University of Wisconsin
Microeconomics Prelim Exam with Solution Sketches
Monday, June 15, 2015: 9AM - 2PM

• There are four parts to the exam. All four parts have equal weight.
• Answer all questions. No questions are optional.
• Hand in 12 pages, written on only one side.
• Write your answers for different parts on different pages. So do not write your answers for questions in different parts on the same page.
• Please place a completed label on the top right corner of each page you hand in. On it, write your assigned number, and the part of the exam you are answering (I,II,III,IV). Do not write your name anywhere on your answer sheets!
• Show your work, briefly justifying your claims. Some solutions might be faster done by drawing a suitable diagram.
• You cannot use notes, books, calculators, electronic devices, or consultation with anyone else except the proctor.
• Please return any unused portions of yellow tablets and question sheets.
• There are five pages on this exam, including this one. Make sure you have all of them.
• Best wishes!
Part I

1. How would the expected profits of the monopolist with linear demand be affected if the vertical intercept of the demand curve were randomly oscillating about a mean? How would the expected profits of a competitive firm be affected if its input costs were randomly oscillating about a mean, but if the firm were unable to react fast enough to change its production quantity, but adjust its inputs?

2. Tono owns $A > 100$ continuously divisible shares of a firm. If he is left with a quantity $a \leq A$ of shares, his salvage value is $V(a)$, where $V(0) = 0$, $V'(0+) > 0$, with $V' > 0 > V''$ on $(0, \infty)$.

(a) Suppose that $A$ is actually random, equal to $\bar{A} + \epsilon$, where $\epsilon$ is random with $E[\epsilon] = 0$ and $\sigma^2 = 0.1$. What approximate nonrandom level of shares $\hat{A}$ would leave Tono just as happy as with $A$?

(b) Assume now that $A$ is certain. Assume that buyer offer to pay $p$ per share for any amount of the firm. Derive a formula for Tono’s supply of shares $Y(p, A)$ to the buyer as a function of $(p, A)$. Be rigorous.

(c) Assume also that $V'' > 0$ on $(0, \infty)$. When positive, show that the supply $Y(p, a)$ is increasing and strictly concave in $p$. Plot supply as a function of $A$ and as a function of $p$. 

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Part II

Two players choose actions from the unit interval. Player $i$'s von-Neumann Morgenstern utility, expressed as a function of his action $x_i \in [0, 1]$ and his opponent’s action $x_j \in [0, 1]$, is

$$u_i(x_i, x_j) = \begin{cases} 
(\theta_i + 3x_j - 4x_i)x_i & \text{if } x_j < \frac{2}{3}, \\
(3x_j - 2)x_i & \text{if } x_j \geq \frac{2}{3}.
\end{cases}$$

First, consider the normal form game $G$, in which $\theta_1 = \theta_2 = 2$.

1. Find all pure Nash equilibria of $G$.

Next, consider the Bayesian game $BG$ in which player 1 is of type $\theta_1 = 1$ or $\theta_1 = 3$, each with probability $\frac{1}{2}$, and player 2 is of type $\theta_2 = 2$ with probability 1.

2. Find all pure Nash equilibria of $BG$. (Hint: Define a map whose fixed points are the equilibrium strategies of player 2.)

Finally, consider the Bayesian game $BG'$ in which player 1 is of type $\theta_1 = -1$ or $\theta_1 = 5$, each with probability $\frac{1}{2}$, and in which player 2 is of type $\theta_2 = 2$ with probability 1.

3. Find all pure Nash equilibria of $BG'$.
Part III

1. Assume linear demand and that all firms have zero costs. Rank these cases according to total profits and market quantity: (i) a monopoly; (ii) a Stackelberg monopoly (one first mover, one second mover); (iii) a Cournot duopoly; (iv) a monopoly with a competitive fringe (a first mover, and free entry of a continuum of second movers).

2. Assume linear demand. Let $K$ identical firms with zero costs be in Cournot competition. Assume that $J$ firms can merge, and that after the merger, they act as a single firm (or a cartel) in Cournot competition with the $K - J$ other firms. Find all values of $J$ and $K \geq J$ for which the merged firms as a cartel earn higher total profits than they did before the merger?

3. Consider three binary action games: (i) the prisoner's dilemma; (ii) the game of chicken; (iii) the battle of the sexes; and (iv) matching pennies. In which of games (i)–(iv) would a player strictly prefer the role of spy, namely, where it is commonly understood that the spy can act after seeing the pure action chosen by the non-spy?
Part IV

Consider the following model of job market signalling. The players are a worker and two firms. The worker’s type \( \theta \in \Theta = \{ \theta_L, \theta_H \}, 0 < \theta_L < \theta_H \), is his ability level. He is type \( \theta_H \) with probability \( p_H \).

The game proceeds as follows: [0] The worker learns his type; [1] The worker chooses an education level \( e \in [0, \infty) \); [2] The two firms observe the worker’s education level and simultaneously make wage offers \( w_1, w_2 \in [\theta_L, \theta_H] \); [3] The worker then chooses which offer to accept, if any.

Firm \( i \)'s payoff for hiring a worker of type \( \theta_A \in \Theta \) at wage \( w \) is \( u_i(w, \theta_A) = \theta_A - w \). Its payoff for not hiring a worker is 0.

The payoff of a type \( \theta_A \in \Theta \) worker is \( u_A(w, e) = w - c_A(e) \), where \( c_A(\cdot) \) is differentiable, increasing, and strictly convex with \( c_A(0) = 0 \). We also assume that \( c_L'(e) > c_H'(e) \) for \( e > 0 \).

1. Describe each player’s pure strategy set in this game.

2. State the requirements that define a pure perfect Bayesian equilibrium with common beliefs in this game.

3. Derive the set worker strategies that can arise in a separating equilibrium. (Diagrams may be helpful here.) For each worker strategy you identify, fully describe an equilibrium in which the worker chooses this strategy.

4. Define a weak notion of forward induction that allows you to eliminate all but one pooling equilibrium, and prove that it does so.

5. Derive the set worker strategies that can arise in a pooling equilibrium. (Diagrams may also be helpful here.) For each worker strategy you identify, fully describe an equilibrium in which the worker chooses this strategy.

6. Define a notion of forward induction that rules out all pooling equilibria, and prove that it does so.

7. Draw a diagram representing a pooling equilibrium that could also be ruled out by the criterion you used in part (iv), and explain why the criterion rules out the equilibrium.

8. Draw a diagram representing a pooling equilibrium that cannot be ruled out by the criterion you used in part (iv), and explain why the criterion does not rule out the equilibrium. (Hint: Assume that \( p_H \) is close to 1.)