

Macroeconomics Preliminary Examination

January 1998

Answer 4 of the following 8 questions. Each question counts equally. You have 3 hours 15 minutes to complete the examination. You are not allowed to use notes, books, or calculators. Good luck.

QUESTION 1

Answer true, false or uncertain, and briefly explain your answer. Explanation determines grade.

1. The link between stock prices and business investment is strong. But stock price movements alone can't explain all of the movement in business investment.

2. Reduced form VAR analysis indicates that technology shocks generate a negative co-movement between output and labor. This is inconsistent with conventional real business cycle models.

3. As a quantitative matter, money shocks generate considerable persistence in output if the Phillips curve is flat.

4. The literature on structural VARs still has not come to grips with the fact that many variables appear to have unit autoregressive roots. Consequently, this literature has little reliable to say about the long run effects of shocks.

5. If there is labor hoarding, estimates of the Solow residual will seriously misstate the rate of technological progress.

EXTRA CREDIT

6. Professor West's 10 year old son likes to say "whatever." (Answer in one word. All answers receive full credit.)

QUESTION 2

Answer both 1 and 2:

1. Suppose a firm that chooses labor input to maximize the expected present discounted value of cash flow, taking price and wage as given:

$$(1) \quad \max \Pi_t = E_t \sum_{j=0}^{\infty} b^j \{ a_0 x_{t+j} - [.5a_1 x_{t+j}^2 + .5a_2 (x_{t+j} - x_{t+j-1})^2] + w_{t+j} x_{t+j} + e_{t+j} x_{t+j} \}$$

Here,  $0 \leq b < 1$  is the discount factor,  $x_t$  is labor input,  $w_t$  is the real wage and  $e_t$  is a white noise cost shock uncorrelated with  $w_t$  at all leads and lags.

The constant returns to scale production function is

$$\text{output in period } t+j = a_0 x_{t+j};$$

output price is set to one in every time period, so in (1)

$$a_0 x_{t+j} = \text{revenue in period } t+j.$$

The term in brackets in (1) is cost in period  $t+j$ . The shock  $e_t$  is observable to the firm but not the econometrician. To simplify the algebra, you may set  $a_0$  to zero, and assume all variables have zero unconditional mean.  $a_1$  and  $a_2$  are strictly positive.

(a) Derive the Euler equation and the decision rule (without solving out for the present value that appears in the decision rule).

(b) Suppose that  $w_t \sim \text{AR}(1)$ ,  $w_t = \phi w_{t-1} + \epsilon_t$ ,  $|\phi| \leq 1$ ,  $\epsilon_t$  the iid innovation in  $w_t$ .

(i) Solve for the decision rule expressing  $x_t$  as a function of lagged  $x_t$  and current and lagged  $w_t$  and  $e_t$ .

(ii) If you regress  $x_t$  on  $x_{t-1}$  and  $w_t$ , which gives a larger (in absolute value) population value for the coefficient on  $w_t$ : (a)  $\phi = .9$ , or (b)  $\phi = 0$ ? Give an economic interpretation of the difference in coefficients.

2. An equilibrium condition and implied fundamentals solution for the martingale model of stock prices are

$$(1) \quad p_t = b E_t (p_{t+1} + d_{t+1})$$

$$(2) \quad p_t^f = \sum_{j=0}^{\infty} b^j E_t d_{t+j},$$

where  $p_t$  is stock price,  $d_t$  dividends,  $E_t$  mathematical expectations conditional on information available to market participants,  $0 < b < 1$  the constant discount factor. Let  $x_t$  be a "rational bubble" of the form

$$x_t = \begin{cases} (x_{t-1} - x^*) / \pi b & \text{with probability } \pi \\ x^* / [(1-\pi)b] & \text{with probability } 1-\pi \end{cases}$$

where  $0 < \pi < 1$ ,  $x^* > 0$ .

(a) Show that  $x_t$  follows an explosive autoregressive process. (An explosive autoregressive process is one whose autoregressive polynomial has a root strictly less than one in modulus.)

(b) Show that  $p_t = p_t^f + x_t$  satisfies (1).

(c) (One paragraph maximum.) What sorts of characteristics of stock prices might be rationalized by the presence of a rational bubble?

## QUESTION 3

### Policy Differences and Growth

Consider a world in which each country can be described as a standard, one sector, competitive growth economy. Assume that each country is populated by a large number of individuals with strictly concave instantaneous utility functions. All individuals discount future utility at the rate  $\beta$ , where  $0 < \beta < 1$ . Thus, the representative worker's utility function is just,

$$\sum_{t=0}^{\infty} \beta^t u(c_t),$$

with  $u$  a strictly concave (and strictly increasing) function.

Output is produced under constant returns to scale, using capital and labor as inputs. In what follows you are to evaluate several claims. Please be as precise as possible (i.e. use formal models or arguments) when justifying your conclusions.

- a) It is claimed that if all countries are at the steady state differences in wasteful (i.e. non-productive and non-utility enhancing) government spending financed by lump sum taxes can explain differences in productivity per worker.
- b) It is claimed that if government spending enters the utility function, the higher the level of government spending (financed by lump sum taxes) the lower the steady state per capita output.
- c) It is claimed that an unexpected once-and-for-all increase in government spending (financed by lump sum taxes) has a larger impact on per capita output if a country is at the steady state than in the case in which the initial level of capital falls short of the steady state.
- d) Let the level of government spending to be the same for all countries. Assume that countries differ according to the fraction of spending that is raised using capital income taxes (the rest is collected from lump-sum taxes). It is claimed that countries that rely more on capital income taxes will have lower levels of output and higher (both before and after tax) interest rates in the steady state.

## QUESTION 4

### Career and Firm Choice

Consider a standard search model in which an individual's income is the sum of a "career specific" draw, denoted by  $x$ , and a "firm specific" draw denoted by  $z$ . Income for an individual who has career draw  $x$  and firm draw  $z$  is  $y=x+z$ . Assume that successive draws of  $x$  are independently drawn from the cdf  $F(x)$ , while successive draws of  $z$  are drawn from the cdf  $G$ . Moreover assume that  $x$  and  $z$  are independent.

At the beginning of a period a worker may draw a new firm match,  $z$ , or she may draw a new pair  $(x,z)$ . However, she is not allowed to draw a new career match  $x$ , while retaining a firm match,  $z$ , from a previous period.

Assume that the worker maximizes expected present discounted value of income.

a) Write Bellman's equation for this problem.

b) Go as far as you can showing that the optimal strategy is of the reservation career/firm variety. More precisely, show that there exist values  $x^*$  and  $z^*$  such that,

i) if the state is  $(x,z)$  with  $x+z \geq x^*+y^*$ , and  $z \geq z^*$ , the worker stops searching.

ii) If the state is  $(x,z)$  with  $z \leq z^*$  and  $x \geq x^*$ , the worker draws a new  $z$ , but retains its career choice.

iii) In the complement of cases i) and ii) the worker draws new career and firm choices.

c) Assume that the optimal policy is as described in b) (even if you could not prove that it is) go as far as you can drawing the empirical implications of the model for a data set containing individual histories of career and firm choice; that is, for each individual you have his/her history of occupation in each firm he/she has worked for.

# Question 5

1. (Barsky and Summers (1988)) One of the basic predictions of economics is that the nominal interest rate should be closely related to the rate of inflation, through the Fisher relation that the nominal interest rate is simply the sum of the real interest rate and the inflation rate. Yet, as first emphasized by Keynes, economic data up to 1930 do not support this relationship. From 1820 to 1920, there is no relation between the inflation rate and the nominal interest rate. In fact, there is a strong positive relationship between nominal interest rates and the price level. This robust empirical finding is known as Gibson's paradox. This problem asks you to use the following model to explain the Gibson paradox.

- Under the gold standard, the government commits to buy or sell gold at an official, fixed price from private agents. Let this fixed price of gold be 1 and the price level (or deflator) be  $p_t$  so that

$$p_t^G \equiv \frac{1}{p_t}. \quad (1)$$

Taking logs and differentiating with respect to time, it is then also true that

$$\frac{\dot{p}_t}{p_t} = -\frac{\dot{p}_t^G}{p_t^G}. \quad (2)$$

- The Gold Market: The amount of gold in the world is exogenously fixed at  $\bar{G}$ , but this gold can be used to back currency issued by the government, which we will call  $G^m$ , or be used by the private sector in commercial and industrial uses, denoted  $G^n$ , so that

$$G_t^m + G_t^n = \bar{G}. \quad (3)$$

The marginal return to gold used in the private sector is a decreasing function of the amount of gold in use. For simplicity, gold does not depreciate, so that the following equation holds in equilibrium:

$$\frac{\lambda_0}{\lambda} - \frac{1}{\lambda} G_t^m + \frac{\dot{p}_t^G}{p_t^G} = r \quad (4)$$

where the left hand side is the marginal return to gold in the private sector,  $\lambda_0$  and  $\lambda$  are positive, and  $r$  is the real interest rate which we will assume is exogenous (by either an open economy assumption or because output is steady at its steady state fixing the real interest rate).

- Money Supply: Because the government backs currency with gold, the money stock is proportional to gold held by the government:

$$M_t = \mu G^m \quad (5)$$
$$\mu > 0.$$

- Money Demand:

$$\frac{M_t}{p_t} = \lambda_0 - \lambda i \quad (6)$$

where the  $\lambda$ 's just happen to be the same as those in equation (4) and  $i$  is the nominal interest rate.

- (a) Interpret equation (4). Why must it hold?
- (b) Combine the above equations to derive the relationship between the price level and  $G^n$  and exogenous parameters. (Hint: Rewrite equation (4) in terms of the nominal interest rate; eliminate money from the money demand equation; combine.) Explain the intuition for this relationship.
- (c) Use the just-derived relationship and equation (6) to derive the relationship between the price level and the nominal interest rate. In the 1800's, the price level was stationary. Explain the Gibson paradox correlation between the price level and the nominal interest rate by 1) assuming that the primary shocks to the economy are exogenous shocks to technology which lead to exogenous fluctuations in the real interest rate and 2) looking across steady states in which the inflation rate is zero.
- (d) Without necessarily using math, how does this model explain the other half of the Gibson paradox: the lack of the usual positive correlation between the nominal rate and the inflation rate?
- (e) (Extra points for previous errors) Can the model explain the Gibson paradox if the stock of gold fluctuates as well?

2. Consider an economy with identical infinitely-lived consumers, each with additively-separable, Von-Neuman-Morgenstern logarithmic utility. The expected present discounted value of the representative agent's utility at time  $t$  is

$$E \left[ \sum_{i=0}^{\infty} \beta^i \log(C_{t+i}) | \Omega_t \right]$$

Firms in this economy are owned by shareholders, and shares are traded in the stock market. The value of the stock market at time  $t$  is given by  $V_t$ , and represents the value of the claim to the aggregate profit stream.

- (a) Solve for the value of the stock market as a function of present and expected future profits ( $\pi_t$ ) and consumption.
- (b) Firms make profits according to how much they sell, which is in turn proportional to the aggregate consumption of households, so that, for all  $t$ ,  $\pi_t = \alpha C_t$ . Consumption evolves stochastically through time, in a potentially serially correlated manner.
  1. Solve for the value of the stock market
  2. Characterize the effects on  $V_t$  of an increase in expected future profits and discuss how expectations do and do not matter for the value of the stock market.
  3. Discuss the premium for holding equity rather than (risk-free) bonds in this economy.
- (c) Suppose now that profits evolve as some certain, known sequence  $\{\pi_t\}_{t=0}^{\infty}$ . Consumers initially hold all stocks, have no other wealth, and can borrow or lend at the constant risk-free rate (the economy is of the small open variety) which happens to equal their common discount rates. There is no labor income.
  1. Solve for  $C_0$  and  $V_0$  in terms of future profits and interpret.
  2. Describe how aggregate consumption, the trade deficit and  $V_t$  covary as profits fluctuate.
  3. Does equity command a premium in this economy?

Question 7

I. The following claims have been made about per capita output growth across countries between 1950-1990.

1. In a cross-country regression of per capita output growth between 1950-1990 on a constant and per capita income in 1950, the regression coefficient on 1950 income is approximately zero and statistically insignificant.

2. In a cross-country regression of per capita output growth between 1950-1990 on a constant, human capital savings rates, physical capital savings rates, population growth rates and per capita income in 1950, the regression coefficient on 1950 income is negative and statistically significant.

3. If the poorest 50% of economies in 1950 (measured in terms of real per capita income) and the richest 50% of economies in 1950 are separately subjected to the regression described in 2., the coefficients on each of the comparable regressors for the two regressions are statistically significantly different from each other.

4. The cross-section variance of per capita output levels is lower in 1990 than 1950.

Interpret these claims in the context of models of economic growth. Are these consistent with a single model? Are the facts mutually contradictory?

Question 8

II. Consider the following overlappings generations model

1. The population consists of a collection of  $I$  families. All family lines are infinitely lived.

2. Agent  $i, t-1$  is born at  $t-1$  and lives two periods. At  $t-1$ , agent  $i, t-1$  receives education. At time  $t$ , agent  $i, t-1$  has one child, works and divides his income  $Y_{i,t}$  between education  $E_{i,t}$  and consumption  $C_{i,t}$ .

3. Agent  $i, t-1$  wishes to maximize

$$\pi_1 \log C_{i,t} + \pi_2 \log E_{i,t}$$

4. The income function for each individual exhibits constant returns to scale in education received from parent.

$$Y_{i,t+1} = c + \phi E_{i,t} + \xi_{i,t+1}$$

$\xi_{i,t+1}$  denotes an individual-specific productivity shock with expected value 1. The shocks may be correlated over time within a family line but are independent across families.

5. Parent's cannot borrow to educate their children; children cannot borrow to educate themselves.

A. What is the time series process which describes family income within a dynasty as described by this model?

B. Give sufficient conditions under which it is true (with probability one) that even though individual  $i, t$  has a higher income than individual  $j, t$ , there will be a time  $t + K$  such that individual  $j, t + K$  has a higher income than individual  $i, t + K$ .

C. Give sufficient conditions such that if individual  $i, t$  has a higher income than individual  $j, t$ , there is a positive probability that individual  $i, t + K$  has a higher income than individual  $j, t + K$  for all  $K$ .