University of Wisconsin
Department of Economics

Macroeconomic Theory Preliminary Exam

January 16, 1997
9:00 am - 12:00 pm

- On the top of EACH yellow sheet, write your ASSIGNED NUMBER, date, and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the questions sheets and yellow pads will be collected. Do not write on the question sheets.

- This is a closed book exam.

- Please solve any two (2) problems. Both will be weighted equally. Each problems is worth 50 points. However, within a problem not all sections are weighted equally. Use your time efficiently!

- The total time allotted is three (3) hours.

- If you get stuck in a problem/section move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- Read the questions clearly. The questions will not be explained. If you think that a question is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.

- Please return unused portion of the yellow tablets.

- There are seven (7) pages in this exam. Please make sure that you got all of them.

- Good luck!
Problem # 1. Energy Shocks, Capacity Utilization and Output

Consider a one sector growth model in capital utilization is variable, and production requires energy as an input. For simplicity (since there could be some profits/nonconvexities at the firm level) we assume that each representative household must operate its own firm. Thus, the representative family solves the following problem,

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t), \quad 0 < \beta < 1,$$

subject to,

$$c_t + x_t + p_t e_t \leq A (\min(k_t, a(s_t)^{-1}e_t) s_t)^e,$$

$$k_{t+1} \leq (1-\delta)k_t + x_t,$$

where it is assumed that effective capital is a Leontieff function of physical capital ($k_t$) and energy ($e_t$). Moreover, both the coefficient in this function, $a(s_t)$, as well as the effective capital stock depend on the rate of utilization, $s_t$. We assume that the function $a(s)$ is given by,

$$a(s) = \nu s^\gamma, \quad \nu > 0, \gamma > 1.$$

The function $u$ is assumed concave in both arguments, increasing in each argument and twice continuously differentiable.

1) (10 points) Define a competitive equilibrium.

2) (15 points) Describe the steady state. In particular, make sure that you explain how an increase in energy prices affect the steady state capital stock per family and the rate of capital utilization.

3) (15 points) Suppose that at time zero there is an unexpected increase in energy prices. Go as far as you can describing the response --both short run and medium-long run-- of the capital stock and the rate of capital utilization. Assume that the economy is, initially, at the pre-shock steady state.

4) (10 points) Suppose that a government can levy lump-sum taxes that are used to subsidize capital purchases (e.g. if the rate of subsidy is $\sigma$, $x$ units of new capital only cost $(1-\sigma)x$ units of consumption). What are the effects of increasing subsidies on capital utilization and energy consumption in the steady state? Following an increase in energy prices should the government subsidize capital purchases?
Problem # 2. Costs of Adjustment and Stochastic Shocks

Consider a standard RBC model, simplified so that labor supply is inelastic, and with a different capital accumulation equation:

\[(2-1) \quad \text{max } E \Sigma_{j=t}^\infty \beta^j U(C_{t+j}) = E \Sigma_{j=t}^\infty \beta^j \ln(C_{t+j}) \]

s.t.

\[(2-2) \quad Y_t = \hat{\lambda}_i K_{t+1}^{1-\delta} \tilde{N}^0 = A_t K_{t+1}^{1-\delta} \]
\[(2-3) \quad Y_t = C_t + I_t \]
\[(2-4) \quad K_{t+1} = K_t \left( \frac{I_t}{K_t} \right)^{1-d}, 0 < d < 1 \Rightarrow \]
\[(2-4') \quad I_t = (K_{t+1})^{1-d} (K_t)^{-d(1-d)} Z_t \]

Notation: \( Y \)=output, \( C \)=consumption, \( I \)=investment, \( K \)=capital. In (2-2), \( \tilde{N} \) is the fixed labor supply, and is absorbed into the definition of the exogenous level of technology \( A_t \).

The novel feature is the capital accumulation equation (2-4), which will lead to an exact closed form solution (see below). Here, \( Z_t \) is an exogenous technology process. Note that \( Z_t \) need not have a mean of one (that is, any constant that might scale the right hand side is absorbed in \( Z_t \)), and that \( Z_t \) is raised to the power \(-1-(1-d)\) only to simplify (2-4').

Recall that the standard accumulation equation is

\[(2-5) \quad K_{t+1} = (1-\delta)K_t + I_t \]

1. (12 points) Compare (2-4) and (2-5):
   a. Under what special circumstances are the two identical?
   b. What economic rationalization might there be for (2-4)?

2. (13 points) Form a Lagrangian, and derive first order necessary conditions for \( C_t \) and \( K_{t+1} \). Then eliminate the Lagrange multiplier by combining the two first order conditions.

3. (12 points) Let lower case letters denote logarithms: \( k_t = \ln(K_t), a_t = \ln(A_t), z_t = \ln(Z_t) \). Show that the following solution satisfies your answer to part 2:

\[(2-6) \quad C_t / Y_t \text{ is constant;} \]
\[(2-7) \quad k_{t+1} = \text{another constant} + (1-\theta+d\theta)k_t + (1-d)a_t - (1-d)z_t \]

Notes: (a) You might want to begin by clearly stating how you plan to answer this question, so that if you end up making an algebraic mistake we will recognize that you nevertheless understand the basic idea. (b) Note that you are not being asked to motivate the solution (2-6) and (2-7), but are merely asked to construct a solution by "guess and verify," with the guess supplied by us. (c) If it
helps in answering, you may assume that $E_t z_{t+1}$ depends only on $z_t$ and $E_t a_{t+1}$ depends only on $a_t$, although (2-6) and (2-7) do not require this. (d) Of course, for (2-6) and (2-7) to hold in equilibrium, they must also be consistent with (2-2), (2-3) and (2-4). You are not asked to show this.

4. (13 points) Take (2-7) as given. Let $a_t$ follow a random walk, $a_t = \text{constant} + a_{t-1} + \text{iid shock}$. Under what conditions on $z_t$ will the capital-output ratio have a secular trend, as it does in the U.S. and many other countries? Do these conditions seem reasonable to you?

5. (Extra Credit--try this only if you've completed and checked the rest of your answers throughout the exam. The person who wrote this question is not sure of the answer himself.) Intuitively, why is the consumption/output ratio $C_t/Y_t$ constant in equilibrium?
Problem #3. Wages and Job Stability

Consider a standard search model in which individuals are risk neutral. The representative agent maximizes the following utility functional,

\[ U = E_t \sum_{j=t}^{\infty} \beta^j y_{tj}, \]

where \( y_t \) is income at time \( t \).

Each "job" is completely described by two characteristics: a wage, \( w \), and a probability of losing the job, \( \alpha \). Thus, there is a probability distribution function over the two dimensional vector \((w, \alpha)\). Assume that each worker draws one realization of \((w, \alpha)\) in each time period, and that successive draws are i.i.d. If a worker accepts an offer \((w, \alpha)\) he/she works with probability one this period --and gets paid \( w \) units of consumption-- and, in every subsequent period, with probability \( \alpha \) he/she can lose the job, in which case he/she becomes unemployed and is eligible to receive \( c \), and with probability \( 1-\alpha \), he/she keeps the job and gets paid \( w \). Note that a person holding a job \((w, \alpha)\) can lose it in every period --except for the first-- with probability \( \alpha \). Assume that workers cannot quit a job that they have accepted.

Unemployed individuals receive \( c>0 \) units of consumption.

1) (15 points) Go as far as you can proving that the optimal strategy on the part of a worker is, conditional on \( \alpha \), of the reservation wage variety.

2) (15 points) Show that the reservation wage decreases with \( \alpha \).

3) (15 points) Assume that \( w \) and \( \alpha \) are independent. Thus, \( \text{Prob}(w \leq x, \alpha \leq y) = F(x)G(y) \) for some cumulative distribution functions \( F \) and \( G \). Go as far as you can describing the average market wage of "high" and "low" \( \alpha \) jobs.

4) (5 points) Suppose now that workers can quit. More specifically, at the beginning of each period (before observing whether they will be laid off or not) an employed worker can quit his/her job. Go as far as you can describing how this setup changes your answers to 1) -3).
Problem #4. Shocks and Business Cycles

1) (25 points) What shocks cause business cycles?

2) (25 points) How are the shocks propagated?

Discuss 1) and 2), referencing evidence from real business cycle and vector autoregressive studies.


Problem # 5. Coordination Problems and Investment in Human Capital

Consider a simple overlapping generations economy in which each generation lives for two periods. There is a continuum of identical individuals in each generation with names in the interval [0,1]. The utility function of the representative individual in generation k is,

\[ \ln(c_t^k) + \beta \ln(c_{t+1}^k), \]

where 0<\beta<1, and \( c^k_t \) is consumption of generation k at time t (of course, generation t only consumes goods t and t+1). Individuals have one unit of labor in each of their two periods. In the first period they have to decide how to allocate their time between working and studying. Wages per unit of human capital depend on the aggregate level of human capital. Thus, if an individual has human capital h and the average level of human capital is H, earnings per unit of time are w(H)h. We assume that all young individuals have human capital equal to 1. The human capital of an old individual who spent fraction n of his/her time in school when young is \( h=g(H,N,n) \), where H was the average level of human capital in the economy when the individual was young, and N was the average fraction of time spent in school by individuals of his/her generation.

Initially assume that this is a closed economy, and individuals are free to borrow or lend. Individuals take both the interest rate and the average levels of human capital, H_n, and time spent in school, N_n, as given and beyond their control. Thus, the budget constraints for generation t are,

\[
\begin{align*}
    c_t^1 + b_{t+1} &\leq w(H_t)(1-n_t) \\
    c_{t+1}^1 &\leq w(H_{t+1})g(H_t,N_t,n_t) + R_t b_{t+1}.
\end{align*}
\]

The aggregate human capital stock at t+1 is just \( H_{t+1} = \int_0^1 g(H_t,N_t,n_t) \, di \), where the integral indicates averaging over members of generation t. Or course, \( N_t = \int_0^1 n_t \, di \).

1) (10 points) Describe an equilibrium for this economy in which all individuals behave alike.

2) (10 points) Assume that \( g(H,N,n) = [H^n + N n^n]^{1/n} \). Try to go as far as you can describing conditions under which two similar economies (in the sense of having the same value of H) can behave differently. Is there a coordination problem in this economy? [Note: If necessary assume that H is sufficiently small]

3) (10 points) Assume that \( w(H) = w_0 H \). Is there a sense in which “good” and “bad” equilibria can be indexed by the interest rate?

4) (10 points) Are all the equilibria that you found in 2) Pareto optimal? Explain. If your answer is negative, is there a government policy that would improve upon the competitive equilibrium.

5) (10 points) Consider the case of an open economy with \( w(H) = w_0 H \). By open economy we mean a constant --and exogenous-- interest rate. Go as far as you can trying to determine if “opening the economy” solves potential coordination problems.