UNIVERSITY OF WISCONSIN
DEPARTMENT OF ECONOMICS

MACROECONOMICS THEORY Preliminary Exam

June 17, 2014

9:00 am - 2:00 pm

INSTRUCTIONS

- **Please place a completed label (from the label sheet provided) on the top right corner of each page containing your answers. To complete the label, write:**

  (1) your assigned number
  (2) the number of the question you are answering
  (3) the position of the page in the sequence of pages used to answer the questions

Example:
MACRO THEORY 6817/13
ASSIGNED # ___________
Qu # __1__ (Page __2__ of __4__):

- **Do not answer more than one question on the same page !**
  When you start a new question, start a new page.
- **DO NOT write your name anywhere on your answer sheets!**
  After the examination, the question sheets and answer sheets will be collected.
- **Please DO NOT WRITE on the question sheets.**
- **Each question counts equally.**
- **Answer all questions.**
- **Answers will be penalized for extraneous material; be concise.**
- **You are not allowed to use notes, books, calculators, or colleagues.**
- **Do NOT use colored pens or pencils.**
- **There are five pages in the exam, including this instruction page—please make sure you have all of them.**

Read the problems carefully and completely before you begin your answer. The problems will not be explained—if a problem seems to be ambiguous, make clarifying assumptions and state them explicitly. Aim for well organized and legible answers that address the question and that demonstrate your command of the relevant economic theory.

- If you get stuck in a problem/section, move on. Partial credit will be granted when it is clear from your work that you were approaching the problem in a generally correct way.

- All scratch paper, unused tablet paper, and exams are to be turned in after the exam. Your proctor will give you directions, listen to your proctor.

- Good luck!

Question 1 (100 total points)

I. (40 points) Consider a Lucas-type asset pricing model, in which a representative agent gets an endowment e_t in period t, where e_t follows a Markov process. However instead of having standard time-separable utility, the representative agent has preferences with external habit persistence. That is, the agent seeks to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t - \lambda C_t),$$

where the initial values e_0 is given, and C_t is aggregate consumption. The consumer takes aggregate consumption as given (driven by an exogenous Markov process) when making his decisions, but in equilibrium $c_t = C_t$. The agent can trade in a full set of contingent securities.

(a) Write down the consumer's Bellman equation and find his optimality conditions.

(b) Define a recursive competitive equilibrium in this environment.

(c) Find the expressions (Euler equations) which determine the equilibrium price P_t of a claim to the endowment process, and R the gross risk free interest rate. What is the stochastic discount factor here?

II. (30 points) Consider a two period, two state version of Thomas and Worrall (1990). There is a risk-neutral moneylender that borrows and lends at rate $R = 1/\beta$, and a risk-averse household with standard preferences with discount factor β. Characterize the optimal contract by minimizing the costs to the moneylender of delivering a level of expected utility to the household. In the second period transfers b_2 must be independent of the state, so the contract solves:

$$P_2(v) = \max_{b_2} -b_2 \ \text{ s.t.} Eu(e + b_2) = v$$

where v is the level of promised utility and e is the endowment where $e_t = e_H$ with probability π and $e_t = e_L$ with probability $1 - \pi$. In the first period, the planner chooses transfers b_s, continuation utility w_s (conditional on the agent's report $e = e_s$):

$$P_1(v) = \max_{\{b_H, b_L, w_H, w_L\}} \pi(-b_H + \beta P_2(w_H)) + (1 - \pi)(-b_L + \beta P_2(w_L))$$

subject to a promise keeping constraint that the contract must deliver v in expected utility, and incentive constraints which ensure that the agent truthfully reports his endowment.

(a) Write down the promise keeping and incentive constraints, and verify that $b_L \geq b_H$ and $w_L \leq w_H$. Interpret your answer.

(b) Verify that the incentive constraint binds when the consumer gets the high endowment.

(c) Show that the household's expected marginal utility increases over time. Interpret your answer.

III. (30 points) Consider the following variations on the basic (McCall) sequential search model from class. In each case, suppose that workers are risk-neutral, unemployed workers get constant payments z, jobs last forever (unless specified), and search always yields at least one job offer drawn in an i.i.d. manner from a distribution $F(w)$.

(a) Suppose that an unemployed worker had the option to recall a previous wage offer. That is, if the worker received an offer of w_0 at some date, he could turn it down and search again, but would have the option at future dates to work at that previously made offer w_0. Find the unemployed worker's Bellman equation and characterize his optimal decision rule. How does it compare to the case without recall?

(b) Suppose that employed workers receive unsolicited offers, that is a worker currently employed at wage w receives an offer w' from a distribution $G(w'|w)$. If the employed worker takes the job at wage w' he must spend one period in transit, earning no income, before beginning the job. Find the Bellman equations of employed and unemployed workers and characterize their decision rules. How does the the possibility of offers on the job change the reservation wage of an unemployed worker?

Question 2 (100 total points)

Part A. (25 points) Consider the following stochastic endowment economy. There are two periods $t = 1, 2$ and two agents $i = 1, 2$. Let y_t^i denote household i's endowment in period t. Endowments in period $t = 1$ are known with certainty, but endowments in period $t = 2$ are uncertain and depend on the future state of the world s which is drawn from a distribution Pr. Both agents have the same utility function:

$$u(c_1^i) + \beta \sum_s u(c_2^i(s)) \Pr(s)$$

where c_t^i is consumption, $\beta > 0$, and u is strictly increasing.

1. Show that an equilibrium allocation is Pareto optimal if agents are allowed to trade a complete set of state contingent claims.

2. Show that an equilibrium allocation may not be Pareto optimal if markets are incomplete.

Part B. (25 points) Consider the following optimal investment problem for a firm:

$$\max_{(D_t, I_t, K_{t+1})_{t=0}^{\infty}} \sum_{t=0}^{\infty} R^{-t} D_t, \quad \text{s.t.}$$

$$D_t = A_t K_t - I_t - \frac{\phi}{2}\left(\frac{I_t}{K_t}\right)^2 K_t, \quad \forall t$$

$$K_{t+1} = (1-\delta)K_t + I_t, \quad \forall t$$

$$K_t \geq 0, \quad \forall t$$

$$K_0 = \bar{K}_0$$

Here, D_t is dividends, I_t is investment, K_t is capital, $R > 1$ is the gross real interest rate, $A_t > 0$ is a measure of profitability, $\delta \in (0, 1)$ is the depreciation rate, $\phi > 0$ is an adjustment cost parameter, and $\bar{K}_0 > 0$ is initial capital.

Let $(D_t^*, I_t^*, K_{t+1}^*)_{t=0}^{\infty}$ be a solution to this problem. Define the firm's market value as:

$$V_t^* = \sum_{s=t}^{\infty} R^{-s+t} D_s^*$$

and Tobin's q as:

$$q_t^* = V_t^* / K_t^*.$$

Show that there are constants b_0 and b_1 such that

$$I_t^* / K_t^* = b_0 + b_1 q_{t+1}^*$$

for each t.

Part C. (25 points) Consider an overlapping generations economy in which production takes place using capital and labor as inputs. Show that an equilibrium allocation can be dynamically inefficient.

Part D. (25 points) Consider a version of the neoclassical growth model with homogeneous households (or, equivalently, a representative household) in which the firm owns the capital stock and makes investment decisions. The firm's shares are traded in a competitive stock market. Show that, in a steady state equilibrium, the stock price equals the capital stock.