Elasticity and Incidence – by Jeff Traczynski, 9/3/07

Elasticity
Elasticity: the percentage change in a variable in response to a given percentage change in another variable.

Elasticity is a measure of the sensitivity of one variable to changes in another variable. Most often, elasticity is used to describe demand curves, though we’ll see that the concept has some applicability to supply curves as well. There are many types of elasticity, each named for the variables involved. We also have point elasticity and arc elasticity, two separate formulas which we use depending on whether we’re calculating the elasticity at a given point or between two given points. So while it may seem like there are a lot of formulas here, the patterns will become clear, so it’s not too much to memorize. We’ll start with the most frequently used type of elasticity, the price elasticity of demand.

Price Elasticity of Demand

\[
\varepsilon = \left| \frac{\Delta Q / Q}{\Delta P / P} \right| = \left| \frac{(\text{percentage change in quantity demanded})}{(\text{percentage change in price})} \right|
\]

The price elasticity of demand (\(\varepsilon\)) shows how much the quantity demanded of a good changes as its own price changes. That is, when the price of a good increases 10%, how much does the quantity demanded fall? The price elasticity of demand is of particular interest to producers who are attempting to determine how to set the price of their products – if they raise their price slightly, will consumers buy nearly as much as before or will they turn to competing products? It is also worth pointing out that we always report price elasticity of demand as a positive number (note the absolute value signs in the above formula), even though we know from our study of the Law of Demand that as price goes up, quantity demanded should go down.

So, here are the two formulas to use when computing price elasticity of demand:

- **Point Elasticity**
  \[
  \varepsilon = \left( -\frac{1}{\text{slope}} \right) \left( \frac{P_A}{Q_A} \right)
  \]

- **Arc Elasticity**
  \[
  \varepsilon = \left( \frac{Q_B - Q_C}{Q_B + Q_C} \right) \left( \frac{P_B - P_C}{P_B + P_C} \right)
  \]

The point elasticity formula allows us to determine the elasticity at point A = (Q_A, P_A), while the arc elasticity formula gives us the responsiveness of quantity demanded to the change in price between points B = (Q_B, P_B) and C = (Q_C, P_C).

Let’s try to make things a little more clear by working through an example. Consider the demand curve D: \(P = 30 - 5Q\) with points A, B, and C as pictured below:
We see that A = (1, 25), B = (3, 15), C = (5, 5), and the slope of our demand curve is -5. We will now compute the point elasticity at each of these three points, as well as the arc elasticity between each pair of points.

\[ \varepsilon_A = \left( \frac{-1}{-5} \right) \left( \frac{25}{1} \right) = 5, \quad \varepsilon_B = \left( \frac{-1}{-5} \right) \left( \frac{15}{3} \right) = 1, \quad \varepsilon_C = \left( \frac{-1}{-5} \right) \left( \frac{5}{5} \right) = 1/5 \] are our point elasticities, while

\[ \varepsilon_{AB} = \left| \frac{1 - 3}{1 + 3} \right| = \left( \frac{1}{2} \right) = 2, \quad \varepsilon_{BC} = \left| \frac{3 - 5}{3 + 5} \right| = \left( \frac{1}{4} \right) = 1/2, \quad \text{and} \]

\[ \varepsilon_{AC} = \left| \frac{1 - 5}{1 + 5} \right| = \left( \frac{2}{3} \right) = 1 \] are our arc elasticities.

You might notice that the point elasticity at point B and the arc elasticity between A and C are both 1, and that B is exactly halfway between A and C. This is not a coincidence. Indeed, the arc elasticity between any two points will always equal to the point elasticity of the point halfway between them. To see this, we will compute the point elasticities at the points (2, 20) (halfway between A and B) and (4, 10) (halfway between B and C).

\[ \varepsilon_{(2,20)} = \left( \frac{-1}{-5} \right) \left( \frac{20}{2} \right) = 2, \quad \text{and} \quad \varepsilon_{(4,10)} = \left( \frac{-1}{-5} \right) \left( \frac{10}{4} \right) = 1/2, \quad \text{so} \quad \varepsilon_{(2,20)} = \varepsilon_{AB} \quad \text{and} \quad \varepsilon_{(4,10)} = \varepsilon_{BC}. \]
This little trick can help save you some work – really, you don’t need the arc elasticity formula if you can correctly find the halfway point and apply the point elasticity formula.

Economists use the price elasticity of demand to divide a demand curve into three regions, as illustrated below:

Demand is called elastic at a point if the price elasticity of demand at that point is strictly greater than 1 \((\varepsilon > 1)\), inelastic if \(\varepsilon < 1\), and unit elastic if \(\varepsilon = 1\). A linear demand curve has exactly one unit elastic point, which is located right in the middle of the line. In our above example, we saw that \(\varepsilon = 1\) at point \(B = (3, 15)\), and \(B\) is the center of the line.

Demand is elastic at all points above the unit elastic point, like point \(A\) from our previous example. We verified this directly when we showed that at point \(A\), \(\varepsilon = 5\). Similarly, demand is inelastic at all points below the unit elastic point, just as we saw that \(\varepsilon = 1/5\) at point \(C\).

What’s the point of dividing the demand curve into these regions? Well, recall that we earlier mentioned how knowing the price elasticity of demand would be useful for producers because it would help them set the price for their products. These regions of the demand curve help us draw the important links between elasticity and total revenue. First, let’s note that the revenue of the seller must equal the expenditure of the consumer. That is, however much money the seller receives must be equal to the amount of money consumers spend on the product. Now, how much do consumers spend on the product? Fortunately, the demand curve gives us this information, as the demand curve shows us the quantity the consumers want to buy at any given price per unit, and total expenditure on the good is equal to the price paid per unit times the number of units bought.

We will illustrate our argument by using the same example as above, so let our demand curve be given by \(D: P = 30 - 5Q\), and consider the points A-E as drawn below:
Consider a market where the price is $25. We know that consumers only demand 1 unit, and we are at point A on the demand curve. Total revenue for the seller is $25 \times 1 = $25, the area highlighted in red. However, if the seller lowers his price to $20 (point B), he will sell 2 units for a total revenue of $20 \times 2 = $40, the area highlighted in light blue. In this case, the seller has increased his revenue by lowering his price. Of course, this will not be true for every price. Consider a market where the price is $10 (point D). Now, the seller has revenue of $10 \times 4 = $40, the area highlighted in pink. If he lowers his price to $5 (point E), then his revenue becomes $5 \times 5 = $25, the area highlighted in blue.

Whenever demand is elastic, total revenue rises when the price falls and falls with the price rises. Whenever demand is inelastic, total revenue rises when the price rises and falls when the price falls. Of course, when we put these two statements together, we realize that on a linear demand curve, revenue is at its maximum at the unit elastic point (point C, total revenue = [15 \times 3] = $45). Indeed, we can see that the grey highlighted area is larger than any of the others – it contains 9 boxes, while the others contain either 8 or 5. Maximizing revenue is one of the most important applications of the price elasticity of demand.

**Income Elasticity of Demand**

The other types of elasticity we will discuss are very similar to the price elasticity of demand, and are also closely related to the determinants of demand which we previously covered. The income elasticity of demand is given by

\[
\varepsilon_i = \frac{\Delta Q}{\Delta I} = \frac{\text{(percentage change in quantity demanded)}}{\text{(percentage change in income)}}
\]

Here, we are interested in how much the quantity demanded of a good changes when the consumer’s income changes. The first thing to notice about this formula is that there is
no absolute value sign, so \( \varepsilon \) can be either positive or negative. For income elasticity, we generally do not use the point elasticity formula, so here’s the arc elasticity formula:

\[
\varepsilon_I = \frac{I_2 - I_1}{I_2 + I_1} \times \frac{Q_2 - Q_1}{Q_2 + Q_1} = \text{Arc Income Elasticity of Demand}
\]

We use this formula when a consumer experiences a change in income from \( I_1 \) to \( I_2 \) and therefore changes his quantity demanded of the good from \( Q_1 \) to \( Q_2 \).

As we know from our discussion of determinants of demand, a consumer’s quantity demanded of a good can go up or down when his income increases depending on what type of good it is. For normal goods, the consumer demands more when his income increases, and the opposite is true for inferior goods. With the income elasticity of demand, the distinction between these two types is reflected in the sign of \( \varepsilon_I \). If \( \varepsilon_I > 0 \), then the good is normal, and if \( \varepsilon_I < 0 \), then the good is inferior\(^1\). A numerical example is worked out below.

Ex: Jen receives a raise in her allowance from $10/week to $15/week, and she also increases the number of sandwiches she purchases from 4/week to 8/week. What is Jen’s income elasticity of demand for sandwiches, and are sandwiches a normal or inferior good for Jen?

Well, let’s apply the above formula and see what we get:

\[
\varepsilon_I = \frac{8 - 4}{8 + 4} \times \frac{15 - 10}{15 + 10} = \frac{1}{3} = \frac{5}{3}, \text{ so sandwiches are a normal good for Jen.}
\]

Cross-Price Elasticity of Demand

\[
\varepsilon_{A,B} = \frac{\Delta Q_A / Q_A}{\Delta P_B / P_B} = \text{(% change in quantity demanded of good A) / (% change in price of good B)}
\]

We don’t use the point elasticity formula with cross-price elasticities, so here’s the arc elasticity formula:

\[
\varepsilon_{A,B} = \frac{Q_{A2} - Q_{A1}}{Q_{A2} + Q_{A1}} \times \frac{P_{B2} - P_{B1}}{P_{B2} + P_{B1}} = \text{Arc Cross-Price Elasticity of Demand}
\]

\(^1\) Some books also make the following distinction: if \( \varepsilon_I > 1 \), then the good is income-elastic or a luxury (like a European vacation), and if \( 1 > \varepsilon_I > 0 \), then the good is income-inelastic or a necessity (like food).
We use this formula when good B changes price from $P_1$ to $P_2$, causing our consumer to change his quantity demanded of good A from $Q_1$ to $Q_2$.

Again, we recall our determinants of demand and note that if the prices of other goods and services (say, good B) change, the demand for good A may also change. We would certainly expect that if Burger King lowered the price of its hamburgers, then demand for McDonald’s hamburgers would probably fall. If an increase in the price of B causes demand for A to rise, then A and B are substitutes, while if an increase in the price of B causes demand for A to fall, A and B are complements. And again, this distinction between the types is reflected in the sign of $\varepsilon_{A,B}$. If $\varepsilon_{A,B} > 0$, then A and B are substitutes, and if $\varepsilon_{A,B} < 0$, then A and B are complements. A numerical example is worked out below.

Ex: Jen buys both CDs and DVDs. Last week, the price of a CD increased from $4 to $6, and Jen responded by purchasing 6 DVDs instead of her usual 10. What is Jen’s cross-price elasticity of demand between CDs and DVDs, and are CDs and DVDs complements or substitutes for Jen?

Let’s try the formula, since that technique worked so nicely for the last example:

\[
\varepsilon_{A,B} = \frac{\frac{Q_{A2} - Q_{A1}}{Q_{A1}}}{\frac{P_{B2} - P_{B1}}{P_{B1}}} = \frac{\frac{6 - 10}{6 + 10}}{\frac{6 - 4}{6 + 4}} = \frac{-1}{4} = -\frac{5}{4},
\]

so these are complements for Jen.

**Price Elasticity of Supply**

\[
\varepsilon_S = \left| \frac{\Delta Q_S}{\Delta P} \right| = \left( \frac{\text{percentage change in quantity supplied}}{\text{percentage change in price}} \right)
\]

Included here for completeness, the price elasticity of supply is rarely directly calculated in practice. The most important thing to remember is that it is computed in exactly the same way as the price elasticity of demand, so we can use the same formulas as above:

**Point Elasticity**

\[
\varepsilon_S = \left| \frac{1}{\text{slope}} \left( \frac{P}{Q} \right) \right|
\]

**Arc Elasticity**

\[
\varepsilon_S = \left| \frac{\frac{Q_{SB} - Q_{SC}}{Q_{SB} + Q_{SC}}}{\frac{P_B - P_C}{P_B + P_C}} \right|
\]

However, we don’t have regions of the supply curve, nor do we have notions of elastic and inelastic goods on the supply side. Perhaps the only noteworthy thing about numerical values of $\varepsilon_S$ is that if the supply curve runs through the origin, then $\varepsilon_S = 1$ at all points.

A final word about elasticity: though we’ve just spent some time talking about how demand curves have elastic and inelastic regions, we haven’t said why the demand (or
supply) of some goods is more elastic than others. There are three main characteristics of products which affect the elasticity of demand and supply:

**Elasticity of Demand:**
1) Existence of close substitutes – if the price of decaffeinated green tea rises, we might think that people are willing to switch to caffeinated green tea even if the price increase is small, so we would expect the demand for decaffeinated green tea to be highly elastic.
2) Necessity vs. Luxury – food is certainly a necessity, while a laptop is not. Thus, the demand for food should be much more inelastic than for laptops.
3) Timeframe – demand is less elastic in the short run than in the long run, as more time gives people more opportunities to find alternatives. For example, if you live 50 miles from your job and the price of gas rises tomorrow, you’re pretty much stuck paying the new higher price. But over the next year, you could find another job, move closer to your current job, start a car pool, figure out the public transportation system, buy a hybrid car – all of which lower your demand for gas.

**Elasticity of Supply:**
1) Alternative uses for inputs – if the price of shirts falls, a company which makes shirts would probably find its machines and labor well-suited to making pants. However, a company which makes airplanes might not be able to easily switch to making another product. Thus, we expect the supply curve for shirts to be relatively elastic, while the supply curve for airplanes is inelastic.
2) Availability of inputs – the inputs for making orange juice are readily available, so if there is a small increase in the price of orange juice, firms can quickly get more inputs and expand production. As inputs become easier to obtain, supply becomes more elastic.
3) Timeframe – just as with demand, as more time passes, supply becomes more elastic.

**Incidence**
Perhaps the most important application of elasticity comes when dealing with excise taxes. Incidence refers to the “burden” of a tax – when the government imposes a tax on a market, who pays it? There are two types of incidence. The legal incidence of a tax falls on the group designated by the government to actually pay the money to the government. For example, it may be the responsibility of consumers to pay the government, as in the case of a sales tax (which is always separately tacked onto your bill). The economic incidence of a tax is the amount of consumer and producer surplus lost due to the imposition of the tax. Economic incidence is often expressed as a percentage to denote what portion of the tax is being paid by consumers versus producers. Unsurprisingly, economists are much more concerned with economic incidence than legal incidence. What may be surprising, however, is that the legal incidence of a tax has no effect on the economic incidence – whether a tax is legally applied to consumers or producers makes no difference in who bears the economic burden of paying it.

To see these ideas in action, consider the following example. Let our curves be given by D: \( P = 100 - 4Q \) and S: \( P = Q \), and apply a $20 excise tax to consumers. This situation is pictured below:
Without a tax, the market has an equilibrium price of $20 and an equilibrium quantity of 20. With the tax in place, the quantity traded falls to $Q_T = 16$ and the price sellers receive falls to $P_{ST} = $16, while the price paid by consumers rises to $P_{CT} = $36. From our previous work with excise taxes, we know that this tax generates $(36 – 16) \times 16 = $320 of revenue. However, the rectangle that we expect to represent the tax revenue has been divided into two smaller rectangles, labeled Consumer Tax Incidence (CTI) and Producer Tax Incidence (PTI). These boxes represent how much of the economic burden of the tax falls on consumers and producers, respectively. Note that if there were no tax on this market, then the entire yellow box would be part of consumer surplus, while the orange box would be part of producer surplus. Instead, the yellow and orange boxes are government tax revenue – surplus lost due to the imposition of the tax. The formulas for calculating CTI and PTI are below:

\[
CTI = (P_{CT} - P^*) \times Q_T = (36 - 20) \times 16 = $256
\]

\[
PTI = (P^* - P_{ST}) \times Q_T = (20 - 16) \times 16 = $64
\]

If we instead wanted to express this as a percentage, we can divide each number by the total tax revenue, so

\[
CTI = 256 / 320 = 80% \text{ and } PTI = 64 / 320 = 20%.
\]

What does all of this have to do with elasticity? Well, an alternative way of calculating the percentages above uses $\varepsilon$ and $\varepsilon_S$ directly, as shown below:

\[
CTI = \frac{\varepsilon_S}{\varepsilon + \varepsilon_S} \text{ and } PTI = \frac{\varepsilon}{\varepsilon + \varepsilon_S}
\]

where the elasticities $\varepsilon$ and $\varepsilon_S$ are calculated at the point $(Q^*, P^*)$, the equilibrium without taxes.

Let’s try out this formula using the equations from our above example:
This elasticity formula allows us to make an important realization: whichever side of the market is more inelastic will bear more of the tax burden. In our above example, the demand curve is less elastic than the supply curve, and consumers bear the majority of the tax burden.

So, this has been a lot of material. Let’s do one final example which ties together many of the above ideas to test your understanding. In 1990, the Democratically-controlled Congress passed a “luxury tax”. The idea was to tax things which the very wealthy purchase so that only the very wealthy would pay the tax. Expensive cars, jewelry, large boats, and other similar items were taxed, and Congress thought they had come up with an effective way to tax the rich. Let’s see if they were right by using our new concepts to analyze the luxury tax. In particular, we’ll consider the market for yachts.

What does the demand curve for yachts look like? Well, they’re certainly luxuries, and there are lots of substitutes for potential yacht buyers to spend their income on – smaller boats, international vacations, etc. So it seems reasonable to believe that the demand for yachts is highly elastic. What about the supply curve for yachts? The labor used to make yachts has some alternative uses; they could make smaller boats, or find jobs welding in other industries. But the other inputs (say, yacht hulls) are not nearly as flexible, and combining this with the fact that getting such inputs (specialized parts, skilled labor) may be difficult might lead us to believe that the supply curve is inelastic.

\[
\varepsilon = \left( \frac{-1}{\text{slope}} \right) \left( \frac{P^*}{Q^*} \right) = \left( \frac{-1}{-4} \right) \left( \frac{20}{20} \right) = 1/4, \text{ and } \varepsilon_s = \left( \frac{1}{\text{slope}} \right) \left( \frac{P_s}{Q_s} \right) = \left( \frac{1}{1} \right) \left( \frac{20}{20} \right) = 1, \text{ so }
\]

\[
CTI = \frac{\varepsilon_s}{\varepsilon + \varepsilon_s} = \frac{1}{1 + (1/4)} = \frac{1}{5/4} = \frac{4}{5} = 80\% \quad \text{and} \quad PTI = \frac{\varepsilon}{\varepsilon + \varepsilon_s} = \frac{(1/4)}{(1/4) + 1} = \frac{1}{5} = 20\%.
\]
Legally, the incidence of this tax fell on consumers, so the demand curve shifts down as pictured above. But look at what has happened with the economic incidence: yacht producers, not consumers, pay the majority of the luxury tax. When we say “yacht producers”, we really mean everyone involved on the production side – so while the owners of yacht factories may be rich, the workers are probably not wealthy. Thus, our theory predicts that the luxury tax would be paid mostly by the working class in this market, not the rich as originally intended. While the yacht market is only one market impacted by this tax, we could make similar arguments about the relative elasticities of supply and demand in other such markets – Lear jets, personal helicopters, fur coats, or Cadillacs, for example. In reality, the luxury tax fell heavily on producers in the vast majority of affected markets, and Congress responded by repealing most of the luxury tax in 1993. The political fallout from the failure of this tax was considerable: the damage done to American firms, particularly in export markets, was used as a campaign issue by Republicans in 1994 as they took control of both houses of Congress for the first time in 40 years. The luxury tax, therefore, stands as an example of just how important economic incidence can be and how a faulty understanding of this concept can have serious political repercussions.