-(1/\theta)} + \gamma \tilde{L}_t]^{-1+\theta/[(1-\theta)/(1-\theta)\theta]} , \\
F_{Lt} &= \alpha \gamma A_t^{1-\theta} \tilde{L}_t^{-1-\theta} [K_t^{1-(1/\theta)} + \gamma L_t^{1-(1/\theta)}]^{-1-\theta/[(1-\theta)/(1-\theta)\theta]} = \alpha \gamma \tilde{L}_t^{-1-\theta} [\tilde{K}_t^{1-(1/\theta)} + \gamma \tilde{L}_t]^{-1+\theta/[(1-\theta)/(1-\theta)\theta]} .
\end{align*}

In (5), the marginal products are rewritten in terms of $\tilde{K}_t$ and $\tilde{L}_t$ to help you answer the questions below.

b. (24 points)

Consider the behavior of the economy as $t \to \infty$. Show that since $L_t=L$ is fixed, $g>0$ and $\theta>1$, as $t \to \infty$

(i) $F_{Kt} \to \alpha (K_t/A_t)^{\theta-1}$ (i.e., $F_{Kt}/[\alpha (K_t/A_t)^{\theta-1}] \to 1$).
(ii) $K_t/A_t \to$ constant.
(iii) $K_{t+1}/K_t \to 1+g$, $Y_{t+1}/Y_t \to 1+g$.
(iv) $F_{Lt}$ grows without bound
(v) $F_{Lt}L_t/Y_t \to 0$.

c. (18 points)

Even if you got lost in the algebra, answer the following, taking previous results as given:

(i) Interpret results b(iv) and b(v) in words.
(ii) Here are data on the share of land income in total income for two countries sometimes modeled as small open economies, Great Britain and Japan:

<table>
<thead>
<tr>
<th></th>
<th>1850</th>
<th>1913</th>
<th>1978</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Britain</td>
<td>30</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Japan</td>
<td>n.a.</td>
<td>42</td>
<td>51</td>
</tr>
</tbody>
</table>

Which of these seems to better fit the results b(iv) and b(v)? Insofar as the data do not mesh with results b(iv) and b(v), in one paragraph maximum note some possible ways the model could be modified.
Problem 2-60 points

This problem involves human capital formation among a population of students. In this problem you are given the bare bones structure of a problem. You are expected to supply a formal model which corresponds to the description and to add whatever assumptions are necessary to allow you to develop an analysis of the problem.

Consider a population of high school students of size $2I$. We assume that each student possesses human capital of $\bar{h}_i$ if he or she graduates from high school, and and a level of $h_i$ if the person drops out; of course, $\bar{h}_i > h_i$.

A. (20 points) Formulate a discrete choice problem to describe individual drop out decisions. Show how this problem can incorporate both private incentives as well as a desire to conform to the expected behavior of others within his or her high school. Be sure the human capital acquired from each choice enters into the decision problem in an appropriate way.

B. (30 points) Suppose that students come in one of two types. For type 1 students, $\bar{h}_i = A$ and $h_i = B$, for type two students, $\bar{h}_i = C$ and $h_i = D$. There are $I$ students of each type.

i. Suppose $A > C$, and $B = D$. Suppose you are a social planner who must choose how to allocate the students across two schools. You must choose between placing all type 1 students in one school and all type 2 students in another school, versus creating two schools each of which is made up of equal numbers of students of each type.

a. Provide an analysis of what factors will determine which allocation rule maximizes the percentage of students which graduate from high school.

b. Does your answer to a. also provide a solution to the question of how to maximize the human capital acquired by the students? Explain.

ii. (10 points) How is your answer to i changed if $B > D$?
Problem 3: 60 Points (10 points for each part of the question)

Each household in the economy is faced with the problem

\[ V(A_t, K_{t-1}) \equiv E \left[ \sum_{s=t}^{\infty} \beta^{s-t} u(K_s, C_s) \right] \]

subject to

\[
\begin{align*}
A_{t+1} & = R(A_t + Y_t - C_t - X_t) \\
K_t & = (1 - \delta) K_{t-1} + X_t
\end{align*}
\]

- \( \beta \) is the discount factor
- \( u(.) \) is the utility flow from consumption \( (C_s) \) and the stock of durable goods \( (K_s) \)
- \( A \) are assets
- \( R \) is the real interest rate
- \( X \) are expenditures on durable goods
- \( \delta \) is the rate at which durable goods depreciate

1. Derive the Euler equation or first order-condition for the consumption of non-durable goods (hint: use dynamic programming). Does the presence of durable goods in this model alter the conclusion that household attempt to smooth the marginal utility of non-durable goods?

2. Suppose that \( \beta R = 1, \delta = 0 \), the utility of non-durable and durable consumption are additively separable, and the utility function is quadratic in durable goods. Solve for the Euler equation for the holding of durable goods. Does the model predict that expenditures on durable goods are smoothed?

3. Is the stochastic process for expenditures on durable goods consistent with this model?

4. Drop the additional assumptions of part two. Suppose that the durable good gives no utility, but is simply an asset that the household can hold. Let

\[ K_t = (1 - \delta_t) K_{t-1} + X_t \]

What must the equilibrium value of \( \delta_t \) be, if \( \delta_t \) is non-stochastic? Interpret.

5. Suppose that \( \delta_t \) is stochastic and has a mean of \( 1 - R \). Suppose that we see in equilibrium that a certain household holds the durable asset. In equilibrium, what can we say about the correlation between this household’s marginal utility of consumption and the return on the durable asset, \( \delta \)?

6. Suppose the economy is an exchange economy with complete markets in which all households are identical. Assume that the aggregate number of units of durable good are constant over time (their value can fluctuate) and that the aggregate endowment of the nondurable good is a constant amount in every period. Describe as much as you can the equilibrium processes for \( \{K_t\} \) (the value of the stock of durable assets) and \( \{\delta_t\} \).
Problem 4 - 60 points  Natural Resource Monopoly, Land Prices and Output

In this problem you will explore the impact of economic liberalization in an economy in which a monopolist controls the supply of a natural resource.

Consider an economy populated by $N$ identical individuals (farmers) and a monopolist who owns the supply of a natural resource. (Pick your friendly dictator here.) All $N+1$ individuals have the same preferences given by,

$$ U = \sum_{t=0}^{\infty} \beta^t u(c_t), $$

where $0 < \beta < 1$ is a discount factor, $c_t$ is consumption at time $t$, and $u$ is a strictly concave, increasing, and twice differentiable utility function. (If necessary make assumptions to guarantee interiority.) Each farmer is endowed with a units of land.

Output is produced using land and natural resources. There is no capital in this economy. There is only one consumption good which is produced according to the following production function,

$$ y \leq zf(a, \gamma n), $$

where $z$ is a productivity factor, $a$ is land per farmer, and $n$ is the amount of natural resource per farmer. Assume that $f$ is strictly concave, twice differentiable, with positive partial derivatives. Consider a closed economy in which farmers can rent --in a spot market-- natural resources. In every period, farmers can buy and sell land, and bonds, rent natural resources, and consume.

We assume that there is a monopolist that owns $n$ units of (perfectly durable) natural resource per farmer. This monopolist decides in each period how much of the natural resource to make available (as before, we always refer to the quantities on a per farmer basis). Thus, in every period, the monopolist announces the total available for that period, say $n^*$. The monopolist maximizes utility (which coincides with the present discounted value of the rents). Given $n^*$, all farmers participate in competitive markets.

a) (10 points) Describe the equilibrium in this economy. Make sure that you make enough assumptions about $f$ to guarantee that an equilibrium exists. Assume that, in equilibrium, $n^* < \hat{n}$.

b) (10 points) How would the equilibrium that you described in a) change if the monopolist sold --once and for all-- part or all of his stock of the natural resource at time zero? [Assume that the monopolist cannot make sales at a latter date]

c) (10 points) An economist at the World Bank indicates that the resource is underutilized, and that opening up the economy to international capital flows (but not to trade in the resource) would solve the problem. Do you agree? [To answer this question assume that the world interest rate, $R^*$, satisfies $\beta R^* = 1$]

d) (10 points) Consider the economy in a). Assume that at time $J > 0$ there is an unexpected permanent increase in $\gamma$, say $\gamma' > \gamma$. What is the effect of this form of technological progress on land prices, the equilibrium supply of the natural resource, its rental price, and total output? Would your answer be the same if the economy was open and the world interest rate, $R^*$, satisfied $\beta R^* = 1$? [Make additional assumptions if necessary]
e) (10 points) Consider the economy in a). Suppose that at \( t=0 \) it is announced that at time \( T > 0 \), the natural resource will be nationalized and the full supply, \( \hat{n} \), will be made available for production. Describe, as precisely as you can, the monopolist's optimal decision about how much to supply to the market at time \( t, n_t^* \), for \( t = 0, 1, \ldots, T-1 \). What are the implications for output, land prices and interest rates from \( t=0 \) on?

f) (10 points) What are your predictions about the effects on output and land prices of technological change of the type discussed in d) after time \( T \). [Basically using the notation we have used so far, \( J > T \)]

*Extra Credit:* Consider the economy in a) and assume that the equilibrium supply is \( n^* < \hat{n} \). An economist at the IMF claims that allowing for a moderate amount of imports of the natural resource, say \( \bar{n} < n^* \), will provide a "competitive fringe" and will bring about an increase in output and a decrease in the price of the natural resource. Discuss