UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

September 1, 2011

9:00 am - 2:00 pm

INSTRUCTIONS

• Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:
  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions.

  Example:

  MACRO THEORY 9/1/11
  ASSIGNED #: 
  Qu # ___1___ (Page ___2___of __4___):

• Do not answer more than one question on the same page!
  When you start a new question, start a new page.

• DO NOT write your name anywhere on your answer sheets!
  After the examination, the question sheets and answer sheets will be collected.

• Please DO NOT WRITE on the question sheets.
• Each question counts equally.
• Answer all questions.
• Answers will be penalized for extraneous material; be concise
• You are not allowed to use notes, books, calculators, or colleagues.
• Do NOT use colored pens or pencils
• There are 8 pages in the exam, including these instruction pages – please make sure you have all of them.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

• If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

• All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

• Good luck!
1. This question concerns the optimal investment decision of a firm. Time flows $t = 0, 1, 2, \ldots$. The firm starts in period $t = 0$ with $k_0$ units of capital and chooses $(k_t)_{t=1}^{\infty}$. If the firm enters period $t$ with $k_t$ units of capital and chooses to bring $k_{t+1}$ units of capital into period $t+1$, it earns a profit $\pi(k_t, k_{t+1})$. The firm wants to maximize the present discounted value of profits given the market (net) interest rate $r > 0$. Finally, each $k_t$ must belong to a finite set $K \subset \mathbb{R}_+$.

1. (10 points) Write down the firm’s decision problem as a maximization problem in terms of sequences.

2. (10 points) What is the firm’s value function? Give a formal definition.

3. (10 points) Write down a functional equation that the firm’s value function must satisfy.

4. (10 points) What is the firm’s policy function? Give a definition in terms of the value function you defined in 2.

5. (10 points) Write down a proposition that relates the policy function you defined in 4 to the decision problem you wrote down in 1.

6. (25 points) Prove your proposition in 5.

7. (25 points) Suppose your adviser gave you $(K, k_0, \pi, r)$ and asked you to solve the problem in 1 on a computer. How would you do so?
2. Points for individual subsections are given in parentheses. Total points = 100. You are given 10 points just for reading the question through. Each of parts (a)-(f) can be answered separately from each of the other parts. So if you get stuck on one part, just go on to the next.

Let IP = industrial production, CPI = price level, UE = unemployment rate, FF = interest rate (measured by Federal funds rate, though again that is not relevant to this question). Data happen to be from the U.S. (also not relevant to this question).

On the attached page is a graph of a set of impulse responses to a 100 basis point (=1%) positive monetary policy shock. There are three specifications, which are labeled in the column headers.

- The last column “R&R baseline approach” uses a complicated scheme to construct a series of monetary policy shocks that are denoted \( \eta_t \). This is done prior to any estimation presented in the graph. The details of the construction of the shocks \( \eta_t \) are not relevant, except to note that the Romer and Romer narrative approach is part of the complicated scheme and that is why the approach is called R&R in the graph. The author runs regressions of the form

\[
\Delta x_t = f + \sum_{j=1}^m \varphi_j \Delta x_{t-j} + \sum_{i=1}^n c \eta_{t-i} + \varepsilon_t
\]

where \( x_t = \log(\text{IP}_t) \) in the top row in the figure, \( x_t = \text{UE}_t \) in the middle row, and \( x_t = \log(\text{CPI}_t) \) in the bottom row. Note that (2-1) is similar to the regression that Romer and Romer run, with a continuous shock variable \( \eta_t \) replacing their 0-1 dummy variable. Observe as well that the growth rate \( \Delta x_t \) is in (2-1) while the response of the level is plotted in the graph.

- The first column “standard VAR approach” uses a five variable VAR with variables

\[
(2-2) \quad X_t = (\log(\text{IP}_t), \text{UE}_t, \log(\text{CPI}_t), \log(\text{commodity price}_t), \text{FF}_t)'.
\]

A monetary policy shock is identified using a Choleski decomposition with the variables in the order given in (2-2), e.g., IP is first and FF is last. A monetary policy shock is defined as the structural innovation in FF.

- The middle column is the same as the first column except that in (2-2) FF, is replaced by the R&R shock \( \eta_t \).

The horizontal axis is measured in months. The vertical axis is percentage points (middle row) or percentage points divided by 100 (top and bottom rows - that is, the “.02” in the top and bottom rows means “2 percent”). The dashed lines are one standard deviation confidence intervals.

(10) a. Suppose that in (2-1) the lag lengths are \( m = 1 \) and \( n = 1 \), so that the regression is \( \Delta x_t = f + \varphi_1 \Delta x_{t-1} + c_1 \eta_{t-1} + \varepsilon_t \).

a1. Write out the formula for the impulse response \( \partial E_t \Delta x_{t+j}/\partial \eta_t \) for \( j = 0, 1, 2 \).

a2. Write out the formula for the impulse response \( \partial E_t x_{t+j}/\partial \eta_t \) for \( j = 0, 1, 2 \). (This question asks for the response of the level \( x_t \); part a1 asked for the response of the difference \( \Delta x_t \).)

(6) b. As usual, write the structural equations of the VAR as

\[
(2-3) \quad X_t = b + BX_t + \Gamma_1 X_{t-1} + \ldots + \Gamma_p X_{t-p} + \nu_t,
\]
where $B$ and the $\Gamma$'s are $5 \times 5$, $b$ and $\nu$ are $5 \times 1$, and $p$ is the VAR lag length. What restrictions does the Choleski decomposition impose on $B$?

(4) c. Consider the panel in the middle column and middle row (VAR with R&R, response of UE). Take as given that the peak response is 0.41, and occurs in month 14.

c1. Suppose the author had plotted a response to a 200 basis point positive shock rather than a 100 basis point positive shock. What would the peak response be, and what month would it occur in?

c2. Suppose the author had plotted a response to a 100 basis point negative shock. What would the peak response be, and what month would it occur in?

(10) d. The magnitude of the response is clearly larger in the "R&R baseline approach" column than in the other two columns. Consider using each of the three models to compute the fraction of variance of a particular one of the three variables, such as IP, that is explained by monetary policy shocks at a particular horizon, such as 20 months. Can one conclude from the graphs alone that this fraction is larger for the "R&R baseline approach" than for other two models? Why or why not?

(35) e. Consider the impulse responses displayed in the "R&R baseline approach" column. Does the pattern make intuitive sense? Which business cycle models is the pattern consistent with?

(25) f. It is a general finding that the narrative approach of R&R yields stronger effects of monetary policy shocks than do VARs such as those in the first column. In light of this analysis, why might this be?
Graph for question 2

standard VAR Approach

VAR with R&R shock

R&R baseline approach

Response of IP

Response of LE

Response of CPI
(100 points) Each agent goes through 3 periods of life: kid, young parent, old parent (and then dies). Each kid is born with a stochastic ability \( a \). The distribution of the kid’s ability is a function of the parent’s ability, i.e. \( a' \sim A(a'|a) \). Each young parent gives birth to one kid and makes decisions for him. A parent (or a grand-parent) cannot purchase insurance against the ability of his children.

The young parent invests \( e \) units of consumption goods and \( n \) units of time in his own human capital accumulation (“on-the-job training”), \( e_k \) units of consumption goods in his kid’s human capital and \( n_k \) units of time in his child’s human capital. Assume that the total amount of time endowed to each individual is 1 in each period. Human capital evolves according to

\[
\begin{align*}
    h'_o &= a(nh)^{\gamma_1}e^{\gamma_2} + (1 - \delta)h \\
    h'_k &= a_k(n_k h)^{\gamma_1}e_k^{\gamma_2} + (1 - \delta)h
\end{align*}
\]

where \( h \) is the human capital of the parent when he is a young adult, \( h'_o \) is the human capital of the parent when he gets old, and \( h'_k \) is the human capital of the child when he grows up and becomes a young parent.

The young parent saves \( s \) for his old age when young, starts off the second period with inter-vivos transfers (transfers made while parent is alive) of \( i \) and begins the third period with bequests \( b \) from his parent. Assume that the young parent makes decisions after she receives her inter-visos transfer from his parent and that the old parent makes decisions after he has receives the bequest \( b \). Preferences are defined as

\[
u(c_y) + \beta E \{ u(c'_o) + \theta V' \},
\]

where \( c_y \) is the consumption of the young parent, \( c'_o \) is the consumption of the old parent, and \( V' \) is the lifetime utility of her child after he grows up and the expectation operator is over future abilities. \( \theta \) is the weight the parent puts on her child’s utility. Assume that we are in a stationary equilibrium.

1. Let \( V(\cdot) \) be the value for the young parent and \( J(\cdot) \) for the old parent, formulate the Bellman equations for the young and old parent. Be careful to clarify the states and controls. (15 points)

2. Briefly outline an argument that demonstrates the value function is concave (Assume that it exists and is given by the unique solution to the Bellman’s equation above). Also outline an argument that demonstrates that the Value function \( V(\cdot) \) is differentiable. (20 points)
3. Derive the first order conditions for the optimal choice of OJT, education, savings and bequests. Derive the envelope conditions. Interpret the Euler equation. (20 points)

4. Now suppose that markets are complete. What do the optimal choices of investment in children look like? Explain how this differs from the incomplete market allocation. (10 points)

5. In the standard Aiyagari-Bewley model, the real interest rate in the steady state is less than the rate of time preference. Does the same result hold in this economy? Explain briefly why you think it holds or does not hold. What if we were to eliminate human capital from the model - how would your answer change? (15 points)

6. Imagine we were to compare the stationary distributions of the incomplete and the complete market economies. Which economy will possess a greater degree of inequality in earnings? Explain. What is the key assumption that generates this result? (20 points)
Consider the Cagan hyperinflation model, which relates $m_t$, the log of the money supply, to $p_t$, the log of the price level, via the relationship

$$m_t - p_t = \gamma (p_{t+1}^e - p_t)$$

where $p_{t+1}^e$ denotes the expected value of $p_{t+1}$ at time $t$. Assume that money supply process is

$$m_t = m_{t-1} + \pi(L) \epsilon_t,$$

where $\pi(L)$ is invertible.

A. (15 points) Characterize the equilibrium price sequence associated with this model when expectations are rational.

B. (15 points) Characterize the equilibrium price sequence when expectations are myopic, specifically that $p_{t+1}^e = \lambda p_t$.

C. (30 points) Under the assumptions of this question, what inferences can be drawn about the validity of the Cagan specification and about the presence or absence of a bubble in prices when price expectations are rational, assuming that one has data on the joint histories of the log of the money supply and the log of the price level?

D. (15 points) Under the assumptions of this question, what inferences can be drawn about the validity of the Cagan specification and about the presence or absence of a bubble in prices when price expectations are myopic in the way described by part B, assuming that one has data on the joint histories of the log of the money supply and the log of the price level?

E. (25 points) Suppose that output is determined by

$$y = \phi (p_{t}^e - p_{t-1})$$

Assume agents have perfect foresight with respect to the money supply process and that no bubbles are present. Will a proportional increase in the log of the money supply in all time periods affect equilibrium output? (By proportional, I mean multiplying the log of the money supply by a positive constant $\nu$.) Explain your answer. Will a constant term added to the log of the money supply in all periods affect output? Explain your answer.