

UNIVERSITY OF WISCONSIN  
DEPARTMENT OF ECONOMICS

**MACROECONOMICS THEORY Preliminary Exam**

July 1, 2011

9:00 am - 2:00 pm

**INSTRUCTIONS**

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers,.** To complete the label, write:
  - (1) **your assigned number**
  - (2) **the number of the question you are answering**
  - (3) **the position of the page in the sequence** of pages used to answer the questions.

<b>Example:</b>	
MACRO THEORY	7/1/11
ASSIGNED # _____	
Qu # <u>  1  </u> (Page <u>  2  </u> of <u>  4  </u> ):	

- **Do not answer more than one question on the same page !**  
When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**  
After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **Each question counts equally.**
- **Answer all questions.**
- **Answers will be penalized for extraneous material; be concise**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils**
- **There are 7 pages in the exam, including these instruction pages – please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.
- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.
- Good luck!

1. Consider the following economy. Time flows  $t = 0, 1, 2, \dots, T$  (where  $T < \infty$ ). There is a unit measure of identical agents who consume a single perishable good in each period. They have utility functions:

$$\sum_{t=0}^T \beta^t u(c_t)$$

where  $c_t$  is consumption in period  $t$ ,  $\beta \in (0, 1)$ , and  $u : \mathbb{R}_+ \rightarrow \mathbb{R}$  is a strictly increasing, strictly concave, and differentiable function. There is a farm that automatically produces  $y_t > 0$  units of the good in each period  $t$ .

Markets open in each period, where agents trade a single perfectly divisible security. This security represents ownership of the farm and its total supply is normalized to one. Thus, an agent who holds  $s_t$  units of the security is entitled to output  $s_t y_t$ . The security's price is denoted  $p_t$ . Agents cannot go short on these securities, so  $s_t$  must be non-negative. At the beginning of the initial period  $t = 0$ , all agents are endowed with an equal share of  $s_0 = 1$ .

1. (25 points) Define a competitive equilibrium for this model.
2. (25 points) Write down a first order difference equation that the equilibrium security price  $p_t$  must satisfy for each  $t$ . Prove your answer.
3. (25 points) What is the final period equilibrium security price  $p_T$ ? Justify your answer.
4. (25 points) Use your answers in 2 and 3 to derive an expression for the equilibrium security price  $p_t$  for each  $t < T$  as a function of  $t$ ,  $T$ ,  $\beta$ ,  $u$ , and  $(y_t)_{t=0}^T$ . Prove your answer.

2. Discuss the role of money and monetary policy in business cycles. Your answer should reference both factual evidence and the analytical framework used to interpret that evidence.

### 3. Properties of Equilibrium Asset Prices

This question explores the empirical implications of the dividend stock price model:

$$p_t = \sum_{j=0}^{\infty} \beta^j E(D_{t+j} | F_t)$$

where

$p_t$  = stock price at time  $t$

$D_t$  = dividend paid at time  $t$

$F_t$  = information available at time  $t$

Throughout the question, assume that the vector  $(p_t, D_t)$  is purely indeterministic and covariance stationary.

1. (30 points) Assume  $D_t = \rho D_{t-1} + \varepsilon_t$ , i.e. dividends obey an AR (1) process. Assume  $F_t = H_t(D)$ . Assume  $\varepsilon_{t+j}$  is independent of  $F_t$ ,  $j > 0$ .
  - a. Derive  $\text{proj}(p_t | H_t(D))$
  - b. Derive the bivariate AR representation of  $(p_t, D_t)$  and characterize the Granger causal relations between prices and dividends.

2. (40 points) Assume  $D_t = \eta_t + \xi\eta_{t-1}$ , i.e. dividends obey an MA(1) process. Assume  $\eta_{t+j}$  is independent of  $F_t$ ,  $j > 0$ . Suppose that  $F_t = H_t(\eta)$ . Compute  $\text{var}(\text{proj}(p_t | H_t(D)))$ . Suppose that  $F_t = H_t(D)$ . Compute  $\text{var}(\text{proj}(p_t | H_t(D)))$ . When are the two variances equal and when are they not equal? Is the condition under which the variances are unequal economically plausible? When the variances are not equal, does this have any implications for the answer to 1-b?

3. (15 points) Replace assumption 2-i with  $F_t = H_\infty(\eta)$ . Retain the MA(1) assumption for the dividend process. What is the variance of excess holding returns? Why?

4. (15 points) Suppose that representative agent maximizes expected lifetime utility

$$E\left(\sum_{j=0}^{\infty} \beta^j (u(C_{t+j})) | F_t\right).$$

Under what circumstances will this model yield the dividend stock price model?

4) (100 points) Each agent goes through 3 periods of life: kid, young parent, old parent (and then dies). Each kid is born with a stochastic ability  $a$ . The distribution of the kid's ability is a function of the parent's ability, i.e.  $a' \sim A(a'|a)$ . Each young parent gives birth to one kid and makes decisions for him. A parent (or a grand-parent) cannot purchase insurance against the ability of his children.

The young parent invests  $e$  units of consumption goods and  $n$  units of time in his *own* human capital accumulation ("on-the-job training"), and  $e_k$  units of consumption goods in his kid's. Assume that the total amount of time endowed to each individual is 1 in each period. Human capital evolves according to

$$h'_o = a(nh)^{\gamma_1} e^{\gamma_2} + (1 - \delta)h$$

$$h' = a_k(h)^{\gamma_1} e_k^{\gamma_2} + (1 - \delta)\bar{h}$$

where  $h'_o$  is the human capital of the parent when he gets old, and  $h'$  is the human capital of the child when he grows up and becomes a young parent.

The young parent saves  $s$  for himself, and the old parent transfers bequests  $b$  to his child who is now a young parent. Assume that the young parent makes decisions *after* she receives her bequests from his parent. Preferences are defined as

$$u(c_y) + \beta \mathbb{E} \{u(c'_o) + \theta V'\},$$

where  $c_y$  is the consumption of the young parent,  $c'_o$  is the consumption of the old parent, and  $V'$  is the lifetime utility of her child after he grows up and the expectation operator is over future abilities.  $\theta$  is the weight the parent puts on her child's utility. Assume that we are in a stationary equilibrium.

1. Let  $V(\cdot)$  be the value for the young parent and  $J(\cdot)$  for the old parent, formulate the Bellman equations for the young and old parent. Be careful to clarify the states and controls. (15 points)
2. Assume concavity of the value function. Briefly outline an argument that demonstrates that the Value function  $V(\cdot)$  is differentiable. (15 points)
3. Derive the first order conditions for the optimal choice of OJT, education, savings and bequests. Derive the envelope conditions. Interpret the Euler equation. (20 points)

4. Now suppose that markets are complete. What do the optimal choices of investment in children look like? Explain how this differs from the incomplete market allocation. (20 points)
5. In the standard Aiyagari-Bewley model, the real interest rate in the steady state is less than the rate of time preference. Does the same result hold in this economy? Explain briefly why you think it holds or does not hold. (15 points)
6. Assume that ability is i.i.d. What is the intergenerational correlation in earnings in the model presented above? How would your answer change if markets were complete? (15 points)