UNIVERSITY OF WISCONSIN

DEPARTMENT OF ECONOMICS

Macroeconomics Preliminary Exam

August 6, 1998

9:00 am - 12:00 noon

- On the top fo EACH yellow sheet, write your ASSIGNED NUMBER, date and name of exam and question number. DO NOT write your name on the yellow pads. After the examination, the question sheets and yellow pads will be collected. DO NOT write on the question sheets.

- You are not allowed to use notes, books, calculators, or colleagues.

- Please solve three (3) and only three (3) out of the four (4) problems.

- The total time allotted is three (3) hours. Each problem has a suggested time of an hour.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- Read the questions carefully. The questions will not be explained. If you think that a question is ambiguous or poorly worded, make the minimum necessary assumptions to make it beautiful and well posed.

- Please return unused portion of the yellow tablets and question sheets.

- There are five (5) pages in this exam (including this cover page). Please make sure that you have all of them.

- Good Luck!
(20) A. Consider a three equation system for inflation $\pi_t$, money growth $\Delta m_t$ and output growth $\Delta y_t$:

1. $\Delta y_t = \beta(\pi_t - E_{t-1}\pi_t) + \delta \Delta y_{t-1} + u_t$ (aggregate supply)

2. $\Delta m_t - \pi_t = \Delta y_t + v_t$ (aggregate demand)

3. $\Delta m_t = \lambda \Delta m_{t-1} + w_t$. (money supply)

In this system: $E_{t-1}$ means expectations conditional on lagged values of the variables in the system; $u_t$, $v_t$ and $w_t$ are iid and mutually independent, with finite variance; $0<\delta,\lambda<1$; $\beta>0$. Inessential constants are omitted to simplify the notation.

(4) 1. What is a possible rationalization of the aggregate supply curve (1)? Comment on both the inflation surprise term, and the term in $\Delta y_{t-1}$.

(6) 2. Does money growth Granger cause output growth, relative to an information set consisting of lagged $\Delta m_t$, $\Delta y_t$ and $\pi_t$?

(4) 3. If one eliminates inflation and expected inflation from the system, the result may be written as the structural VAR

4. $X_t = BX_t + \Gamma X_{t-1} + e_t$.

where $X_t = (\Delta y_t, \Delta m_t)'$. What are $B$, $\Gamma$ and $e_t$? (Hint: $B$ has zeroes on the diagonal, and the two elements of $e_t$, which are linear combinations of the shocks $u_t$, $v_t$ and $w_t$, are uncorrelated with one another.)

(6) 4. Consider a variance decomposition for money growth, for a 1 period horizon. Is the fraction of the variance of money growth due to money growth shocks larger or smaller in the three equation system (1)-(3) or the two equation system (4)?

B. Answer true, false or uncertain, and explain your answer. Explanation determines grade. Note that the first question is worth more points than the second and third.

(10) 1. Some economists have recommended that the Fed aim solely to stabilize prices. This may have a great effect on inflation, but probably won't affect output behavior much,

(5) 2. Shifts in preferences (for leisure versus consumption, for example) are a potentially important driving force for business cycles.

(5) 3. Since most prices and wages are adjusted infrequently, business cycle models should incorporate price and wage stickiness.
Open Economy and Adjustment Costs (1 hour)

Consider a standard one sector growth model in which the planner has access to an international bond market. In this market the gross interest rate is fixed and equal to 1+r. To increase the capital stock the economy must allocate resources that could be otherwise consumed ---this, of course, is just investment--- and it must pay installation costs. In particular, if the beginning of the period capital per worker is \( k \), and the economy invests \( x \) per capita, then next period's capital per worker will be \( k' = (1-\delta)k + x \). If this period's per capita output is denoted by \( y \), the amount available after investment (for consumption and debt repayment) is \( y - x - (\varphi/2)(k' - k)^2 \). This formulation captures the idea that installation ---and even scrapping--- is costly. Assume that \( \varphi \geq 0 \).

The technology is represented by a constant returns to scale production function, \( F(K, N) \), where \( K \) is total capital, and \( N \) is employment. To simplify, assume that labor supply is inelastic and that the per capita production function is just \( f(k) \), where \( k \) is capital per worker.

The representative household has preferences given by,

\[
\sum_{t=0}^{\infty} \beta^t u(c_t),
\]

where \( 0 < \beta < 1 \), and \( u \) is a "nice" concave function.

a) Formulate the planner's problem for this economy.

b) Describe the steady state. Denote the steady state capital per worker by \( k^* \). How does it compare with the steady state capital per worker when \( \varphi = 0 \)? Is it possible to gain evidence on adjustment costs under the assumption that the data are generated by steady states?

c) Assume that \( \varphi = 0 \), and that \( \beta(1+r)=1 \). Let \( k_0 < k^* \) be arbitrary. Go as far as you can describing the dynamics of capital and consumption.

d) Assume that \( \varphi > 0 \), and that \( \beta(1+r)=1 \). Let \( k_0 < k^* \) be arbitrary. Go as far as you can describing the dynamics of capital and consumption.

Note: A single second order difference equation of the form \( a(x_{t+2} - x_{t+1}) + bG(x_{t+1}) + c(x_{t+1} - x_t) = 0 \), can always be transformed to a system of two first order equations by defining \( z_{t+1} = x_{t+2} - x_{t+1} \). Then, the previous equation is equivalent to,

\[
az_{t+1} + bG(x_{t+1}) + cz_t = 0, \\
x_{t+1} = z_t + x_t.
\]

e) Suppose that the economy is subject to random shocks (i.e. the technology is \( \varepsilon_t f(k_t) \) for some random variable \( \varepsilon_t \)). Is it true that in this case, economies subject to adjustment costs will display more volatility in consumption relative to economies with no adjustment costs? What about the volatility of output?

f) Bonus: Extra Credit. Go as far as you can describing the price of installed capital (i.e. a firm) in the economy with adjustment costs.
This question requires you to set up a formal model to analyze a policy question. Be as explicit as possible in your analysis and introduce mathematical assumptions as you see fit. Credit will be given according to how well you are able to do the formal modelling and analysis.

Suppose that the state of Wisconsin has $N$ graduating high school seniors. Each graduating senior has an initial individual educational level, $e_n$, which is a function of his innate ability, the quality of the high school he attended, and individual effort.

The state of Wisconsin has to determine how to allocate these $N$ students across $K$ colleges. (Assume that the $K$ colleges have exactly $N$ slots available in them.) Assume that the quality of the colleges, not accounting for student composition, is identical. Hence the college human capital each student receives, $E_n$, is a function of his own initial educational level, the initial levels of the other students at the college, and his own effort.

Formally analyze the problem of allocating students across colleges if the goal of the state of Wisconsin is to maximize aggregate college human capital, i.e. $\sum E_n$. Can you give conditions under which students are segregated by initial educational levels?
4. Answer all 8 parts of this question.

Consider a household maximizing a quadratic, intertemporally-separable preferences in an infinite-horizon, discrete-time setting with a constant real interest rate equal to the discount rate \( R \beta = 1 \) and no credit market imperfections:

\[
Max_{(c_s)_t} \mathbb{E}_s \left[ \sum_{s=t}^{\infty} \beta^{s-t} (ac_s - bc_s^2) \right]
\]

subject to

\[
\sum_{s=t}^{\infty} R^{t-s} (c_s - y_s) \leq A_t.
\]

Assume that the labor income process is exogenous and is given by \( y_t = p_t + \varepsilon_t \) where \( E_{t-1}[\varepsilon_t] = 0 \) and \( p_t = p_{t-1} + u_t \) and \( E_{t-1}[u_t] = 0 \) so that \( E_{t-1}[p_t] = p_{t-1} \).

1. Derive the Euler equation for consumption.

2. If \( \varepsilon_t \) is identically and always zero, will consumption track labor income for this household? Why or why not?

3. Derive the consumption level as a function of \( A_t \), and the net present value of future labor income. What is the marginal propensity to consume out of assets, \( A_t \)? Out of the net present value of future labor income?

4. Define savings as asset plus labor income less consumption: \( s_t = \frac{R}{R} A_t + y_t - c_t \). Let \( \alpha_1 \) be the marginal propensity to save out of out of assets, \( A_t \). Out of transitory shocks to labor income? Out of permanent shocks to labor income?

5. Suppose now that you had data on a cross-section of household's income, \( y_{i,t} \), expectations from one period ago about current income, \( E_{t-1}[y_{i,t}] \), and expectations from the current period about next period's income, \( E_{t}[y_{i,t+1}] \), all accurately measured. In addition you have a noisy measure of each household's consumption, \( c_{i,t} \). Suppose you ran the regression

\[
s_{i,t} = \alpha_0 + \alpha_1 (E_{i,t}[y_{i,t+1}] - y_{i,t}) + \alpha_2 (E_{i,t}[y_{i,t+1}] - E_{t-1}[y_{i,t}]) + \nu_{i,t}.
\]

Assuming that the estimation is consistent, what values would you like to test for \( \alpha_0 \), \( \alpha_1 \), and \( \alpha_2 \) to test the theory?

6. Assuming the model is true, can we consistently estimate this equation using cross-sectional data rather than time-series data? Why is or is not this the case when in general we cannot estimate an Euler equation consistently using cross-sectional data?

7. In practice, the model is rejected. The parameters are estimated as \( \alpha_1 \approx -0.5 \) and \( \alpha_2 \approx 0.15 \). Might these estimates be consistent with an alternative model in which we allow for precautionary saving?

8. Suppose that the income process includes a deterministic age-specific growth rate— a deterministic age profile— so that young consumers get higher income growth than old consumers. Does this invalidate the above test as a test of the marginal propensity to save out of shocks to income?