UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY  Preliminary Exam

June 8, 2015

9:00 am - 2:00 pm

INSTRUCTIONS

- Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:
  
  (1) your assigned number  
  (2) the number of the question you are answering  
  (3) the position of the page in the sequence of pages used to answer the questions

  Example:

  MACRO THEORY 6/8/15
  ASSIGNED # __________
  Qu # ___ (Page __ of __):

- **Do not answer more than one question on the same page!**
  
  When you start a new question, start a new page.

- **DO NOT write your name anywhere on your answer sheets!**
  
  After the examination, the question sheets and answer sheets will be collected.

- **Please DO NOT WRITE on the question sheets.**
- Each question counts equally.
- Answer all questions.
- Answers will be penalized for extraneous material; be concise.
- You are not allowed to use notes, books, calculators, or colleagues.
- Do NOT use colored pens or pencils.
- There are four pages in the exam, including this instruction page—please make sure you have all of them.

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!
Question 1 (100 total points)

1. (60 points) This problem considers the effects of an increase in population, say due to immigration. Consider an optimal growth model in an economy with population growth at rate $n$, so that we write household preferences in terms of consumption per capita $c_t = C_t/N_t$ as:

$$\sum_{t=0}^{\infty} [\beta(1+n)]^t u(c_t)$$

We can write evolution equation of capital per capita $k_t = K_t/N_t$ as:

$$(1 + n)k_{t+1} = (1 - \delta)k_t + f(k_t) - c_t.$$ 

(a) Write down the Bellman equation for the social planner. Under what conditions will the Bellman equation have a unique solution which is strictly increasing, strictly concave, and differentiable?

(b) Find the optimality conditions and characterize the dynamics of the economy. Under what conditions will there be a unique steady state (in per capita variables)?

Now suppose the economy is initially at this steady state, and then consider the effects of the following changes. Assume that the initial population is $N_0 = 1$. In addition, suppose that $u(c) = c^{1-\gamma}/(1-\gamma)$ and $f(k) = k^\alpha$ for $\gamma < 1$ and $0 < \alpha < 1$.

(c) Suppose that there is a one-time unforeseen jump in population at date $t$, so that $N_{t-1} = (1 + n)^{t-1}$, but then $N_t = (1 + n)N_{t-1} + \Delta$, for $\Delta > 0$ and then $N_{t+s} = N_t(1 + n)^s$ for $s = 1, 2, \ldots$. What happens to consumption and capital at the time of the change and in the following periods? (Answer this part qualitatively, using phase diagrams). What happens to consumption and capital (both their levels and growth rates) in the long run? (Answer this part analytically, finding expressions.)

(d) Now suppose instead that there is an unforeseen permanent increase in the population growth rate at date $t$, so that $N_{t-1} = (1 + n)_{t-1}$, but then $N_t = (1 + n')N_{t-1}$ for $n' > n$ and then $N_{t+s} = N_t(1 + n')^s$ for $s = 1, 2, \ldots$. What happens to consumption and capital at the time of the change and in the following periods? (Answer this part qualitatively, using phase diagrams). What happens to consumption and capital (both their levels and growth rates) in the long run? (Answer this part analytically, finding expressions.)

(e) Now consider a decentralization of the optimal allocation as a competitive equilibrium. What will be the short-run and long-run effects of a change in population on wages? How does this depend on whether the change is a one-time increase in population versus a change in the population growth rate?
2. **(20 points)** Consider a variation on the basic McCall search model. Workers are risk-neutral, unemployed workers get constant payments $z$, and search yields at least one job offer. Workers are allowed to quit, but they would not otherwise separate from firms. That is, at any date an employed worker would have the option to quit his job, become unemployed, and search for a new job. Wages are not known with certainty when a worker accepts a job. That is, unemployed workers receive offers of an expected wage $w$ drawn from $F(w)$. But once employed, the jobs pay $w + \Delta$ with probability 1/2 and $w - \Delta$ with probability 1/2 for $\Delta > 0$. The uncertainty is all resolved once the job has begun. That is, wages are constant over time but uncertain at the time of acceptance. Find the Bellman equations for employed and unemployed workers and characterize their optimal decision rules. Would employed workers ever quit?

3. **(20 points)** Suppose that a representative agent has preferences:

$$E \sum_{t=0}^{\infty} \beta^t \log c_t$$

over the single nonstorable consumption good (“fruit”). Her endowment of the good is governed by a Markov process with transition function $F(x', x)$. What is the equilibrium price/dividend ratio of a claim to the entire consumption stream? How does it depend on the distribution of consumption growth? Interpret your answer.
Question 2 (100 total points)

Part A. (50 points) Consider an environment where time is discrete, the horizon is infinite, and there is no uncertainty. Time is indexed \( t = 0, 1, 2, \ldots \), and there is a single consumption good in each period. A long-lived asset is a perfectly durable object that produces \( d_t \) units of the consumption good in each period \( t \). This question is about how the price of such an asset \( p_t \) relates to its “dividends” \( d_t \) and \( q_t \), the price of one-period risk free bonds traded in period \( t \).

1. (10 points) Explain what it means for the asset price \( p_t \) to contain a bubble.
2. (20 points) Write down a standard, frictionless endowment economy model populated by households who never die, and show that in such a model the equilibrium asset price \( p_t \) will not contain a bubble.
3. (20 points) Write down an exchange economy model populated by overlapping generations of households, and show that in such a model the equilibrium asset price may contain a bubble.

Part B. (50 points) Consider an infinite horizon economy where time flows as \( t = 0, 1, 2, \ldots \). There is a continuum of identical households indexed \( j \in [0, 1] \) and a government which is benevolent but lacks the ability to commit to a tax policy. In each period \( t \), households first simultaneously choose how much to work \( l^j_t \in [0, 1] \) and then the government chooses a tax rate on labor income \( \tau_t \in [0, 1] \). Everyone can observe and recall tax rates \( \tau_t \) and aggregate labor supply \( L_t = \int_0^1 l^j_t dj \) from previous periods. Households are, however, anonymous, so each household \( j \)'s individual labor supply \( l^j_t \) cannot be observed by anyone other than household \( j \). Household \( j \)'s utility is

\[
\sum_{t=0}^{\infty} \beta^t \left\{ c^j_t - \frac{1}{2} (l^j_t)^2 + 2G_t \right\}
\]

where \( 0 < \beta < 1 \) and

\[
c^j_t = (1 - \tau_t)l^j_t, \quad G_t = \tau_t L_t.
\]

a. (20 points) Define a sustainable equilibrium (S.E.) for this model.

b. (30 points) Write down a set of equalities and inequalities that are necessary and sufficient for a sequence \( (\bar{l}_t, \bar{\tau}_t)_{t=0}^{\infty} \) to be a S.E. outcome. Fully justify your answer.