Monopoly

Here’s the picture of a basic monopoly. Above, we have our cost curves (MC and ATC), and our demand curve. Unlike perfect competition, you’ll note that demand is downward sloping for a monopolist. Why might this be? Well, in a PC model, we assumed that the firm was, in some sense, small relative to the whole market. This is why the PC firm is a price-taker – its actions do not affect the aggregate market. So for a PC firm, the demand curve is a perfectly horizontal line, as a firm can sell as much as it wants at the price which prevails in the market. However, this would be a very poor assumption to make in the case of a monopoly. Obviously, a monopolist is not “small” relative to the supply side of the market – indeed, the monopolist is the only supplier of its product. Thus, the demand which the monopolist faces is the market demand for its product.

The above means that we must consider how to find the marginal revenue curve for a monopolist. Unlike a PC firm, the MR curve is not simply a flat line equal to the price. This is because when the monopolist wants to sell an additional unit, it must lower its price in order to do so, as the demand curve it faces is downward sloping. Thus, the MR curve is also downward sloping. The simple way to remember the MR equation is that it has the same y-intercept and twice the slope of the demand curve. Be careful that you have the equation in “P=” form before you do this – otherwise, your answer will be very wrong.

ex: if demand is given by \( P = 40 - 3Q \), the marginal revenue is given by \( P = 40 - 6Q \).
Mathematical digression: why is this relationship between demand and marginal revenue true? Well, marginal revenue is the change in total revenue divided by the change in quantity. Let demand be given by \( D: P = aQ + b \). Thus we have:

Total Revenue (TR) = \( PQ = (aQ + b)Q = aQ^2 + bQ \)

Now we want to take the derivative of TR with respect to Q, so we have:

\[
dTR/dQ = 2aQ + b = MR.\]

Comparing this against our original demand equation, we see that the marginal revenue equation does indeed have the same y-intercept and twice the slope. If this digression helps you see why this relationship is true, great. If not, don’t worry about it, but do remember that it’s “same y-intercept, twice the slope”.

Monopolists (like every other firm) set MR = MC, marginal revenue equal to marginal cost, to determine the optimal quantity to produce (\( Q^* \)). In order to find the price the monopolist will charge, we then go up to the demand curve to see how much consumers are willing to pay for quantity \( Q^* \). The monopolist may charge this price because it is the only supplier in the market, so there are no other firms to help bring the price down to the minimum of ATC as is the case in perfect competition.

Now that we have \( P^* \) and \( Q^* \), we can easily find ATC* as in the above picture. Thus, we can now find monopoly profits, consumer surplus, and deadweight loss.
Monopoly profits are in green. Profits are calculated in the same way as we did before in perfect competition. We find the difference between the total revenue and total cost boxes, or we consider the difference between the price which the monopolist receives for its product and its average total costs, multiplied by the quantity it sells. Thus, we have:

\[ \text{Profits} = (P^* - \text{ATC}^*) \times Q^* \]

Consumer surplus is the blue area above. CS is defined as before – it’s the area below the demand curve and above the price which consumers pay for the product. Here, the price which consumers pay is clearly \( P^* \), so we have:

\[ \text{CS} = \frac{[(y\text{-intercept of } D) - P^*] \times Q^*}{2} \]

Deadweight loss is in grey above. Here, there is no straightforward equation for DWL, because the bottom of the area is actually curved. This means that in order to calculate it directly, we would have to integrate the area (and those of you who have done this before know that integrating over an area between lines is even trickier than traditional integration relative to the x-axis). So while you will not be expected to find this area numerically in the general case, you should still be able to find it on the diagram.

Let’s work through a numerical example so you can see how this works. This example is the sort of problem which shows up on tests a lot due to its mathematical simplicity.

**ex:** (Constant marginal costs, zero fixed costs)

Let demand be given by \( D: P = 100 - Q \), \( MC = 20 \). Also, assume that the firm has no fixed costs \( (FC = 0) \). Find the monopoly profits, CS, and DWL.

First, we find the equation for MR. As we have the equation for demand, we know that MR has the same y-intercept but twice the slope. So MR: \( P = 100 - 2Q \).

We know that \( MC = 20 \), but we also know that in order to calculate profits, we’re going to need the ATC curve. So how do we find it? Well, let’s try to build it from the information we have. Consider the first unit. We know that there are no fixed costs, and \( MC = 20 \), so the total cost of making the first unit is 20, so the ATC is 20. Now consider the second unit. Again, no fixed costs, and the cost of making the second unit is 20. So the total cost for making two units is 40, so the ATC is again 20. Thus, we see that in the case of constant marginal costs and zero fixed costs, the ATC curve is the same as the MC curve. This is the critical insight of this example, as it allows us to calculate the desired quantities with simple areas. So armed with this knowledge, we can now draw the picture below to represent this situation. And from here, it is straightforward to compute profits, CS, and DWL. Thus we have:

\[ \text{Profits} = (60 - 20) \times 40 = $1600 \]
\[ \text{CS} = \frac{[(100 - 60) \times 40]}{2} = $800 \]
\[ \text{DWL} = \frac{[(60 - 20) \times (80 - 40)]}{2} = $800 \]
So now that we’ve dealt with the idea of monopoly, what are the important features of natural monopoly? Here’s the official definition.
Natural Monopoly: a monopoly created and sustained by economies of scale over the relevant range of output for the industry.

The phrase “over the relevant range of output” is the operative one here. In a subtle and important sense, natural monopoly is really a statement about demand and industry costs, not market structures. For an example, consider the above picture, and assume that the product is supplied by a monopolist. If demand for this product is given by D1, then the industry is a natural monopoly. If demand is D2, then we have a regular monopoly.

An equivalent way to think about natural monopolies is through the cost structure. Remember that MC must run through the minimum of ATC – so if ATC is falling, then MC is below ATC. This means that if we think about the total costs of the industry for producing a given quantity Q*, then the costs are lowest with only one firm producing the output.

The above picture illustrates this idea. If one firm produces quantity Q*, then the total costs for this industry are the red areas. If two firms each produce Q*/2, then we have the same total quantity of output, but the total costs of the industry are the red areas plus the orange areas. This is because the costs for the first firm are the left hand red and orange areas, and as the second firm is identical to the first, the costs for the industry are twice the costs of the first firm. The important thing to take away from this is that the total industry costs rise as the number of firms increases if ATC is falling over the relevant range of output. Thus, it is most efficient for society if there’s only one producer in this industry – so having a monopoly in this industry is therefore “natural”. A common example of natural monopoly is public utilities (electricity, water, cable) due to the high cost of building a network of pipes or wires.
What do we do with natural monopolies? We could find \( P^* \) and \( Q^* \) and profits, and that’s all done in exactly the same way as we did it above for a regular monopoly. The main topic related to natural monopolies is that of regulation. Under regulation schemes, the government forces the natural monopolist to provide its product to consumers at a price fixed by the government. There are two prices which we wish to consider here – price = ATC (average cost pricing) and price = MC (marginal cost pricing). Here, the subscripts denote the type of regulation imposed on the market.

Each type of regulation has its pluses and minuses. Under average cost pricing, \( \text{ATC} = P \), so the profits of the natural monopolist are zero. However, we can see from the picture that we still have deadweight loss, as we are not producing the socially optimal quantity of output. This DWL, as before, must be the area between the demand curve, \( Q_{\text{ate}} \), and MC. Remember, the socially optimal quantity of output is equal to the quantity which is produced by a perfectly competitive industry, where \( P = MC \). Therefore, under marginal cost pricing, we are producing the optimal quantity of output. However, notice that the firm is then making negative profits equal to \((\text{ATC}_{mc} - P_{mc}) \times Q_{mc}\), so they do not want to stay in business.

So what do we do? Well, if we set \( P = \text{ATC} \), then our firm will stay in business, but we lose out by having DWL. If we set \( P = \text{MC} \), then we have no DWL, but we can only keep our monopoly in business by giving them a subsidy equal to their negative profits (as this would make their profits zero). And in order to have the money to subsidize the monopoly, we have to put taxes on other markets to raise revenue. So which one we ultimately pick should depend on the DWL we may impose on other markets.\(^1\) In most

\(^1\) I say “may” here because of the possibility of using Pigouvian taxes on markets with externalities. Don’t worry about this for now – we’ll cover this topic in the last part of the course.
cases, governments tend to go with average cost pricing because getting the public to approve subsidies for monopolists is often politically difficult. So while marginal cost pricing is the efficient solution in that it produces no DWL in this market, average cost pricing is the most common practical solution applied to real natural monopolies. If you read your utility bills closely, you’ll notice that there are occasionally notices about “public hearings” concerning rates – these are opportunities for the utility company to present evidence and demonstrate their costs to government officials, while the public can comment on the effects any rate changes would have on their lives. Thus, while natural monopolies are not common in the real world, the proper regulation of such firms is an important function of the government.