Buying First or Selling First in Housing Markets*

Espen R. Moen[†], Plamen T. Nenov[‡], and Florian Sniekers[§]

Abstract

Housing transactions by moving owner-occupiers take two steps – purchase of a new property and sale of the old unit. This paper shows how the transaction sequence decision of moving homeowners depends on, and in turn, affects housing market conditions in an equilibrium search model of the housing market. Moving homeowners prefer to buy first whenever there are *more* buyers than sellers in the market. This behavior leads to multiple steady state equilibria and self-fulfilling fluctuations in prices and time-on-market. Equilibrium switches create large fluctuations in the housing market, which are broadly consistent with stylized facts about the housing cycle.

Keywords: housing market, search frictions, order of transactions, strategic complementarity, self-fulfilling fluctuations

JEL Codes: R21, R31

*We want to thank Elliot Anenberg, Jan Eeckhout, Yong Suk Lee, and seminar participants at a number of seminars and conferences for valuable comments and suggestions. We want to thank Henning Bunzel and Rune Vejlin for providing us with access to the Danish property ownership and sales registers. Florian Sniekers gratefully acknowledges financial support by the Netherlands Organization for Scientific Research (NWO).

†Norwegian Business School (BI), e-mail: espen.r.moen@bi.no.

‡Norwegian Business School (BI), e-mail: plamen.nenov@bi.no.

§University of Amsterdam, Vrije Universiteit Amsterdam and Tinbergen Institute, e-mail: f.j.t.sniekers@uva.nl.

1 Introduction

A large number of households move within the same local housing market every year. Many of these moves are by owner-occupiers who buy a new property and sell their old housing unit. However, it takes time to transact in the housing market, so a homeowner that moves may end up either owning two units or being forced to rent for some period, depending on the sequence of transactions. Either of these two alternatives may be costly.[1] There is anecdotal evidence that the incentives to "buy first" (buy the new property before selling the old property) or "sell first" (sell the old property before buying the new one) may depend on the state of the housing market.[2] If the transaction sequence decisions of moving owner-occupiers in turn affect housing market conditions, there could be powerful equilibrium feedbacks with important consequences for housing market dynamics.

In this paper we study a tractable equilibrium model of the housing market, which features trading delays and a transaction sequence decision for moving owner-occupiers. We show that the transaction sequence choices of moving owners can have powerful effects on the housing market and can lead to large fluctuations in the stock of houses for sale, time-on-market, transaction volume, and prices.

In the model, agents continuously enter and exit a local housing market. They have a preference for owning housing over renting, and consequently. The housing market is characterized by a frictional trading process, and the speed at which buyers and sellers find a trading partner is affected by the tightness in the market – the ratio of buyers to sellers. As an owner-occupier, an agent

[1]The following quote from Realtor.com, an online real estate broker, highlights this issue: "If you sell first, you may find yourself under a tight deadline to find another house, or be forced in temporary quarters. If you buy first, you may be saddled with two mortgage payments for at least a couple months." (Dawson (2013))

[2]A common realtor advice to moving homeowners is to "buy first" in a "hot" market, when there are more buyers than sellers and prices are high or expected to increase, and "sell first" in a "cold" market, when there are more sellers than buyers and house prices are depressed or expected to fall. Anundsen and Røed Larsen (2014) provide survey evidence from Norway for such a behavior by owner-occupiers. In Section 2 we provide evidence for this link using data for the Copenhagen housing market.

may be hit by an idiosyncratic preference shock and become "mismatched" with his current house, in which case he wants to move internally in the same housing market. To do that, the mismatched owner has to choose optimally the order of transactions – whether to buy a new housing unit first and then sell his old unit (buy first), or sell his old unit first and then buy (sell first). Given trading delays, the agent would then become a double owner (owning two housing units) or a forced renter (owning no housing) for some time, which is costly. The expected time in such a state depends on the time-on-market for sellers and buyers, respectively.

If the costs incurred by a double owner or a forced renter are high relative to the costs of mismatch, the mismatched owner prefers buying first over selling first whenever there are *more* buyers than sellers in the market, or more generally, when the buyer-seller ratio is high. The reason is that a moving owner wants to minimize the delay between the two transactions, and hence do the most time-consuming transaction first. When there are more buyers than sellers, the expected time-on-market is low for a seller and high for a buyer. Hence the agent prefers to buy first. Conversely, if there are more sellers than buyers, the expected time-on-market is high for sellers and low for buyers, and the agent wants to sell first.

The order of transactions by moving owner-occupiers affects the buyer-seller ratio. Specifically, when mismatched owner-occupiers buy first, they crowd the buyer side of the market, and so, the market ends with more buyers than sellers in steady state. Conversely, when all mistmatched owners sell first, there are more sellers than buyers in steady state. Since a high (low) buyer-seller ratio is consistent with the incentives of mismatched owners to buy first (sell first), the interaction between the behavior of mismatched owners and the buyer-seller composition of the housing market creates a *strategic complementarity* in their transaction sequence decisions, which in turn may lead to multiple steady state equilibria. In one steady state equilibrium (a "Sell first" equilibrium), mismatched owners prefer to sell first, the market tightness is low and the expected time-on-market for sellers is high. In the other steady state equilibrium (a "Buy first" equilibrium), mismatched owners prefer to buy first, the market tightness is high and the expected time-on-market for sellers is low.

In our analysis, we first assume that prices are fixed across steady states, or alternatively, that the rental price is equal to the house price times the discount rate, in which case prices do not influence the transaction sequence decision. After explaining this channel and its implications, in a separate section, we endogenize house prices. First, we assume that the steady state price level is an increasing function of the buyer-seller ratio. We show that even with a rental price that is constant, multiplicity exists as long as the responsiveness of the house price to the buyer-seller ratio is not too high. After clarifying this countervailing effect, we show that there can exist multiple equilibria when house prices are (endogenously) determined by Nash bargaining, and hence, both differ across trading pairs and respond to changes in the buyer-seller ratio. Specifically, the main channel that drives equilibrium multiplicity in our benchmark model – the strategic complementarity in the agents' transaction sequence decisions, and the stock-flow conditions that determine the equilibrium market tightness – is still present in that environment. We also show that there can be equilibrium multiplicity even in an environment with competitive search where agents can trade off prices and time-on-market (queue length).

We also analyze switches between the "Buy first" and "Sell first" equilibria. Such switches lead to fluctuations in the housing market. Specifically, moving from the "Buy first" to the "Sell first" equilibrium is associated with an increase in the for-sale stock and time-on-market for sellers, and a drop in transactions. This behavior is broadly consistent with evidence on the housing cycle. In particular, explaining the negative comovement between the for-sale stock and transactions has been a challenge for search-based models of the housing market (Diaz and Jerez, 2013). As we show in a simple numerical example, the fluctuations generated by these switches can be substantial. Also, when prices are determined by Nash bargaining, we show that there can be substantial house price fluctuations arising from these equilibrium switches.

Finally, we show that when house prices respond to changes in the buyer-seller ratio, there can exist equilibria with self-fulfilling fluctuations in prices and market tightness. Since mismatched owners are more likely to buy first (sell first) when they expect price appreciation (depreciation), they end up exerting a destabilizing force on the housing market. For example, if agents expect prices to depreciate, they are more likely to sell first. However, this decreases the buyer-seller ratio, which in turn drags down house prices, and thus, confirms the agents' expectations.[3]

Related literature. The paper is related to the growing literature on search models of the housing market initiated by the seminal work of Wheaton (1990), and particularly, to the literature on search frictions and housing market dynamics (Krainer (2001), Novy-Marx (2009), Caplin and Leahy (2011), Diaz and Jerez (2013), Head, Lloyd-Ellis, and Sun (2014), Ngai and Tenreyro (2014), Guren and McQuade (2013), Anenberg and Bayer (2015), Ngai and Sheedy (2015), Piazzesi, Schneider, and Stroebel, 2015).[4] However, most of this literature abstracts away from the transaction sequence choices of moving owners by assuming that the actions of buying and selling are independent of each other.

In Wheaton (1990), mismatched owners must also both buy and sell a housing unit. However, the model implicitly assumes that the cost of becoming a forced renter with no housing is prohibitively large, so that mismatched owners always buy first. As we show in our paper, allowing mismatched owners to endogenously choose whether to buy first or sell first has important consequences for the housing market.

Diaz and Jerez (2013) calibrate a model of the housing market in the spirit of Wheaton (1990) where mismatched owners must buy first, as well as a model where they must sell first. They show that each of the two models can explain

[3]The working paper version of our paper (Moen, Nenov, and Sniekers (2015)) provides a discussion of the institutional details of transacting for several countries. There, we argue that our model captures essential elements of housing transactions for many countries, including Denmark, Norway, the Netherlands, and the United States. In these countries, the institutional set-up for the process of housing transactions is such that homeowners are concerned about the order of buying and selling, at least to some extent.

[4]The paper is also broadly related to the Walrasian literature on house price dynamics and volatility (Stein (1995), Ortalo-Magne and Rady (2006), Glaeser, Gyourko, Morales, and Nathanson (2014), He, Wright, and Zhu (2015)).

some aspects of housing market cycles, which points to the importance of a model that can accommodate an explicit transaction sequence choice.

Ngai and Sheedy (2015) model an endogenous moving decision based on idiosyncratic match quality as an amplification mechanism of sales volume. The paper argues that the endogenous participation decisions of mismatched owners is important for explaining key patterns in the data during the housing boom of the late 90s and early 2000s. In our model we assume that mismatched owners always participate and instead focus on their transaction sequence decisions. The implications we draw from our analysis are, therefore, complementary to the insights in their paper.

Anenberg and Bayer (2015) is a recent contribution that is close to our paper, particularly in terms of motivation. The paper studies a rich quantitative model of the housing market to explain the large volatility of internal moves by existing owners. The authors calibrate their model to data for Los Angeles and show how shocks to the flow of new buyers can be amplified through the decisions of existing owners and can lead to substantial fluctuations in prices and volume. Similar to our model, existing owners in their model also have to both buy and sell. Unlike our model, however, existing owners simultaneously search both on the buyer and seller side, so they do not choose to bias their search decisions. Specifically, with some probability, both a buy and a sell offer may arrive in the same period, in which case an owner has to choose to conduct one of the two transactions and forfeit the possibility of conducting the other. Otherwise, whether a buy offer arrives before a sell offer or vice versa is exogenous from the point of view of individual owners.

In contrast, in our model, mismatched owners choose which side of the market to enter and whether to receive only offers from buyers or from sellers. This endogenous choice influences market tightness and the buyer and seller times-on-market, which in turn influence the choices of other existing owners. The strategic complementarity resulting from this feedback between market tightness and the decisions of mismatched owners leads to multiplicity, self-fulfilling fluctuations, and housing market volatility in our model. Therefore, our theoretical focus and the different mechanism that we explore make our paper complementary to this study.[5]

The paper is also related to the literature on multiple equilibria and self-fulfilling fluctuations as the result of search frictions. Multiple equilibria in that literature arise mainly from increasing returns to scale in matching (Diamond (1982)) or from the interactions between several frictional markets (Howitt and McAfee (1988)).[6] In contrast, multiplicity in our model arises in a single market with constant returns to scale in matching. Other sources of multiplicity in models with search frictions include an indeterminacy in the division of the match surplus (Howitt and McAfee (1987), Farmer (2012), Kashiwagi (2014)) or the interaction between the outside option of matched market participants and their endogenous separation decisions (Burdett and Coles (1998), Coles and Wright (1998), Burdett, Imai, and Wright (2004), Moen and Rosén (2013), Eeckhout and Lindenlaub (2015)). In our paper, the division of the match surplus is determined by a fixed price, by Nash bargaining, or by competitive search, so that the indeterminacy of a bilateral monopoly is not exploited. Also, separation is exogenous in our framework.

Finally, our extension in Section 5.3 to a model of the housing market with competitive search relates the paper to recent models of competitive search in housing and asset markets (Diaz and Jerez (2013), Lester, Visschers, and Wolthoff (2013), Lester, Rocheteau, and Weill (2015), Albrecht, Gautier, and Vroman (2016)).

2 Motivating Facts

We provide some motivating facts about the transaction sequence decisions of owner-occupiers for Copenhagen, Denmark. We combine information from the Danish ownership register with a record of property sales for each year. The

[5] Maury and Tripier (2014) study a modification of the Wheaton (1990) model, in which mismatched owners can buy and sell simultaneously. However, they do not consider the feedback from buying and selling decisions on the stock-flow process and on market tightness. This feedback is key for the mechanisms we explore in our paper.

[6] Similar papers include Drazen (1988), Diamond and Fudenberg (1989), Mortensen (1989), Howitt and McAfee (1992), Boldrin, Kiyotaki, and Wright (1993), Mortensen (1999), Kaplan and Menzio (2013), Chéron and Decreuse (2014), and Sniekers (2014), among others.

unique owner and property identifiers give us a matched property-owner data set, which we use to keep track of the transactions of individuals over time.

We use the ownership records of individual owners over time to identify owner-occupiers who buy and sell in Copenhagen.[7] We then use the property sales record to determine the agreement dates (the dates the sale agreement is signed) and closing dates (the dates the property formally changes ownership) for the two transactions. We use those to measure the time difference between the sale of the old property and purchase of the new property. If the difference is positive, the owner-occupier buys first, if it is negative – she sells first.

Figure 1 shows the distribution of the difference between the agreement dates (Panel 1a) and closing dates (Panel 1b) for owner-occupiers who buy and sell in Copenhagen during our sample period. There is substantial dispersion in the difference between agreement dates, which suggests that a large fraction of moving owners cannot synchronize the two transactions on the same date. Examining the difference in closing dates shows a similar pattern. Even though the distribution is more compressed in that case, since homeowners try to a greater extent to synchronize the closing dates, a large fraction of homeowners face a time difference of a month or more in between closing the two transactions. Overall, these distributions suggest that for moving owner-occupiers the time difference between transactions can be substantial, confirming the anecdotal evidence cited in the introduction.

[Figure 1 here]

Another important observation is that the two distributions are right-skewed, so moving homeowners tend to buy first during our sample period. This is confirmed when we examine the time series behavior of the fraction of homeowners that are identified as buying first in a given year, as Figure 2 shows. As the figure shows, the fraction of owners that buy first is not constant over time but exhibits wide variations going from a low of around 0.3 in 1994

[7]The Appendix contains detailed information on the data used and on the procedure for identifying owner-occupiers that buy and sell. Given the way we identify these owner-occupiers, we have a consistent count for the number of owners who buy first or sell first in a given year for the years 1993 to 2008.

to a high of 0.8 in 2006 and then back to a low of around 0.4 in 2008. This series also closely tracks house prices, which suggests that the decisions to buy first may be related to the state of the housing market.

[Figure 2 here]

A closer examination of the period 2004-2008 strengthens this conjecture. Specifically, Figure 3 illustrates the fluctuations in key housing market variables like the for-sale stock, seller time-on-market, transaction volume and prices for Copenhagen in the period 2004-2008. It also includes our constructed fraction of buy first owners for Copenhagen in the period 2004-2008. During the first half of this period seller time-on-market (TOM), and the for-sale stock are low, while transaction volume and the fraction of buy first owners are high. There is a switch in all of these series around the 3rd quarter of 2006 and a quick reversal during which seller time-on-market and the for-sale stock increase rapidly, while the fraction of moving owners that buy first drops. Transaction volume is also lower during the second half of this period. Prices increase during the first half of the period and then decline.

[Figure 3 here]

We interpret the three exhibits as showing that there is a non-trivial transaction sequence choice for owner-occupiers, that the time difference between the two transactions can be substantial, and that the decision to buy first or sell first is related to the state of housing markets. These facts motivate our theoretical study below.

2.1 Why an explicit transaction sequence choice?

Before proceeding with our model, we make the following important conceptual point. Suppose that rather than explicitly choosing how to conduct the sequence of transactions, moving owners always enter both sides of the market simultaneously, and simply take whichever trading opportunity comes first.

Thus, they are *observed* to "buy first" whenever they happen to meet a seller before a buyer and vice versa. We call this a simultaneous search strategy.

Suppose that the market is frictional and there are trading delays. With standard assumptions on the matching technology, the rate at which a buyer meets a seller is decreasing in the buyer-seller ratio in the market, while the rate at which a seller meets a buyer is increasing in the buyer-seller ratio. Hence, a simultaneous search strategy implies that the observed fraction of agents that buy first is decreasing in the buyer-seller ratio. The (steady state) average time-on-market is the inverse of the meeting rate of sellers, and hence, is also decreasing in the buyer-seller ratio. It follows that if the agents use a simultaneous search strategy, the fraction of owners that are observed to buy first and the seller time-on-market should move in tandem – one should observe fewer owners "buying first" whenever seller time-on-market is low and vice versa. However, this is counterfactual, in view of Figure 3.[8]

This simple example shows (without reference to the optimization decision of agents) that to be consistent with the data, mismatched owners must explicitly choose to (predominantly) search only on one side of the market, thus steering the sequence of their transactions, rather than to search simultaneously on both sides and having the sequence of their transactions be determined by the exogenous arrival of trading counterparties. Moreover, as we show in our model, when agents' optimizing decisions are taken into account, under a naturally satisfied parametric assumption on preferences, mismatched owners rationally choose to bias their search towards one side of the market in a way that leads to aggregate behavior that is consistent with the data.

[8]To show this formally, below we denote the buyer-seller ratio by θ, the rate at which sellers meet buyers by $\mu(\theta)$, where $\mu(\theta)$ is increasing in θ, and the rate at which a buyer meets a seller by $q(\theta) = \frac{\mu(\theta)}{\theta}$, where $q(\theta)$ is decreasing in θ. If owners follow a simultaneous search strategy, in steady state, the ratio of owners observed to buy first relative to those observed to sell first is

$$\frac{q(\theta)}{\mu(\theta)} = \frac{1}{\theta}. \tag{1}$$

Therefore, this ratio is decreasing in θ. Since θ and sellers' (average) time-on-market, $\frac{1}{\mu(\theta)}$, are negatively related, the fraction of owners observed to buy first and seller time-on-market should move in tandem as θ changes.

3 Model

In this section, we set up the basic model of a housing market characterized by trading frictions and re-trading shocks that will provide the main insights of our analysis.

Preferences. Time is continuous. The housing market consists of a unit measure of durable housing units that do not depreciate, and a unit measure of households, which we refer to as agents. The agents are risk neutral and can borrow and lend freely at interest rate $r > 0$. When an agent buys a house and becomes a homeowner, he receives a flow utility of $u > 0$. We say that the homeowner is *matched*. With a Poisson rate γ the matched homeowner is hit by a taste shock, and becomes *mismatched* with his current housing unit. In that case the homeowner obtains a flow utility of $u - \chi$, for $0 < \chi < u$. A mismatched owner has to move to another house to become matched again.

A mismatched owner can choose to *sell first* (and become a *mismatched seller*) – selling the housing unit he owns first and then buying a new one. Alternatively, he can choose to *buy first* (becoming a *mismatched buyer*) – buying a new housing unit first and then selling his old one.[9] We also assume that a mismatched owner cannot synchronize the two transactions (the selling and buying). For example, he cannot exchange houses with another mismatched owner, as there is no double coincidence of housing wants among owners.[10] Instead, he has to conduct the two transactions in a sequence. A mismatched

[9]The mismatched owner can also choose to not enter the housing market and stay mismatched. However, that action is not taken in the equilibria we will consider. Also, see our discussion in Section 2.1 for why an explicit choice of buying first versus selling first is necessary for explaining the patterns we observe in the data. In Section 5.4, we explicitly allow a mismatched owner to search as a buyer and seller simultaneously, subject to a fixed time endowment, and show that restriction to either only buying first or selling first is without loss of generality in this case.

[10]This is similar to the lack of double coincidence of wants used in money-search models (Kiyotaki and Wright (1993)). In reality, some moving homeowners may be able to synchronize the two transactions as is also evident from Figure 1. Allowing some mismatched owners to synchronize their buy and sell transactions would reduce the tractability of the model without changing its qualitative predictions. Notice that allowing for simultaneous search as both a buyer and seller does not mean that the agent can synchronize the two transactions, only that he chooses to receive offers from both potential buyers and sellers.

buyer ends up holding two housing units simultaneously for some period. In this case we say that he becomes a *double owner*. Similarly, a mismatched seller ends up owning no housing. In that case he becomes a *forced renter*.

The utility flows during the transaction period (when the agent is a double owner or a forced renter) are key for our results. We assume that a double owner receives a flow utility of $u_2 < u$, while a forced renter receives a flow utility of $u_0 < u$. These flows do not include the cost of renting a house for a forced renter, or rental income from renting out the second house for the double owner (as will be clear below, we assume that the double owner rents out the second housing unit). For the double owner, u_2 includes maintenance costs, costs (pecuniary and non-pecuniary) associated with renting out the second house, reflecting unmodeled frictions in the rental market, costs associated with bridge loans (over and above the interest rate), reflecting unmodeled frictions in the financial market, etc. For the forced renter, u_0 includes relocation costs to a new temporary quarter, inconveniences and monetary costs associated with short-term renting, such as, for example, rent-back agreements, where a former owner rents his old house at a premium from the new owner, and other unmodeled frictions in the rental market, costs of storage of furniture, etc. As will be clear below, a driving assumption in our analysis is that u_2 and u_0 are less than $u - \chi$, i.e., the flow utilities to an agent during the transaction period are lower than when living as mismatched in his own house. For tractability, we also assume that a double owner does not experience mismatching shocks. This ensures that an agent will not hold more than two housing units in equilibrium.

Agents are born (enter) and die (exit) at the same rate g. New entrants start out their life without owning housing, and receive a flow utility $u_n < u$. Also, we assume that $u_n \geq u_0$, so that forced renters do not obtain a higher utility flow than new entrants. After a death/exit shock, an agent exits the economy immediately and obtains a reservation utility normalized to 0. If he owns housing, his housing units are taken over by a real-estate firm, which immediately places them for sale on the market.[11] Real-estate firms are owned

[11]For simplicity, we assume that exiting agents are not compensated for their housing. In Section 5.4, we discuss the case where real-estate firms compensate them upon exit.

by all the agents in the economy with new entrants receiving the ownership shares of exiting agents. Given the exit shock, agents effectively discount future flow payoffs at a rate $\rho \equiv r + g$. We will directly use ρ later on.

Finally, agents without a house rent a dwelling. A landlord can simultaneously rent out a unit and have it up for sale. Hence, double owners rent out one of their units, as do real-estate firms. The rental price is denoted by R.

Trading frictions and aggregate consistency. The housing market is subject to trading frictions. These frictions are captured by a standard constant returns to scale matching function $m\left(B\left(t\right), S\left(t\right)\right)$, mapping a stock $B\left(t\right)$ of searching buyers and a stock $S\left(t\right)$ of searching sellers to a flow m of new matches. We assume that there is random matching, so different types of agents meet with probabilities that are proportional to their mass in the population of sellers or buyers.[12] We define the market tightness in the housing market as the buyer-seller ratio, $\theta\left(t\right) \equiv \frac{B(t)}{S(t)}$. Additionally, $\mu\left(\theta\left(t\right)\right) \equiv m\left(\frac{B(t)}{S(t)}, 1\right) = \frac{m(B(t),S(t))}{S(t)}$ is defined as the Poisson rate with which a seller meets a buyer. Similarly, $q\left(\theta\left(t\right)\right) \equiv \frac{m(B(t),S(t))}{B(t)} = \frac{\mu(\theta(t))}{\theta(t)}$ is the rate with which a buyer meets a seller.

Beside the market tightness, $\theta\left(t\right)$, which will be relevant for agents' equilibrium payoffs, we keep track of a number of stock variables. Specifically, those include the new entrants (denoted by $B_n\left(t\right)$), matched owners ($O\left(t\right)$), mismatched buyers ($B_1\left(t\right)$), mismatched sellers ($S_1\left(t\right)$), double owners ($S_2\left(t\right)$), forced renters ($B_0\left(t\right)$) and the housing units sold by real-estate firms ($A\left(t\right)$). Therefore, the total measure of buyers is $B\left(t\right) = B_n\left(t\right) + B_0\left(t\right) + B_1\left(t\right)$ and the total measure of sellers is $S\left(t\right) = S_1\left(t\right) + S_2\left(t\right) + A\left(t\right)$. Since the total population is constant and equal to 1 in every instant, it follows that

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1. \tag{2}$$

Also, since the housing stock does not shrink or expand over time, the following

[12] Directed search is discussed in Section 5.3.

housing ownership condition holds in every instant,

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{3}$$

House price and rental price determination. We begin our analysis by assuming that the house price p is fixed and does not vary with the market tightness θ (or that the rental price is equal to the house price times the discount rate ρ – see the end of this subsection). However, in the equilibria we consider, the price p lies in the bargaining set of all actively trading pairs. We progressively relax this assumption by assuming that p varies with θ in a reduced form-way in Section 5.1 and by assuming that prices are determined by symmetric Nash bargaining in Section 5.2 or in a competitive search equilibrium in Section 5.3. The main insights of our analysis hold in those environments as well, although at a significant reduction in tractability.[13]

In this paper, we do not explicitly model the rental market. Since there are equally many houses as there are agents in the economy, and all houses are either occupied by the owner or rented out, the supply of houses for rent is equal to the demand for houses for rent independently of the price (as long as all agents prefer to own rather than to rent) and independently of the transaction sequence of the agents. Thus, if the rental market is competitive, the rental price is indeterminate. In what follows, we assume that the rental price is constant, independently of θ. Furthermore, given the assumption that

[13]Although the assumption that house prices are independent of θ is made for convenience, it may also be an equilibrium outcome in some environments. Given that the price is assumed to lie in the bargaining sets of all trading pairs, it can be derived as the market clearing price in a competitive market with frictional entry of traders. In particular, as in Duffie, Garleanu, and Pedersen (2005) or Rocheteau and Wright (2005), the total measure of participants in that competitive market is determined by the matching function $M(B, S)$. The transaction price in our case will be indeterminate, and this opens up for a price that is independent of θ. Also, under certain conditions, a unique fixed price that does not vary with tightness or across trading pairs can be microfounded as resulting from bargaining between heterogeneous buyers and sellers, in which the buyer has full bargaining power but does not know the type of the seller. As shown in Appendix F, take-it-or-leave-it offers from buyers under private information about the seller's type can generate a fixed price that is equal to the present discounted value of rental income.

a real-estate firm can rent out a housing unit without costs, we require that

$$\rho p \geq R, \tag{4}$$

as otherwise the real estate firms would want hold on to their houses forever.

An interesting special case arises when $R = p\rho$. As we show below, in this case, the (steady state) house price does not influence the sequence of transactions. A higher house price makes it more attractive to sell first due to discounting, while a higher rental price makes it more attractive to buy first. If $R = \rho p$, the two effects cancel out.

4 Steady State Equilibria

We start by characterizing steady state equilibria of this economy.

4.1 Value functions

We use the notation V^x, for the value function of a new entrant ($x = Bn$), a forced renter ($x = B0$), a mismatched buyer or seller ($x = B1$ or $x = S1$), a double owner ($x = S2$) and real-estate firm holding one housing unit ($x = A$). Finally, we denote the value function of a matched owner by V. Given these notations, we have a standard set of Bellman equations for the agents' value functions in a steady state equilibrium.

First of all, for a mismatched buyer we have

$$\rho V^{B1} = u - \chi + q(\theta) \max\left\{-p + V^{S2} - V^{B1}, 0\right\}, \tag{5}$$

where $u - \chi$ is the flow utility from being mismatched. Upon matching with a seller, a mismatched buyer purchases a housing unit at price p, in which case he becomes a double owner, incurring a utility change of $V^{S2} - V^{B1}$.

A double owner has a flow utility of $u_2 + R$ while searching for a counterparty. Upon finding a buyer, he sells his second unit and becomes a matched owner. Therefore, his value function satisfies the equation[14]

$$\rho V^{S2} = u_2 + R + \mu(\theta)\left(p + V - V^{S2}\right). \tag{6}$$

The value function of a mismatched seller is analogous to that of a mismatched buyer apart from the fact that a mismatched seller enters on the seller side of the market first and upon transacting becomes a forced renter. Therefore,

$$\rho V^{S1} = u - \chi + \mu(\theta)\max\left\{p + V^{B0} - V^{S1}, 0\right\}. \tag{7}$$

Finally, for a forced renter we have

$$\rho V^{B0} = u_0 - R + q(\theta)\left(-p + V - V^{B0}\right). \tag{8}$$

The other value functions are straightforward and are given in the Appendix.

4.2 Optimal choice of mismatched owners

In a steady state equilibrium, the optimal decision of mismatched owners depends on the simple comparison

$$V^{B1} \gtreqless V^{S1}. \tag{9}$$

We can substitute for V^{B0} and V^{S2} from equations (8) and (6) into the value functions for a mismatched buyer and seller to obtain

$$V^{B1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+q(\theta)} + \frac{q(\theta)\left(u_2 - (\rho p - R)\right)}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V\right\}, \tag{10}$$

[14] We present the value functions of double owners and forced renters assuming that they always trade at the price p, since that will always be the case in the steady state equilibria we consider. For example, for the case of a double owner we have $V + p \geq \frac{u_2+R}{\rho}$. The Appendix provides a set of sufficient conditions for this to hold.

and

$$V^{S1} = \max\left\{\frac{u-\chi}{\rho}, \frac{u-\chi}{\rho+\mu(\theta)} + \frac{\mu(\theta)(u_0 + (\rho p - R))}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V\right\}. \tag{11}$$

We define the effective utility flow for a forced renter as $\tilde{u}_0 \equiv u_0 + \Delta$, and for a double owner as $\tilde{u}_2 \equiv u_2 - \Delta$, where

$$\Delta \equiv \rho p - R. \tag{12}$$

In the special case, in which $R = \rho p$, $\tilde{u}_0 = u_0$ and $\tilde{u}_2 = u_2$. Hence in that case, the housing price does not influence the flow value (including incomes/expenses from renting) of double owners or forced renters.

We focus on the empirically relevant and realistic case, in which being mismatch gives a higher flow value than being double owner or forced renter:

Assumption A1: $u - \chi \geq \max\{\tilde{u}_0, \tilde{u}_2\}$**.**

Anecdotal evidence points to the mismatch state as not particularly costly for the majority of homeowners. As Ngai and Sheedy (2015) argue, mismatch may be so small for many homeowners that they prefer not to move and save on the transaction costs. Also, a comparison between the difference in agreement and closing dates from Figure 1 shows that moving homeowners tend to minimize the delay between the closing of the two transactions. Many transactions either occur simultaneously or within a short period. This suggests that delays between transactions are particularly costly for moving homeowners.

In the special case with $R = \rho p$, Assumption A1 can be written as $u - \chi > \max\{u_0, u_2\}$. If, in addition, $u_0 = u_2 = c$, this boils down to $u - \chi \geq c$.

Define $D(\theta) \equiv V^{B1} - V^{S1}$ as the difference in value between buying first and selling first. Assuming that it is optimal for both a mismatched buyer and mismatched seller to transact, we have

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))}\left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2) - \tilde{u}_0 + \tilde{u}_2\right]. \quad (13)$$

In the case where $\tilde{u}_0 = \tilde{u}_2 = c$, equation (13) simplifies to

$$D(\theta) = \frac{(\mu(\theta) - q(\theta))(u - \chi - c)}{(\rho + q(\theta))(\rho + \mu(\theta))}. \quad (14)$$

In this simple case, buying first is preferred whenever $\mu(\theta) > q(\theta)$. The (expected) time-on-market for a buyer and a seller are $\frac{1}{q(\theta)}$ and $\frac{1}{\mu(\theta)}$, respectively. Therefore, buying first is preferred, if and only if, time-on-market is higher for a buyer than for a seller. Intuitively, a mismatched owner has to complete two transactions on both sides of the market. Since it is more costly to be a double owner or a forced renter than to be mismatched, a mismatched owner cares more about the expected time to conduct the second transaction, and hence, wants to minimize the delay between the two transactions. A low seller time-on-market thus favors buying first.

We now formally characterize the optimal action of a mismatched owner given a steady state market tightness θ. We adopt the notation $\theta = \infty$ for the case where the buyer-seller ratio is unbounded. We define

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2}{u - \chi - \tilde{u}_0}. \quad (15)$$

Note that if $\tilde{u}_2 = \tilde{u}_0$, then $\tilde{\theta} = 1$, while if $\tilde{u}_2 > \tilde{u}_0$, then $\tilde{\theta} < 1$, and vice versa if $\tilde{u}_2 < \tilde{u}_0$.[15] The following lemma fully characterizes the incentives of mismatched owners to buy first or sell first given a steady state market tightness θ.

Lemma 1. *Let $\tilde{\theta}$ be as defined in (15). Then for $\theta \in (0, \infty)$, $\theta > \tilde{\theta} \iff V^{B1} > V^{S1}$ and $\theta = \tilde{\theta} \iff V^{B1} = V^{S1}$.*

[15] In what follows, we will additionally assume that at $\theta = \tilde{\theta}$, both $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so that a mismatched owner is strictly better off from transacting at $\theta = \tilde{\theta}$. This removes uninteresting steady state equilibria in which mismatched owners never transact. Assumption A2 in the Appendix gives a sufficient condition for this.

Proof. See Appendix C. □

Lemma 1 shows that, in general, as θ increases, the incentives to buy first are strengthened. For sufficiently high (low) values of θ, buying (selling) first dominates selling (buying) first.

4.3 Steady state flows and stocks

We turn next to a description of the steady state equilibrium stocks and flows of this model. The full set of equations for these flows are included in the Appendix. Here we just make some important observations on the stock-flow process. First, combining the population and housing ownership conditions (2) and (3) we get that

$$B_n(t) + B_0(t) = A(t) + S_2(t). \tag{16}$$

Since there are equally many agents and houses, the stocks of agents without a house (forced renters and new entrants) must be equal to the stock of double owners and real-estate firms. This identity implies that in a candidate steady state equilibrium where all mismatched owners buy first (so that there are no forced renters), the market tightness, denoted by $\overline{\theta}$ satisfies

$$\overline{\theta} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n} > 1. \tag{17}$$

Similarly, if $\underline{\theta}$ denotes the market tightness in a candidate steady state where all mismatched owners sell first (so that there are no double owners), we have

$$\underline{\theta} = \frac{B_n + B_0}{A + S_1} = \frac{A}{A + S_1} < 1. \tag{18}$$

Therefore, $\underline{\theta} < 1 < \overline{\theta}$. This points to possibly wide variations in market tightness arising from changes in the behavior of mismatched owners. Lemma 2 characterizes the steady state market tightnesses, $\overline{\theta}$ and $\underline{\theta}$.

Lemma 2. *Let $\overline{\theta}$ and $\underline{\theta}$ denote the steady-state market tightness when all mismatched owners buy first and sell first, respectively. Then $\overline{\theta}$ and $\underline{\theta}$ are unique. Moreover, $\overline{\theta} > 1$, $\underline{\theta} < 1$, and $\overline{\theta}$ is increasing and $\underline{\theta}$ is decreasing in γ.*

Proof. See Appendix C. □

It is illustrative to consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. One can show that (see proof of Lemma 2),

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \overline{\theta} = 1 + \kappa, \tag{19}$$

and

$$\lim_{\gamma \to 0, g \to 0, \frac{\gamma}{g} = \kappa} \underline{\theta} = \frac{1}{1 + \kappa}. \tag{20}$$

Thus, the more important mismatched owners are in housing transactions (the higher is $\kappa = \gamma/g$), the larger the variation in market tightnesses from changes in mismatched owners' actions.

4.4 Equilibrium characterization

We now combine the observations on the optimal choice of mismatched owners and the steady state stocks from the previous two sections to characterize equilibria of our model.

Proposition 1. *Consider the above economy. Let $\tilde{\theta}$ be defined by condition (15), and $\overline{\theta}$ and $\underline{\theta}$ be defined by (17) and (18), with $\overline{\theta}$, $\underline{\theta} \in (0, \infty)$.*

1. *If $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, the model exhibits multiple steady state equilibria: an equilibrium with $\theta = \overline{\theta}$, in which mismatched owners buy first (a "Buy first" equilibrium); an equilibrium with $\theta = \underline{\theta}$, in which mismatched owners sell first (a "Sell first" equilibrium); and an equilibrium with $\theta = \tilde{\theta}$, in which the mismatched owners randomize between buying and selling.*

2. *If $\tilde{\theta} < \underline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners buy first.*

3. *If $\tilde{\theta} > \overline{\theta}$, there exists a unique steady state equilibrium in which all mismatched owners sell first.*

Proof. See Appendix C. □

Therefore, depending on the flow payoffs $\tilde{u}_0$ and $\tilde{u}_2$, there can exist multiple equilibria or a unique equilibrium. Note that the mixing equilibrium is unstable, in the sense that a small deviation from $\tilde{\theta}$ implies that all mismatched owners want to buy first if the deviation is upwards, and to sell first if the deviation is downwards.

In the special case, with $\tilde{u}_0 = \tilde{u}_2 = c$, multiple equilibria always exists:

Corollary 1. *Consider the above economy and suppose that $\tilde{u}_0 = \tilde{u}_2 = c$. Then there exist three steady state equilibria: one with $\theta = \underline{\theta}$, in which mismatched owners sell first; one with $\theta = \overline{\theta}$, in which mismatched owners buy first; and another with $\theta = 1$, in which mismatched owners are indifferent between buying first and selling first, and half of them buy first.*

Proof. See Appendix C. □

Intuitively, the equilibrium multiplicity arises because the feedback from the transaction sequence decisions of mismatched owners to the steady state equilibrium market tightness creates a form of *strategic complementarity* in their actions. When mismatched owners are buying first, the steady state buyer-seller ratio is high, so that it is individually rational for any mismatched owner to buy first. Conversely, when mismatched owners are selling first, the steady state buyer-seller ratio is low, and it is individually rational to sell first.

Since $\tilde{\theta}$ depends on flow payoffs of mismatched owners, shocks to these payoffs can lead to equilibrium switches.[16] Apart from payoff shocks, equilibrium switches may also occur because of changes in agents' beliefs. Next, we

[16]As an example of such a payoff shock, suppose that the payoff of a double owner, u_2, includes costs associated with a mortgage that finances the downpayment on his new property prior to the sale of his old. When financial markets function normally, these costs are relatively low. Suppose that in that case $\tilde{u}_2 > \tilde{u}_0$ and $\tilde{\theta} < \underline{\theta}$, and the "Sell first" equilibrium does not exist. Now suppose that there is a shock to financial markets, so that a bridging mortgage becomes very costly, and thus $\tilde{u}_2 < \tilde{u}_0$ and $\tilde{\theta} > \overline{\theta}$. As a result, after the shock, buying first is no longer optimal and the "Buy first" equilibrium no longer exists.

discuss the implications of such equilibrium switches for transaction volume, time-on-market, and the stock of houses for sale.

4.5 Equilibrium switches

To simplify the analysis, we consider the limit economy introduced in Section 4.3, where $g \to 0$ and $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, $\overline{\theta} = 1 + \kappa$, and $\underline{\theta} = \frac{1}{1+\kappa} = \frac{1}{\overline{\theta}}$. Suppose that the economy starts in a "Buy first" equilibrium. In that case

$$\theta = \overline{\theta} = \frac{\overline{B}}{\overline{S}} = \frac{B_n + B_1}{A + S_2} = \frac{B_n + B_1}{B_n}, \tag{21}$$

where $\overline{B}$ and $\overline{S}$ denote the stocks of buyers and sellers in the "Buy first" equilibrium. Suppose that the whole stock of mismatched owners, B_1, decide to sell first rather than buy first, and so, moves to the seller side of the market. In that case, the new market tightness becomes

$$\theta' = \frac{B'}{S'} = \frac{B_n}{B_n + B_1} = \underline{\theta},$$

where B' and S' denote the stocks of buyers and sellers immediately after the switch. Hence, the tightness jumps directly to its new steady state equilibrium value with no dynamic adjustment in θ.

What are the implications of this switch? First of all, clearly average time-on-market for sellers, $\frac{1}{\mu(\theta)}$, increases. Second, consider the ratio of the stock of sellers before and after the switch. That ratio is exactly $\underline{\theta}$, which is less than 1. Therefore, there is an increase in the for-sale stock, since some of the previous buyers become sellers. Finally, transaction volume may also fall depending on the shape of the matching function. Specifically, suppose that we have a Cobb-Douglas matching function, so $m(B, S) = \mu_0 B^{\alpha} S^{1-\alpha}$, for $0 < \alpha < 1$, and consider the ratio of transaction volumes before and after the switch

$$\frac{\mu\left(\overline{\theta}\right)}{q\left(\underline{\theta}\right)} = \frac{\mu_0 \overline{\theta}^{\alpha}}{\mu_0 \underline{\theta}^{\alpha-1}} = (1 + \kappa)^{2\alpha - 1}.$$

Hence, transaction volume falls after the equilibrium switch if $\alpha > \frac{1}{2}$ and increases if $\alpha < \frac{1}{2}$. The reason is that for $\alpha > \frac{1}{2}$ buyers are more important than sellers in generating transactions. When mismatched owners switch from buying first to selling first, this leads to a reduction in the number of buyers and an increase in the number of sellers, and hence, to a fall in the transaction rate. Genesove and Han (2012) estimate a value of $\alpha = 0.84$. At that value, transaction volume would drop after the switch.

Consequently, a switch from a "Buy first" to a "Sell first" equilibrium implies behavior for key housing market variables – the for-sale stock, average time-to-sell, and transaction volume – that is broadly consistent with evidence on housing cycles (Diaz and Jerez (2013), Guren (2014)). This behavior is also consistent with the evidence on the housing cycle in Copenhagen as shown in Figure 3. The negative comovement between the for-sale stock and transactions is of particular interest, since search-based models of the housing market usually have trouble generating this comovement (Diaz and Jerez, 2013).

4.6 Quantitative relevance

In this section we provide a simple numerical example to assess the quantitative relevance of our mechanism. Specifically, we examine the switch from the "Buy first" to the "Sell first" equilibrium in the limit economy.

We use the calibration of Head, Lloyd-Ellis, and Sun (2014) to determine plausible values for the rate of mismatch, γ, and the entry and exit rate, g. We use the annual fraction of owners that move across U.S. counties, which according to the Census Bureau is around 3.2 percent, to determine g. The fraction of owners that stay in the same county conditional on moving is 60 percent, or around 1.5 times more owners move within the same county than across counties. Therefore, we set $g = 0.035$ and $\gamma = 0.0525$. This implies an average duration for homeowners of around 11.4 years and an annual turnover rate of 8%. These flows give us values of the market tightness in the limit economy of $\lim \overline{\theta} = 2.5$ and $\lim \underline{\theta} = 0.4$, respectively. Finally, we assume that the matching function is Cobb-Douglas so that $\mu(\theta) = \mu_0 \theta^{\alpha}$. We choose

$\alpha = 0.84$, following Genesove and Han (2012).[17]

With these numbers, switching from the "Buy first" to the "Sell first" equilibrium in the limit economy leads to a decrease in the market tightness by around 85%. This is associated with a 150% increase in the for-sale stock, and a 350% increase in average time-to-sell. In addition, the transaction rate falls by around 50% immediately after the switch.

5 Endogenous prices

In this section we analyze equilibrium multiplicity without the fixed price assumption of the previous section.

5.1 Prices depend on market tightness

When deriving conditions for multiplicity, we assumed that prices were exogenous, and hence unaltered by the transaction sequence decision of the agents. Now, we allow steady state housing prices to depend on θ. We thus write $p = p(\theta)$, with $p'(\theta) > 0$. We may also write the rent R as a function of θ, $R = R(\theta)$, with $R'(\theta) \geq 0$. Recall that the flow values of double owners and forced renters depend on $\Delta = \rho p - R$. Thus, we can write $\Delta = \Delta(\theta)$. We assume that $\Delta'(\theta) \geq 0$ (otherwise, prices responding to θ do not create any countervailing effects).

As we already pointed out in Section 4.1, in the case with $R = \rho p$, $\Delta'(\theta) = 0$, and so, assuming that house prices are exogenous is without loss of generality, since different (steady state) prices do not influence the flow value of double owners or forced renters. However, a countervailing effect arises if $\Delta'(\theta) > 0$. Specifically, from Equation (13), the decision whether to buy or to

[17] The small values of γ and g suggest that the focus on the limit economy in Section 4.5 is reasonable. See the working paper version of this paper for a comparison of tightnesses in the limit economy and away from the limit. Also note that the value of μ_0 is irrelevant for the numerical results in this section. In Section 5.2, where we discuss the quantitative implications for house prices, we use a value of $\mu_0 = 4$, which gives a seller time-on-market of 6 weeks in a "Buy first" equilibrium.

sell first depends on the sign of the following expression:

$$\tilde{D}(\theta) = \frac{\theta - 1}{\theta} (u - \chi - u_2 + \Delta(\theta)) + u_2 - u_0 - 2\Delta(\theta).$$

We normalize $u_2 - u_0$ so that $\tilde{D}(1) = 0$, which requires that $u_2 - u_0 = 2\Delta(1)$.

In order for the "Buy first" and "Sell first" equilibria to exist, we must have that:

$$\frac{\overline{\theta} - 1}{\overline{\theta}} [u - \chi - u_2 + \Delta(\overline{\theta})] \geq 2(\Delta(\overline{\theta}) - \Delta(1)), \tag{22}$$

and

$$\frac{1 - \underline{\theta}}{\underline{\theta}} [u - \chi - u_2 + \Delta(\underline{\theta})] \geq 2(\Delta(1) - \Delta(\underline{\theta})). \tag{23}$$

In the two conditions above, the left-hand side broadly reflects the strategic complementarity effect identified in Section 4. A higher θ makes buying more time-consuming and selling less time-consuming, and since the agent's utility flow is higher when mismatched compared to being a double owner or a forced renter, this favors buying first.

The right-hand sides of (22) and (23) reflect how a higher value of θ changes the difference in the flow values of a forced renter compared to a double owner. Note that if $R \leq \rho p$, it is beneficial for the agent, everything else equal, to buy late and sell early. We refer to this as a *discounting effect*. A higher θ increases p, and unless R increases at the same rate, this strengthens the discounting effect and makes it more attractive to sell first. Due to our normalization, at $\theta = 1$, the discounting effect is exactly balanced by a difference in the utility flows, u_2 and u_0. However, for $\theta > 1$, the discounting effect becomes stronger, which favors selling first to buying first. Similarly, for $\theta < 1$, the discounting effect becomes weaker, which favors buying first to selling first.

Therefore, our multiple equilibrium result requires that the house price (or more generally, Δ) should not be too sensitive to changes in θ, so that the discounting effect is weaker than the strategic complementarity effect. Notice that the two conditions always hold if u_2 (and u_0, given the normalization) is sufficiently low. In that case, the strategic complementarity effect always dominates for values of θ that are consistent with a steady state equilibrium.

5.2 Prices determined by Nash bargaining

In this section we assume that prices are determined by symmetric Nash bargaining, so that buyers and sellers split the match surplus equally between them. Therefore, there is no longer a single transaction price, p, but prices depend on the types of trading counterparties. To simplify the stock-flow conditions, we again consider a limit economy with small flows, where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is constant, and where $\overline{\theta} = 1 + \kappa$ and $\underline{\theta} = \frac{1}{1+\kappa}$. Additionally, for analytical tractability, we consider parameter restrictions which ensure that all trading surpluses are positive in both a "Buy first" and "Sell first" candidate equilibrium, except for the surplus between mismatched buyers and sellers. We give the relevant parameter restrictions in the Appendix. Also, with Nash bargaining, "symmetry", in the sense that mismatched owners are indifferent between buying first and selling first if $\theta = 1$, holds if $u_n - u_0 = u - u_2$.[18] For tractability we assume this is the case in what follows.

Consider a "Buy first" steady state equilibrium candidate with a market tightness of $\theta = \overline{\theta} > 1$. In that candidate equilibrium, the sellers with positive measure are the double owners and real-estate firms, while the buyers with positive measure are the mismatched owners and new entrants. In the limit economy, the outflow rate of mismatched agents is equal to the outflow rate of new entrants, so $B_1/B_n = \gamma/g = \kappa$. Hence, the shares of new entrants and mismatched buyers in the pool of buyers are $1/\overline{\theta}$ and $1 - 1/\overline{\theta}$, respectively. Furthermore, in the limit, as there is no death, the shares of real-estate firms and double owners in the pool of sellers are also $1/\overline{\theta}$ and $1 - 1/\overline{\theta}$, respectively.

Given these shares and since buyers and sellers split the match surplus evenly, the value function of a mismatch buyer is (given $\rho \to r$ in the limit)

$$rV^{B1} = u - \chi + \frac{1}{2}q(\overline{\theta}) \left[\frac{1}{\overline{\theta}}\Sigma_{AB1} + (1 - \frac{1}{\overline{\theta}})\Sigma_{S2B1} \right],$$

[18]The reason is that the real-estate firms are different from the other agents, as they receive no utility from owning a house, and their gain from transacting is the price, which is a transfer, and hence, does not affect match surplus. As a result, the equilibrium allocation becomes asymmetric, and tilts towards the "Buy first" equilibrium even with $u_2 = u_0$.

where $\Sigma_{AB1} = V^{S2} - V^{B1} - V^{A}$ is the match surplus when a mismatched buyer meets a real-estate firm, and $\Sigma_{S2B1} = V - V^{B1}$ is the match surplus when a mismatched buyer meets a double owner.

Consider a mismatched owners who deviates (permanently) and sells first.[19] Since a meeting between a mismatched buyer and a mismatched seller is assumed to lead to negative surplus, the value function of a deviator is simply

$$rV^{S1} = u - \chi + \frac{1}{2}\mu\left(\overline{\theta}\right)\frac{1}{\overline{\theta}}\Sigma_{S1Bn},$$

and so, the difference between the value of buying first and selling first, $D(\overline{\theta}) = V^{B1} - V^{S1}$, can be written as

$$D(\overline{\theta}) = \frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r + \frac{1}{2}q\left(\overline{\theta}\right)}\left(\frac{u_n - u_0}{r + \frac{1}{2}q\left(\overline{\theta}\right)} - \frac{1}{\overline{\theta}}\frac{u - u_2}{r + \frac{1}{2}\mu\left(\overline{\theta}\right)}\right). \tag{24}$$

Given our assumptions on utility flows, $D\left(\overline{\theta} = 1\right) = 0$ for $\kappa = 0$. An increase in $\overline{\theta}$ (equivalently, an increase in κ) leads to a increase in $D\left(\overline{\theta}\right)$, since the expression in parenthesis increases. This increase comes from two effects. First, $\mu\left(\overline{\theta}\right)$ increases and $q\left(\overline{\theta}\right)$ decreases, so the second term in the parenthesis decreases (given that $u_2 < u - \chi < u$) and the first term increases (since then $u_n > u_0$). This effect is tightly linked to the strategic complementarity effect in our benchmark model in Section 4. Specifically, as before, an increase in $\overline{\theta}$ increases the value of buying first given a lower expected time-on-market for double owners, while it decreases the value of selling first, given a higher expected time-on-market for forced renters.

Second, the fraction of new entrants and real-estate firms, $1/\overline{\theta}$, decreases. Therefore, buyers are more likely to meet double owners and sellers are more likely to meet mismatched buyers. However, the trading surplus for a buyer is higher when matched with a double owner compared to a match with a real-estate firm. Similarly, the trading surplus is lower for a seller when matched with a mismatched buyer compared to a new entrant. This compositional

[19] Studying permanent deviations is without loss of generality, since a temporary deviation can dominate a permanent deviation iff no deviation dominates the permanent deviation.

effect on both sides of the market strengthens the incentives to buy first.[20, 21]

Consider a "Sell first" equilibrium candidate with a market tightness of $\theta = \underline{\theta} < 1$. In that candidate equilibrium, the sellers with positive measure are the mismatched owners and real-estate firms, while the buyers are the forced renters and new entrants. In the limit economy, the shares of forced renters and new entrants in the pool of buyers are $\underline{\theta}$ and $1 - \underline{\theta}$, respectively. These are also the respective shares of real-estate firms and mismatched owners.

In this equilibrium candidate, the gain from deviating to (permanently) buying first for a mismatched owner is $D(\underline{\theta}) = V^{B1} - V^{S1}$, which is given by

$$D\left(\underline{\theta}\right) = \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \left(\underline{\theta} \frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)} - \frac{u - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\right). \tag{25}$$

Given our assumptions on the utility flows, $D(\underline{\theta} = 1) = 0$ for $\kappa = 0$. Decreasing $\underline{\theta}$ (increasing κ) decreases D, and hence, makes it more attractive to sell first.

We conclude that the model with Nash bargaining exhibits multiple equilibria, as stated in the following proposition.

Proposition 2. *Consider the limit economy with prices determined by symmetric Nash bargaining. Suppose that conditions B1-B4 given in the Appendix hold. Then there exists a steady state equilibrium, in which all mismatched owners buy first and the equilibrium market tightness converges to* $\overline{\theta} = 1 + \kappa$. *Also, there exists a steady state equilibrium, in which all mismatched owners sell first and the equilibrium market tightness converges to* $\underline{\theta} = \frac{1}{1+\kappa}$.

Proof. See Appendix D. □

As already mentioned, we have to make restrictions on parameters (conditions B2-B4 in the Appendix), to ensure that our assumptions regarding the

[20] Albrecht, Anderson, Smith, and Vroman (2007) explore a similar compositional effect. However, entry of buyers and sellers is exogenous in their model and there is no transaction sequence decision.

[21] When trading between a mismatched buyer and seller is not profitable for $\overline{\theta}$ close to 1, the discounting effect arising from higher prices is dominated by both the strategic complementarity and compositional effects. Thus, $D\left(\overline{\theta}\right)$ in (24) unambiguously increases in $\overline{\theta}$. See Section 5.2.1 below for a numerical example where the discounting effect may dominate when $\overline{\theta}$ becomes sufficiently large.

matching sets are satisfied in equilibrium. These restrictions are only sufficient and the same matching sets may emerge for more general parameter values. Also, we conjecture that if these restrictions are not satisfied, and other matching sets emerge, these may also have multiple steady state equilibria with the structure described in Proposition 2.[22]

5.2.1 Numerical example

To gain some additional insight into when multiple equilibria are possible with endogenously determined prices, and to show that conditions B2-B4 are only sufficient for multiplicity, we consider a simple numerical example, in which conditions B2-B4 need not be satisfied.[23] Figure 4 plots the difference $D(\theta) = V^{B1} - V^{S1}$ in a candidate "Sell first" equilibrium for $\theta \leq 1$ and a candidate "Buy first" equilibrium for $\theta > 1$. The figure illustrates two important points.

First, multiple steady state equilibria can be sustained even if the steady state market tightness fluctuates substantially between the candidate "Buy first" and "Sell first" equilibria. Using the stock-flow calibration from Section 4.6 produces substantial variation in market tightness across the two steady state equilibria as illustrated by the red and blue vertical lines on the Figure. This in turn means that there can be substantial price effects from switches across the two steady state equilibria. Specifically, for this numerical example, average house prices decline by around 32% between the "Buy first" and the "Sell first" equilibrium. Our mechanism can thus also lead to quantitatively significant fluctuations in prices when those are endogenously determined.

Second, unlike the benchmark case with exogenously fixed prices, endogenously determined prices imply that for sufficiently low (sufficiently high) market tightness, buying first starts to dominate selling first (and vice versa). This reflects the discounting effect discussed in section 5.1, which can dominate the strategic complementarity effect, so that multiple equilibria cease to exist.

[22]Simulations confirm that multiple equilibria exist also for parameter values that do not satisfy conditions B1-B4.

[23]One can also relax the symmetry condition B1. Relaxing B1 changes the value of θ for which a mismatched owner is indifferent between buying first and selling first. Thus, it essentially shifts the curve in Figure 4 to the left or to the right.

Nevertheless, as the numerical example illustrates, with Nash bargaining, the price response must be very large for the discounting effect to start dominating. Specifically, in the example in Figure 4, average house prices must decline by more than 50% between a candidate "Buy first" and "Sell first" equilibria for the discounting effect to dominate.

[Figure 4 here]

5.3 Competitive search

In competitive search equilibrium, sellers post prices, and buyers direct their search towards the sellers they find most attractive, taking into account that better terms of trade mean a longer expected waiting time before trade occurs. The market splits up in submarkets, and the different agents choose which submarket to enter. As shown in Garibaldi, Moen, and Sommervoll (2016), the most patient buyers (who are most willing to trade off a short waiting time for a low price) will search for the most impatient sellers (who are most willing to trade off a low price for a short waiting time). Analogously, the least patient buyers search for the most patient sellers.

In the Appendix we derive sufficient conditions on parameter values for multiple competitive search equilibria to exist. The economic mechanisms play out in a slightly different way with competitive search than with random search and bargaining. With bargaining, the strategic complementarity works through the market tightness – if more agents buy first, this increases the buyer-seller ratio in the market, and makes it more attractive to buy first. In competitive search equilibrium, the strategic complementarities are more involved. Single agents trade off waiting time and terms of trade. In that environment, more mismatched buyers influence the buyer-seller ratio in the submarket(s) for mismatched buyers. This does not directly affect the mismatched sellers, as they search in different submarkets. However, the asset values of all agents change, thereby influencing the value of selling first and the speed at which mismatched sellers transact.

In the Appendix we characterize competitive search equilibria when the cost of being mismatched, χ, is low, and so is the flow utility of being a double owner, u_2. Also, as in 5.2 we assume that new entrants enjoy a strictly higher flow utility than forced renters: $u_n > u_0$.

Proposition 3. *Consider the economy with competitive search, and suppose that χ and u_2 are small and that $u_0 < u_n$. Then the economy exhibits multiple equilibria. In one equilibrium, all mismatched owners buy first. In another equilibrium, all mismatched owners sell first.*

Proof. See Appendix E. □

We have no reason to believe that the condition on χ is necessary to obtain multiple equilibria. However, without it the model becomes less tractable, as it is not clear from the outset what market constellations will then be realized.

5.4 Additional extensions

Our benchmark model assumes that a mismatched owner has to choose to enter the housing market either as a buyer or as a seller. In Appendix F, we examine the optimal behavior of a mismatched owner that can choose to be both a buyer and a seller at the same time. We show that a mismatched owner strictly prefers to either only enter as a buyer or as a seller for any $\theta \neq \tilde{\theta}$, where $\tilde{\theta}$ is defined as in Equation (15). Intuitively, since the decision to enter as both a buyer and seller depends ultimately on the value from entering as a buyer only and the value from entering as a seller only, whenever entering as a buyer only is dominated by entering as a seller only, then entering as both a buyer and seller is also dominated by entering as a seller only, and vice versa.

Additionally, we show that our main results continue to hold even assuming that homeowners are compensated for the value of their housing units when they exit the economy when g is sufficiently small. In that case a modified version of Equation (15) determines the critical value of $\tilde{\theta}$ below which a mismatched owner is better off selling first. In the limit, as $g \to 0$, that modified version converges to the value of $\tilde{\theta}$ from Equation (15).

6 House Price Fluctuations

In this section, we examine the implications of expected changes in the house price for the behavior of mismatched owners. We then construct dynamic equilibria with self-fulfilling fluctuations in prices and tightness. In the entire section we study our benchmark case, in which $R = \rho p$.

Consider a simple, exogenous process for the price p. With rate λ, the house price p changes to a permanent new level p_N.[24] We compare the utility from buying first relative to selling first for a mismatched owner before the price change. If the price change occurs between the two transactions, the mismatched owner will make a capital gain of $p^N - p$ if he buys first and a capital loss of the same amount if he sells first. If the shock happens before the first or after the second transaction, it will not influence the decision to buy first or sell first. To simplify the exposition we assume that $u_0 = u_2 = c$.[25]

The price risk associated with the transaction sequence decision creates asymmetry in the payoff from buying first or selling first. Specifically, at $\theta = 1$, the difference between the two value functions $D(\theta) = V^{B1} - V^{S1}$ is

$$D(1) = \frac{\mu(1)}{(\rho + q(1) + \lambda)(\rho + \mu(1) + \lambda)} 2\lambda (p_N - p). \tag{26}$$

An expected price decrease, leads to a higher value of V^{S1} relative to V^{B1}, even if matching rates for a buyer and a seller are the same. Consequently, $V^{S1} > V^{B1}$ even for some values of $\theta > 1$. If the expected price decrease (increase) is sufficiently large, so that $D(\overline{\theta}) < 0$ ($D(\underline{\theta}) > 0$), then selling (buying) first will dominate buying (selling) first for any value of θ that is consistent with equilibrium.

Proposition 4. *Consider the modified economy with an exogenous house price change. Then for every $\lambda > 0$ and $\theta \in [\underline{\theta}, \overline{\theta}]$, a mismatched owner prefers to*

[24] Since we assume that $p = \frac{R}{\rho}$, one can think of a permanent change in the equilibrium rental rate to R_N, which leads to a house price change to $p_N = \frac{R_N}{\rho}$.

[25] We assume that θ remains constant over time, so the only change occurs in the price p. Also, for this exercise, we implicitly assume that $\gamma \to 0$, so that V is independent of p.

sell first for sufficiently low values of p_N. Analogously, a mismatched owner prefers to buy first for sufficiently high values of p^N.

Proof. See Appendix C. □

So far we have shown that the model may have multiple steady state equilibria with different market tightnesses. Now we show that it also admits dynamic equilibria with self-fulfilling fluctuations in prices and tightness.

Suppose that $X(t) \in \{0,1\}$ follows a two-state Markov chain. $X(t)$ starts in $X(t) = 0$ and with Poisson rate λ transitions permanently to $X(t) = 1$. The realization of $X(t)$ plays the role of a sunspot variable. The price in state $X(t) = 1$ is given by a smooth function $p_1 = f(\theta_1)$ (as in section 5.1). The price in state 0 is implicitly given by a smooth function $p_0 = f(\theta_0, \lambda(p_1 - p_0))$, increasing in both arguments, and with $f(\theta, 0) \equiv f(\theta)$. We take these relationships as exogenous and reduced-form to illustrate the equilibrium consequences of the interaction of housing prices and market liquidity conditions with the transaction decisions of mismatched owners. Again we look at a limit economy as γ and g go to zero, keeping $\kappa = \gamma/g$ fixed.

We consider an equilibrium in which the economy starts out in a "Buy first" regime ($X(t) = 0$), in which 1) mismatched owners prefer to buy first and the market tightness is $\theta_0 = \overline{\theta}$, and 2) agents expect that with rate λ, the economy permanently switches to a "Sell first" regime with market tightness $\theta_1 = \underline{\theta}$. In that second regime, 1) mismatched owners strictly prefer to sell first, and 2) agents expect that the economy will remain in the "Sell first" regime forever. Since $\overline{\theta} > \underline{\theta}$, it follows that $p_0 > p_1$.[26] It is straightforward to show that as $\lambda \to 0$, the payoffs from buying first and selling first converge to the payoffs without regime switching. Hence, in the limit, buying first in state 0 is an equilibrium strategy if $\overline{\theta} > \tilde{\theta}$, while selling first is an equilibrium strategy in state 1 if $\underline{\theta} < \tilde{\theta}$, where $\tilde{\theta}$ is defined by Proposition 1. Hence, as summarized in

[26]Suppose $p_0 \leq p_1$. Then $p_0 = f(\bar{\theta}, \lambda(p_1 - p_0)) \geq f(\bar{\theta})$. But then $p_0 \geq f(\bar{\theta}) > f(\underline{\theta}) = p_1$, a contradiction.

the next proposition, the regime-switching equilibrium exists whenever

$$1 + \kappa > \frac{u - \chi - u_2}{u - \chi - u_0} > \frac{1}{1 + \kappa}. \tag{27}$$

Proposition 5. *Consider the limit economy with $g \to 0$, $\gamma \to 0$ and $\frac{\gamma}{g} = \kappa$, and with the sunspot process described above. Suppose further that equation (27) is satisfied. Then there is a $\overline{\lambda}$, such that for $\lambda < \overline{\lambda}$, there exists a dynamic equilibrium characterized by two regimes $x \in \{0, 1\}$. In the first regime, $\theta_0 = \overline{\theta}$ and mismatched owners buy first. In the second regime, $\theta_1 = \underline{\theta}$, $p_1 < p_0$, and mismatched owners sell first. The economy starts in regime 0 and transitions to regime 1 with rate λ.*

Proof. Follows from the discussion above. □

Intuitively, if agents expect the change in regimes to occur sufficiently soon (λ is high), then in light of Proposition 4, it can be optimal for mismatched owners to sell first in the first regime despite the high market tightness, speculating on regimes changing in between their two transactions. This, however, is inconsistent with equilibrium. Therefore, a regime-switching equilibrium exists only for (sufficiently) low values of λ.

7 Concluding Comments

The transaction sequence decision of moving owner-occupiers depends on housing market conditions, such as the expected time-on-market for buyers and sellers and expectations about future house price appreciation. However, these decisions in turn exert important effects on the buyer-seller ratio of the housing market. This creates a coordination problem for moving owner occupiers, resulting in multiple equilibria. Equilibrium switches are associated with large fluctuations in the for-sale stock, time-on-market, transactions, and prices.

The tractable equilibrium model that we study in this paper to show these effects is deliberately simplified, and so lacks heterogeneity in many important dimensions. In particular, there is no heterogeneity in the costs of being a double owner versus a forced renter, which are likely to vary substantially across households and also to vary over time in response to aggregate shocks. In addition, we assumed constancy of the rate of mismatch and entry into and exit from the market. Nevertheless, endogenous fluctuations in γ and g are likely to additionally amplify and propagate aggregate shocks. Enriching the model along these dimensions will allow for a detailed quantitative model of the housing market, which can be taken to the data. We view this as an important step for future research.

References

Albrecht, J., A. Anderson, E. Smith, and S. Vroman (2007): "Opportunistic Matching in the Housing Market," *International Economic Review*, 48(2), 641–664.

Albrecht, J., P. A. Gautier, and S. Vroman (2016): "Directed Search in the Housing Market," *Review of Economic Dynamics*, 19, 218–231.

Anenberg, E., and P. Bayer (2015): "Endogenous Sources of Volatility in Housing Markets: The Joint Buyer-Seller Problem," mimeo.

Anundsen, A. K., and E. Røed Larsen (2014): "Strategic sequencing behavior among owner-occupiers: The role played by sell-first movers in a housing recovery," *Journal of European Real Estate Research*, 7(3), 295–306.

Boldrin, M., N. Kiyotaki, and R. Wright (1993): "A dynamic equilibrium model of search, production, and exchange," *Journal of Economic Dynamics and Control*, 17(5), 723–758.

Burdett, K., and M. Coles (1998): "Separation cycles," *Journal of Economic Dynamics and Control*, 22(7), 1069 – 1090.

Burdett, K., R. Imai, and R. Wright (2004): "Unstable relationships," *Frontiers in Macroeconomics*, 1(1).

Caplin, A., and J. Leahy (2011): "Trading Frictions and House Price Dynamics," *Journal of Money, Credit, and Banking*, 43, 283–303.

Chéron, A., and B. Decreuse (2014): "Matching with phantoms," mimeo.

Coles, M., and R. Wright (1998): "A dynamic equilibrium model of search, bargaining, and money," *Journal of Economic Theory*, 78(1), 32–54.

Dawson, M. (2013): "Buy or Sell First?," available from: http://www.realtor.com/home-finance/real-estate/general/buy-or-sell-first.aspx.

Diamond, P. (1982): "Aggregate Demand Management in a Search Equilibrium," *Journal of Political Economy*, 90, 881–894.

Diamond, P., and D. Fudenberg (1989): "Rational expectations business cycles in search equilibrium," *The Journal of Political Economy*, 97(3), 606–619.

Diaz, A., and B. Jerez (2013): "House Prices, Sales, and Time on the Market: A Search Theoretic Framework," *International Economic Review*, 54:3, 837–872.

Drazen, A. (1988): "Self-fulfilling optimism in a trade-friction model of the business cycle," *The American Economic Review*, 78(2), 369–372.

Duffie, D., N. Garleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73:6, 1815–1847.

Eeckhout, J., and I. Lindenlaub (2015): "Unemployment cycles," mimeo.

Farmer, R. E. A. (2012): "Confidence, Crashes and Animal Spirits," *The Economic Journal*, 122(559), 155–172.

Garibaldi, P., E. R. Moen, and D. E. Sommervoll (2016): "Competitive On-the-job Search," .

Genesove, D., and L. Han (2012): "Search and Matching in the Housing Market," *Journal of Urban Economics*, 72, 31–45.

Glaeser, E. L., J. Gyourko, E. Morales, and C. G. Nathanson (2014): "Housing Dynamics: An Urban Approach," *Journal of Urban Economics*, 81, 45–56.

Guren, A. (2014): "The Causes and Consequences of House Price Momentum," mimeo.

Guren, A., and T. McQuade (2013): "How Do Foreclosures Exacerbate Housing Downturns?," mimeo.

He, C., R. Wright, and Y. Zhu (2015): "Housing and liquidity," *Review of Economic Dynamics*, 18(3), 435 – 455.

Head, A., H. Lloyd-Ellis, and H. Sun (2014): "Search, Liquidity and the Dynamics of House Prices and Construction," *American Economic Review*, 104:4, 1172–1210.

Hosios, A. (1990): "On the Efficiency of Matching and Related Models of Search and Unemployment," *Review of Economic Studies*, 57:2, 279–98.

Howitt, P., and R. McAfee (1987): "Costly search and recruiting," *International Economic Review*, 28(1), 89–107.

——— (1988): "Stability of equilibria with externalities," *The Quarterly Journal of Economics*, 103(2), 261–277.

——— (1992): "Animal spirits," *The American Economic Review*, 82:3, 493–507.

Kaplan, G., and G. Menzio (2013): "Shopping externalities and self-fulfilling unemployment fluctuations," Working Paper No. 18777, NBER.

Kashiwagi, M. (2014): "Sunspots and self-fulfilling beliefs in the U.S. housing market," *Review of Economic Dynamics*, 17(4), 654 – 676.

Kiyotaki, N., and R. Wright (1993): "A Search-Theoretic Approach to Monetary Economics," *American Economic Review*, 83:1, 63–77.

Krainer, J. (2001): "A Theory of Liquidity in Residential Real Estate Markets," *Journal of Urban Economics*, 49, 32–53.

Lester, B., G. Rocheteau, and P.-O. Weill (2015): "Competing for Order Flow in OTC Markets," *Journal of Money, Credit, and Banking*, 47, 77–126.

Lester, B., L. Visschers, and R. Wolthoff (2013): "Competing with Asking Prices," mimeo.

Maury, T.-P., and F. Tripier (2014): "Search strategies on the housing market and their implications on price dispersion," *Journal of Housing Economics*, 26, 55–80.

Moen, E., P. Nenov, and F. Sniekers (2015): "Buying First or Selling First in Housing Markets," *CEPR discussion paper No. 10342*.

Moen, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105:2, 385–411.

Moen, E. R., and Å. Rosén (2013): "On-the-Job Search and Moral Hazard," *Journal of the European Economic Association*, 11(6), 1404–1431.

Mortensen, D. (1989): "The persistence and indeterminacy of unemployment in search equilibrium," *The Scandinavian Journal of Economics*, pp. 347–370.

——— (1999): "Equilibrium unemployment dynamics," *International Economic Review*, 40(4), 889–914.

Ngai, L. R., and K. D. Sheedy (2015): "Moving House," CEPR Discussion Paper No. 10346.

Ngai, L. R., and S. Tenreyro (2014): "Hot and Cold Seasons in the Housing Market," *American Economic Review*, 104(12), 3991–4026.

Novy-Marx, R. (2009): "Hot and Cold Markets," *Real Estate Economics*, 37:1, 1–22.

Ortalo-Magne, F., and S. Rady (2006): "Housing Market Dynamics: On the Contribution of Income Shocks and Credit Constraints," *Review of Economic Studies*, 73, 459–485.

Piazzesi, M., M. Schneider, and J. Stroebel (2015): "Segmented Housing Search," NBER Working Paper No. 20823.

Rocheteau, G., and R. Wright (2005): "Money in Search Equilibrium, in Competitive Equilibrium, and in Competitive Search Equilibrium," *Econometrica*, 73:1, 175–202.

Sniekers, F. (2014): "Persistence and volatility of Beveridge cycles," mimeo.

Stein, J. (1995): "Prices and trading volume in the housing market: a model with down-payment effects," *Quarterly Journal of Economics*, 110, 379–406.

Wheaton, W. (1990): "Vacancy, Search, and Prices in a Housing Market Matching Model," *Journal of Political Economy*, 98:6, 1270–1292.

Figure 1: Distribution of the time difference between “sell” and “buy” agreement dates (a) and closing dates (b) for homeowners who both buy and sell in Copenhagen (1993-2008).

(a) (b)

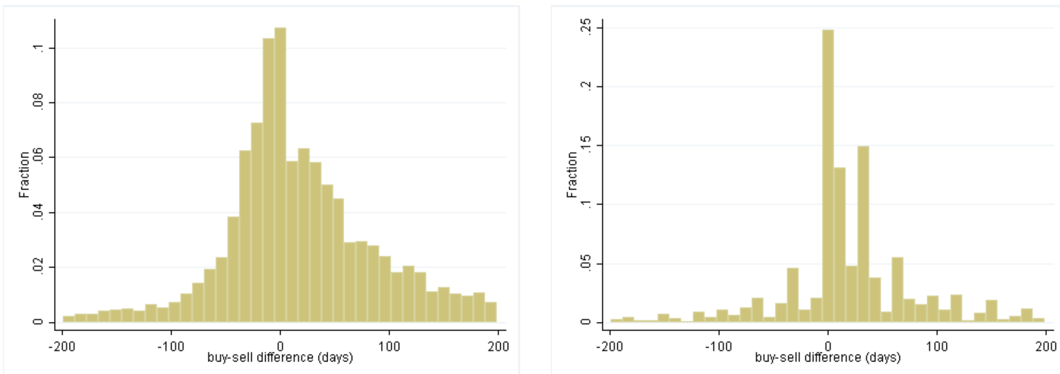
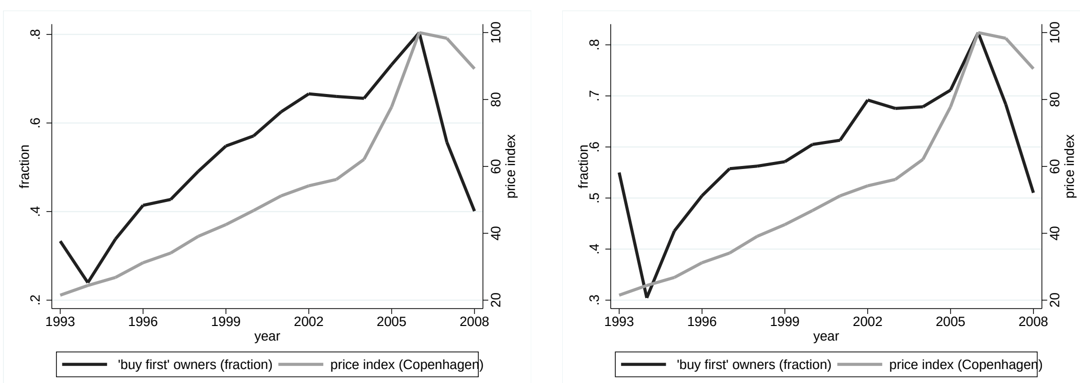


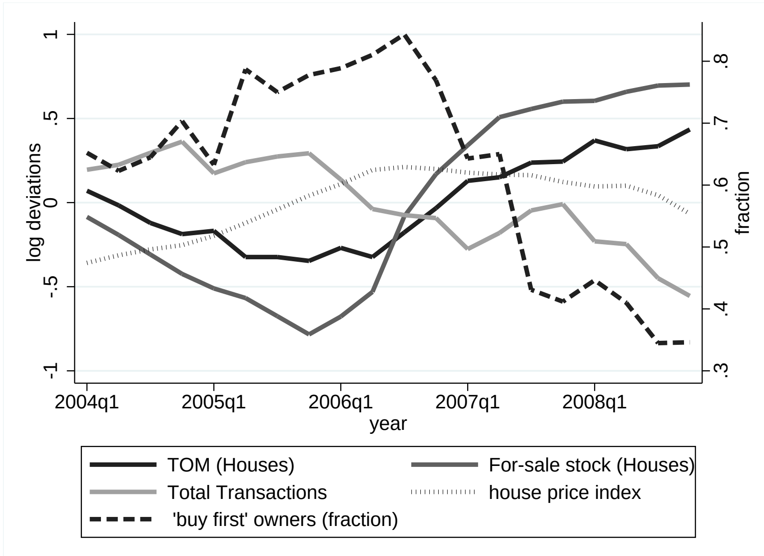
Figure 2: Fraction of owners who “buy first” and housing market conditions in Copenhagen (1993-2008). Panel (a) is based on agreement dates, and panel (b) is based on closing dates.

(a) (b)



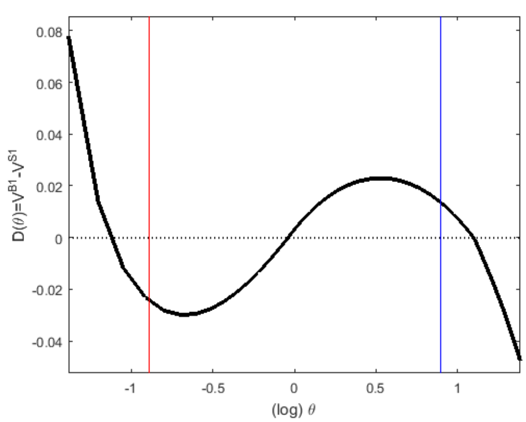
Notes: Own calculations based on registry data from Statistics Denmark. See the Appendix for a description on how we identify an owner that buys and sells in Copenhagen (a buyer-and seller) as a “buy first” (“sell first”) owner. We compute annual counts of the number of “buy first” and “sell first” owners by looking at the year of the first transaction for each of these owners. The fraction of “buy first” owners is then the proportion of buyer-and-seller owners that buy first. For panel (a) the identification of an owner as buy first/sell first is based on the difference in the two agreement dates. For the second, it is based on the difference in the two closing dates. The price index is a repeat sales price index for single family houses for Copenhagen (Region Hovedstaden) constructed by Statistics Denmark.

Figure 3: Housing market dynamics, Copenhagen 2004-2008



Notes: Data sources: seller time-on-market (TOM) and for-sale stock from the Danish Mortgage Banks' Federation (available at http://statistik.realkreditforeningen.dk/BMSDefault.aspx) (in log-deviations from sample mean); total transactions and price index from Statistics Denmark (in log-deviations from sample mean after controlling for seasonality effects). The fraction of buy first owners is from own calculations based on registry data from Statistics Denmark. See the Appendix for details.

Figure 4: A plot of $D(\theta)$ against (log) θ. Steady state market tightnesses in a "Buy first"/"Sell first" equilibrium in blue/red.



Notes: Preference parameters used for the example: $u = 2$, $u_0 = 1.55$, $u_2 = 1.85$, $u_n = 1.7$, $\chi = 0.14$. Rental price $R = 0.1$. In addition, we use the calibration from Section 4.6 for the steady state market tightnesses and a value of $\mu_0 = 4$ for the matching efficiency parameter of the matching function.

Appendix [For Online Publication]

A. Data Description

We use two data sets. The first (EJER) is an ownership register which contains the owners (private individuals and legal entities) of properties in Denmark as of the end of a given calendar year. The data set contains unique identifiers for owners (which, unfortunately, cannot be matched with other datasets beyond EJER for different years). It also contains unique identifiers for each individual property. The second data set (EJSA) contains a record of all property sales in a given calendar year. The majority of transactions include information on the sale price, sale (agreement), and takeover (closing) dates. Furthermore, they contain the property identifiers used in the EJER data-set, which allows for linking of the two datasets. The first data set is available from 1986 (recording ownership in 1985) until 2010 (recording ownership at the end of 2009), while the second is available from 1992 to 2010. Therefore, we effectively use data from 1991 (for ownership as of January 1, 1992) to 2009 (for ownership as of January 1, 2010).

We focus on the Copenhagen urban area (Hovedstadsområdet). We take the definition of the Copenhagen urban area as containing the following municipalities (by number): 101, 147, 151, 153, 157, 159, 161, 163, 165, 167, 173, 175, 183, 185, 187, 253, 269.[27]

We restrict attention to private owners and also to the primary owner of a property in a given year (whenever a property has more than one owners). Furthermore, we examine transactions where the new owner is a private individual and which have a non-missing agreement date. We drop properties that are recorded to transact more than once in a given year. We also remove property-year observations for which no owner is recorded. This leaves us with a total of 3312520 property-year observations. These comprise 199812 unique properties and 345943 unique individual owners over our sample period.

To identify an individual owner as a buyer-and-seller we rely on the infor-

[27]Due to a reform in 2007, which merged some municipalities and created a new one, we omit municipality 190 for consistency.

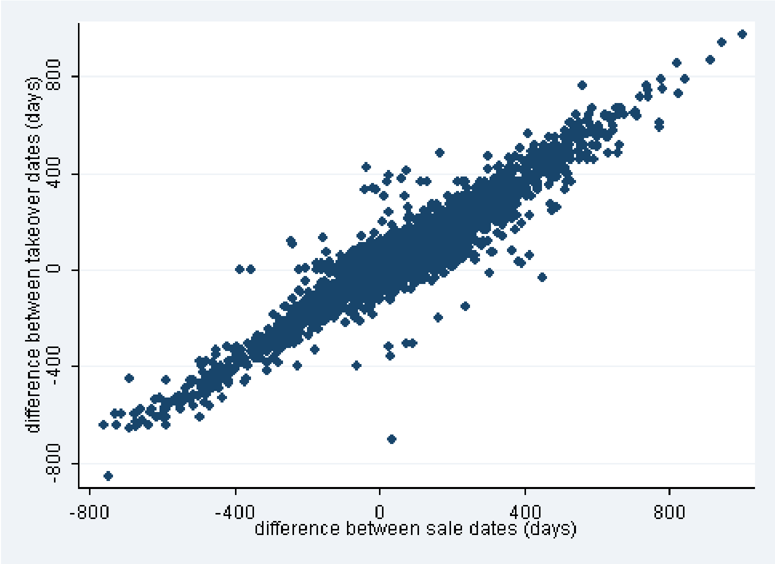
mation from the ownership register across consecutive years. First of all, we use the information on ownership over consecutive years to determine the counterparties for each recorded transaction in our sample. We then identify an individual owner as a buyer-and-seller if he is recorded to buy a new property and sell an old property within the same year or over two consecutive years. An old property is defined as a property which an individual is registered as owning over at least 2 consecutive years.[28] Also, we do not count individuals that are recorded as holding two properties for two or more consecutive years, which we treat as purchases for investment purposes.

We conduct this for individuals that are recorded as owning at most 2 properties at the end of any calendar year in our sample. This comprises the large majority of individual owners in our sample. In particular, in a given year in our sample from 1991-2009 there are on average only around 0.4% of individual owners who own more than two properties in the Copenhagen. Therefore, the majority of individuals hold at most 1 or 2 properties over that period. In particular, on average, around 1.6% of individual owners hold two properties at the end of a calendar year in our sample. Interestingly, around 5% of the recorded owners of two properties at the end of a calendar year are also identified as a buyer-and-seller according to our identification procedure described above with that number going up to almost 14% at the peak of the housing boom in 2006.

For each individual owner that has been identified as buyer-and-seller, we compute the time period (in days) between the agreement data for sale of the old property and the agreement date for the purchase of the new property. Similarly, we compute the time period (in days) between the closing date that of the buyer-and-seller's old property by the new owner and the closing date for his new property. We then denote a buyer-and-seller for which the time period between agreement dates is negative (sale date is before purchase date) as "selling first" and a buyer-and-seller for which the time period is positive

[28]We make this restriction in order not to misclassify as a buyer-and-seller an individual who acquires a house, for example as a bequest (which is not recorded as a transaction), which he ends up selling quickly and then buys a new house with the proceeds from the sale. Adding back those agents has a very small effect on the pattern we uncover.

Figure 5: Difference in agreement dates vs. difference in closing dates, Copenhagen (1993-2008)



(sale date is after purchase date) as “buying first”. We also do the same classification but based on closing dates rather than agreement dates. Given the way we identify a buyer-and-seller, we have a consistent count for the number of owners who “buy first” vs. “sell first” in a given year for the years 1993 to 2008.

In principle, and as Figures 1 and 2 show, working with either of the two identifications produces similar results. This is not surprising given that the time difference between the agreement dates and closing dates are highly correlated with a correlation coefficient of 0.9313. Figure 5 visualizes this strong correlation by plotting a scatter plot of the two time differences.

B. Equilibrium concept and parameter restrictions for the benchmark model

First of all, the steady state value functions for a new entrant, a matched owner, and a real estate firm satisfy the following equations:

$$\rho V^{Bn} = u_n - R + q(\theta)\left(-p + V - V^{Bn}\right), \tag{28}$$

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right), \tag{29}$$

and

$$\rho V^{A} = R + \mu(\theta)\left(p - V^{A}\right). \tag{30}$$

Importantly, in every steady state equilibrium, V satisfies $V \geq \tilde{V}$, where $\tilde{V} = \frac{u}{\rho+\gamma} + \frac{\gamma}{\rho+\gamma}V^m$, with $V^m = \frac{u-\chi}{\rho}$. Hence, $\tilde{V}$ is the value of a matched owner who never transacts. Therefore, $V \geq \tilde{V} = \frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}$ in any steady state equilibrium.

Parameter restrictions

Sufficient conditions for new entrants, forced renters and double owners to prefer transacting and becoming matched owners are given by

$$\frac{u_n - R}{\rho} \leq \tilde{V} - p, \tag{31}$$

$$\frac{u_0 - R}{\rho} \leq \tilde{V} - p, \tag{32}$$

and

$$\frac{u_2 + R}{\rho} \leq \tilde{V} + p. \tag{33}$$

Since $u_n \geq u_0$, we can disregard (32), as it is implied by (31). Conditions (31) and (33) imply restrictions for the values of the house price, p, that are sufficient for these agents to be willing to transact at p, namely

$p \in \left[\frac{u_2}{\rho} - \tilde{V} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]$.

From (30) a real estate firm is willing to transact iff $p \geq \frac{R}{\rho}$. Therefore, equilibrium is defined for a house price p, that satisfies

$$p \in \left[\max\left\{\frac{u_2}{\rho} - \tilde{V}, 0\right\} + \frac{R}{\rho}, \tilde{V} - \frac{u_n}{\rho} + \frac{R}{\rho}\right]. \tag{34}$$

For $u - \chi \geq \max\{u_0, u_2\}$, which is the condition we will use to characterize equilibria, it follows that $\frac{u_2}{\rho} - \tilde{V} < 0$ and so the set for prices is given by

$$p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]. \tag{35}$$

Finally, a sufficient condition for $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$ at $\theta = \tilde{\theta}$, with $\tilde{\theta}$ as defined in (15) is

Assumption A2: $\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu(\tilde{\theta})} + \frac{\mu(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\tilde{u}_0 + \frac{\mu(\tilde{\theta})q(\tilde{\theta})}{(\rho+\mu(\tilde{\theta}))(\rho+q(\tilde{\theta}))}\left(\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho}\right)$. Note that $\frac{u}{\rho} - \frac{\gamma}{\rho+\gamma}\frac{\chi}{\rho} \leq V$, $\forall\theta$, so the right hand side of this expression is lower than the value of V^{S1} at $\theta = \tilde{\theta}$.

Steady state flow conditions

Before moving to our formal definition, it is necessary to describe the flow conditions that the aggregate stock variables defined in Section 3 must satisfy. We have that in a steady state equilibrium, given a market tightness θ, the steady state values of B_n, B_0, B_1, S_1, S_2, O, and A must satisfy the following system of flow conditions:

$$g = (q(\theta) + g) B_n, \tag{36}$$

$$\mu(\theta) S_1 = (q(\theta) + g) B_0, \tag{37}$$

$$\mu(\theta) S_2 + q(\theta)(B_n + B_0) = (\gamma + g) O, \tag{38}$$

$$\gamma x_b O = (q(\theta) + g) B_1, \tag{39}$$

$$\gamma x_s O = (\mu(\theta) + g) S_1, \tag{40}$$

$$q(\theta) B_1 = (\mu(\theta) + g) S_2, \tag{41}$$

$$g(O + B_1 + S_1 + 2S_2) = \mu(\theta) A, \tag{42}$$

$$x_b + x_s = 1, \tag{43}$$

where x_b, and x_s are the equilibrium fractions of mismatched buyers and sellers, respectively. Apart from these conditions, the aggregate variables must satisfy the population constancy and housing ownership conditions (2) and (3). Finally, the equilibrium market tightness θ, satisfies

$$\theta = \frac{B}{S} = \frac{B_n + B_0 + B_1}{S_1 + S_2 + A}. \tag{44}$$

Equilibrium definition

We define a steady state equilibrium for this economy in the following way:

Definition 1. A steady state equilibrium consists of a house price p, equilibrium rental rate R, value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, market tightness θ, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, and aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A such that:

1. The house price $p \in \left[\frac{R}{\rho}, \tilde{V} - \frac{u_0}{\rho} + \frac{R}{\rho}\right]$;

2. The equilibrium rental rate $R \in [0, u_0]$;

3. The value functions satisfy equations (5)-(8) and (28)-(30) given θ, p, and R;

4. Mismatched owners choose $x \in \{b, s\}$, to maximize $\overline{V} = \max\left\{V^{B1}, V^{S1}\right\}$

and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\left\{x_i = b\right\} di,$$

where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

5. The market tightness θ solves (44) given B_n, B_0, B_1, S_1, S_2, O, and A;

6. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (36)-(42) given θ and mismatched owners' optimal decisions reflected in x_b and x_s.

C. Omitted Proofs (Benchmark model)

Proof of Lemma 1

First of all, note that the function $D\left(\theta\right)$, defined in (13) crosses zero only at $\theta = \tilde{\theta}$. To see this, notice that

$$\lim_{\theta \to 0} D\left(\theta\right) = \frac{\tilde{u}_2 - (u - \chi)}{\rho} < 0,$$

and

$$\lim_{\theta \to \infty} D\left(\theta\right) = \frac{u - \chi - \tilde{u}_0}{\rho} > 0.$$

Away from these two limiting values, $D\left(\theta\right) > 0$, whenever

$$\left(1 - \frac{1}{\theta}\right)\left(u - \chi - \tilde{u}_2\right) - \tilde{u}_0 + \tilde{u}_2 > 0,$$

which is equivalent to $\tilde{\theta} < \theta$. Therefore, $D\left(\theta\right) > 0$ iff $\theta \in \left(\tilde{\theta}, \infty\right)$ and $D\left(\theta\right) < 0$ iff $\theta \in \left(0, \tilde{\theta}\right)$. Therefore, $D\left(\theta\right) = 0$, iff

$$\left(1 - \frac{1}{\theta}\right)\left(u - \chi - \tilde{u}_2\right) - \tilde{u}_0 + \tilde{u}_2 = 0,$$

or $\theta = \tilde{\theta}$. Note that $D(\theta)$ fully summarizes the incentives of a mismatched owner to buy first/sell first apart from at $\theta = 0$ and $\theta = \infty$. To see this, let

$$\begin{aligned}\tilde{V}^{B1} &= \frac{u-\chi}{\rho+q(\theta)} + \frac{q(\theta)\,\tilde{u}_2}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V - \frac{u-\chi}{\rho} \\ &= \frac{q(\theta)}{\rho+q(\theta)}\left(\frac{\tilde{u}_2}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)}V - \frac{u-\chi}{\rho}\right),\end{aligned}$$

and

$$\begin{aligned}\tilde{V}^{S1} &= \frac{u-\chi}{\rho+\mu(\theta)} + \frac{\mu(\theta)\,\tilde{u}_0}{(\rho+\mu(\theta))(\rho+q(\theta))} + \frac{q(\theta)\,\mu(\theta)}{(\rho+\mu(\theta))(\rho+q(\theta))}V - \frac{u-\chi}{\rho} \\ &= \frac{\mu(\theta)}{\rho+\mu(\theta)}\left(\frac{\tilde{u}_0}{\rho+q(\theta)} + \frac{q(\theta)}{\rho+q(\theta)}V - \frac{u-\chi}{\rho}\right).\end{aligned}$$

The functions $\tilde{V}^{B1}$ and $\tilde{V}^{S1}$ give the difference between the value of transacting and never transacting for a buyer first and seller first, respectively.

By Assumption A2, at $\tilde{\theta}$, $V^{S1} > \frac{u-\chi}{\rho}$ and $V^{B1} > \frac{u-\chi}{\rho}$, so at $\theta = \tilde{\theta}$, $\tilde{V}^{B1} > 0$, and $\frac{\tilde{u}_2}{\rho+\mu(\theta)} + \frac{\mu(\theta)}{\rho+\mu(\theta)}V - \frac{u-\chi}{\rho} > 0$. Furthermore, this latter inequality holds for any $\theta > \tilde{\theta}$, and so $\tilde{V}^{B1} > 0$ for any $\theta > \tilde{\theta}$. Therefore, for any $\theta > \tilde{\theta}$, a mismatched buyer is better off transacting than not transacting. Similarly, for $\theta < \tilde{\theta}$ the mismatched seller is better off transacting than not transacting.

Therefore, for $\theta \in (0, \infty)$, if $D(\theta) > 0$, a mismatched owners is better off buying first (and transacting) compared to selling first (and transacting or not transacting) and similarly, if $D(\theta) < 0$, a mismatched owner is better off selling first (and transacting) compared to buying first (and transacting or not transacting). At $D(\theta) = 0$, he is indifferent between buying first (and transacting) and selling first (and transacting).

Finally, clearly if $\theta \to \infty$, $\tilde{V}^{B1} \to 0$, so $V^{B1} \to \frac{u-\chi}{\rho} = V^{S1}$. Similarly, if $\theta \to 0$, $\tilde{V}^{S1} \to 0$, and so $V^{S1} \to \frac{u-\chi}{\rho} = V^{B1}$. $\square$

Proof of Lemma 2

We show that $\overline{\theta}$ solves

$$\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta+\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)=\frac{1}{g}+\frac{1}{\gamma}, \tag{45}$$

and $\underline{\theta}$ solves

$$\left(\frac{1}{\mu(\theta)+g}+\frac{1}{\gamma}\right)\frac{1}{\theta}=\frac{1}{g}+\frac{1}{\gamma}. \tag{46}$$

These two equations arise from the flow conditions and population and housing conditions if all mismatched owners buy first and sell first, respectively. For the case where mismatched owners buy first ($x_s = 0$), the stock-flow conditions are

$$g = (q(\theta)+q)\,B_n,$$

$$\gamma O = (q(\theta)+g)\,B_1,$$

$$q(\theta)\,B_1 = (\mu(\theta)+g)\,S_2,$$

$$g = (\mu(\theta)+g)\,A,$$

$$B_n + B_1 + S_2 + O = 1,$$

and

$$B_n = A + S_2.$$

It follows that $B_n = \frac{g}{q(\theta)+g}$ and $A = \frac{g}{\mu(\theta)+g}$, or $A = \frac{q(\theta)+g}{\mu(\theta)+g}B_n$, so $S_2 = \frac{g}{q(\theta)+g} - \frac{g}{\mu(\theta)+g}$. Therefore, from the equation for θ, we have that $B_1 = (\theta-1)\,B_n$ and so $O = \frac{1}{\gamma}(q(\theta)+g)(\theta-1)\,B_n$. Substituting into the population constancy condition, we have that

$$\theta B_n + B_n - \frac{q(\theta)+g}{\mu(\theta)+g}B_n + \frac{1}{\gamma}(q(\theta)+g)(\theta-1)\,B_n = 1,$$

which, after substituting for B_n and re-arranging we can write as

$$\left(\frac{1}{q(\theta)+g}+\frac{1}{\gamma}\right)\theta+\left(\frac{1}{q(\theta)+g}-\frac{1}{\mu(\theta)+g}\right)=\frac{1}{g}+\frac{1}{\gamma}.$$

This is exactly equation (45). At $\theta = 1$, the left-hand side equals

$$\frac{1}{q(1)+g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Furthermore, note that $\left(\frac{1}{q(\theta)+g} + \frac{1}{\gamma}\right)\theta$ is strictly increasing in θ and also unbounded. Similarly, $\left(\frac{1}{q(\theta)+g} - \frac{1}{\mu(\theta)+g}\right)$ is strictly increasing in θ as well. Therefore, the left-hand side of (45) is strictly increasing in θ, unbounded, and lower than the right-hand side for $\theta = 1$. Therefore, it has a unique solution for $\theta > 1$. We call this solution $\overline{\theta}$. Furthermore, by the Implicit Function Theorem, it immediately follows that $\overline{\theta}$ is increasing in γ.

For the case where mismatched owners sell first ($x_s = 1$) the stock-flow conditions become

$$g = (q(\theta) + q) B_n,$$

$$\mu(\theta) S_1 = (q(\theta) + g) B_0,$$

$$\gamma O = (\mu(\theta) + g) S_1,$$

$$g = (\mu(\theta) + g) A,$$

$$B_n + B_0 + S_1 + O = 1,$$

and

$$B_n + B_0 = A.$$

It follows that $A = \frac{g}{\mu(\theta)+g} = B_0 + B_n$, $S_1 = \frac{1-\theta}{\theta} A$ and $O = \frac{1}{\gamma}(\mu(\theta)+g)\frac{1-\theta}{\theta}A$. Therefore, substituting for these in the population constancy condition, we have that

$$\frac{1}{\theta}A + \frac{1}{\gamma}(\mu(\theta)+g)\frac{1-\theta}{\theta}A = 1.$$

Substituting for A, we obtain an equation for θ of the form

$$\left(\frac{1}{\mu(\theta)+g} + \frac{1}{\gamma}\right)\frac{1}{\theta} = \frac{1}{g} + \frac{1}{\gamma},$$

which is equation (46). At $\theta = 1$, the left-hand side equals

$$\frac{1}{\mu(1) + g} + \frac{1}{\gamma} < \frac{1}{g} + \frac{1}{\gamma}.$$

Note also that $\left(\frac{1}{\mu(\theta)+g} + \frac{1}{\gamma}\right)\frac{1}{\theta}$ is strictly decreasing in θ and goes to 0 as $\theta \to \infty$. Also it asymptotes to ∞ as $\theta \to 0$. Therefore, the equation has a unique solution for $\theta < 1$. We call this solution $\underline{\theta}$. By the Implicit Function Theorem, it immediately follows that $\underline{\theta}$ is decreasing in γ.

Finally, the limiting values for $\overline{\theta}$ and $\underline{\theta}$ in (19) and (20) follow directly from taking these limits in equations (45) and (46). □

Proof of Proposition 1

Clearly, Lemma 2 that determines the values of $\overline{\theta}$ and $\underline{\theta}$ is independent of the agents' payoffs. With regard to Item 1, a direct application of Lemma 1 shows that if $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$, then at $\theta = \underline{\theta}$ a mismatched owner is (weakly) better off selling first, at $\theta = \overline{\theta}$ he is (weakly) better off buying first, and at $\theta = \tilde{\theta}$ he is indifferent. If mismatched owners are indifferent, they can randomize, such that $\theta = \tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$. This follows since the stock-flow conditions ensure a continuous relation between the fraction of agents buying first, x_b, and θ, and observing that $\theta = \underline{\theta}$, iff $x_b = 0$ and, $\theta = \overline{\theta}$, iff $x_b = 1$, so that for $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, $x_b \in [0, 1]$. Also, by Assumption A2, they are strictly better off from transacting than not transacting. Consequently, agents' actions are optimal given θ and the steady state value of θ is consistent with agents' actions. Considering Item 2, by the same logic a steady state equilibrium in which mismatched owners buy first and $\theta = \overline{\theta}$ exists. To see that it is the only symmetric steady state equilibrium, remember from Lemma 1 that mismatched owners only sell first for $\theta < \tilde{\theta}$, which contradicts $\tilde{\theta} < \underline{\theta}$. The same logic applies to Item 3. □

Proof of Corollary 1

For $\tilde{u}_0 = \tilde{u}_2 = c$, $\tilde{\theta} = 1$. Because Lemma 2 shows that $\overline{\theta} > 1$ when all mismatched owners buy first, and $\underline{\theta} < 1$ if they sell first, $\tilde{\theta} \in \left[\underline{\theta}, \overline{\theta}\right]$. Then an application of Proposition 1 gives the buy first equilibrium and the sell first equilibrium. To see that $\theta = 1$ if half of the mismatched owners buy first, take $x_s = x_b = \frac{1}{2}$. We have

$$\gamma \frac{1}{2} O = (q(\theta) + g) B_1, \tag{47}$$

and

$$\gamma \frac{1}{2} O = (\mu(\theta) + g) S_1. \tag{48}$$

At $\theta = 1$, $\mu(\theta) = q(\theta) = \mu(1)$, so $B_1 = S_1$. Also, $B_n = A = \frac{g}{\mu(1)+g}$ and $B_0 = S_2 = S_1 \frac{\mu(1)}{\mu(1)+g}$. Finally, population constancy implies that

$$2S_1 \frac{\mu(1)}{\mu(1)+g} + 2S_1 + 2S_1 \frac{\mu(1)+g}{\gamma} = \frac{\mu(1)}{\mu(1)+g},$$

which is satisfied for some $S_1 \in \left(0, \frac{1}{2}\right)$. Because mismatched owners are indifferent at $\theta = 1$ for $\tilde{u}_0 = \tilde{u}_2 = c$ by Lemma 1, an equilibrium in which half of the mismatched owners buy first exists at $\theta = 1$. $\square$

Proof of Proposition 4

Consider the difference between the two value functions, $D(\theta) = V^{B1} - V^{S1}$ assuming that the mismatched owner transacts in both cases.

$$D(\theta) = \frac{\mu(\theta)\left(1 - \frac{1}{\theta}\right)\left(u - \chi - c + \lambda\left(\overline{V}_N - v^{B0}\right)\right)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)} + \frac{\frac{\lambda\mu(\theta)\left(1-\frac{1}{\theta}\right)q(\theta)}{(r+\mu(\theta))(r+q(\theta))}[\rho V - c] + \mu(\theta)\left(1 + \frac{1}{\theta}\right)\lambda(p_N - p)}{(\rho + q(\theta) + \lambda)(\rho + \mu(\theta) + \lambda)}. \tag{49}$$

Consider the case of $1 < \theta \leq \overline{\theta}$, so $\overline{V}_N = V_N^{B1}$. If $\overline{V}_N = V_N^{B1}$, where V_N^{B1} denotes the value of buying first after the price change, this difference simplifies further to

$$D(\theta)=\frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+q(\theta)}\right)(u-\chi-c)+\left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \quad (50)$$

Suppose that $p_N < p$ and define θ_{B1}^{PR} as the solution to

$$\frac{\theta_{B1}^{PR}-1}{\theta_{B1}^{PR}+1}\left(1+\frac{\lambda}{\rho+q\left(\theta_{B1}^{PR}\right)}\right)=\frac{\lambda(p-p_N)}{(u-\chi-c)}. \quad (51)$$

Therefore, θ_{B1}^{PR} is the value of θ that leaves a mismatched owner indifferent between buying first and selling first he anticipates a price change of $p_N - p$ and a market tightness of $\theta > 1$ after the price change. Note that θ_{B1}^{PR} is increasing in $p - p_N$ if $\theta_{B1}^{PR} \geq 1$. Therefore, a sufficient condition for mismatched owners to prefer to sell first, given $1 < \theta \leq \overline{\theta}$, is that $\theta_{B1}^{PR} > \overline{\theta}$.

Similarly, consider the case of $\underline{\theta} \leq \theta < 1$, so $\overline{V}_N = V_N^{S1}$, where V_N^{S1} denotes the value of selling first after the price change. In that case the difference in value functions can be written as

$$D(\theta)=\frac{\mu(\theta)\left[\left(1-\frac{1}{\theta}\right)\left(1+\frac{\lambda}{\rho+\mu(\theta)}\right)(u-\chi-c)+\left(1+\frac{1}{\theta}\right)\lambda(p_N-p)\right]}{(\rho+q(\theta)+\lambda)(\rho+\mu(\theta)+\lambda)}. \quad (52)$$

Suppose that $p_N > p$ and define θ_{S1}^{PR} as the solution to

$$\frac{\theta_{S1}^{PR}-1}{\theta_{S1}^{PR}+1}\left(1+\frac{\lambda}{\rho+\mu\left(\theta_{S1}^{PR}\right)}\right)=\frac{\lambda(p-p_N)}{(u-\chi-c)}. \quad (53)$$

Similarly, to the case of θ_{B1}^{PR}, θ_{S1}^{PR} is increasing in $p - p_N$ if $\theta_{S1}^{PR} \leq 1$. Then, a sufficient condition for mismatched owner to prefer to buy first, given $\underline{\theta} \leq \theta < 1$ is that $\theta_{S1}^{PR} < \underline{\theta}$. □

D. A model with prices determined by Nash bargaining

We show our main analytical result for the model with Nash bargaining under the following parametric assumptions:

Assumption B1: $u_2 - u_0 = u - u_n$.

Assumption B2: $r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi > 0$.

Assumption B3: $r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - (u - \chi)\right) + \frac{1}{2}\mu_0\chi\right]$.

Assumption B4: $\frac{\gamma}{g} \leq \kappa^*$,

where $\kappa^* > 0$ is determined below. The first assumption ensures that buying first and selling first are equally attractive at $\theta = 1$. Assumptions B2-B4 ensure that the trading pattern described in the main text emerges in equilibrium.

Proof of Proposition 2

Below, we use the notation Σ_{ij} to denote the surplus from trade between agents of type i and type j.

"Sell first" equilibrium existence.

We first show that a "Sell first" equilibrium exists with $\theta = \underline{\theta} < 1$. We proceed in two steps. First, we show that no mismatched owner has an incentive to deviate and buy first when $\theta = \underline{\theta} < 1$. This is verified under the conjecture that $\Sigma_{ij} \geq 0$ for all buyer-seller pairs except for Σ_{S1B1}. Second, we verify the conjecture on the different surpluses.

Step 1. In the limit economy with small flows of a "Sell first" equilibrium candidate, the faction of buyers who are forced renters is given by

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{B_0}{B} = \lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{q\left(\underline{\theta}\right) - \mu\left(\underline{\theta}\right)}{g + q\left(\underline{\theta}\right)} = 1 - \underline{\theta},$$

where $\underline{\theta} = \frac{1}{1+\kappa}$. Similarly,

$$\lim_{g\to 0,\gamma\to 0,\frac{g}{\gamma}=\kappa} \frac{A}{S} = \underline{\theta}.$$

Thus,

$$\begin{aligned} rV^{B0} &= u_0 - R + \frac{1}{2}q\left(\underline{\theta}\right)\left(\frac{A}{S}\Sigma_{AB0} + \frac{S_1}{S}\Sigma_{S1B0}\right) \\ &= u_0 - R + \frac{1}{2}q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{AB0} + \left(1-\underline{\theta}\right)\Sigma_{S1B0}\right), \end{aligned}$$

and similarly,

$$rV^{Bn} = u_n - R + \frac{1}{2}q\left(\underline{\theta}\right)\left(\underline{\theta}\Sigma_{ABn} + \left(1-\underline{\theta}\right)\Sigma_{S1Bn}\right),$$

so

$$V^{Bn} - V^{B0} = \frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)}. \tag{54}$$

Also,

$$\begin{aligned} rV^{A} &= R + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\frac{B_n}{B}\left(V - V^{Bn} - V^{A}\right) + \frac{B_0}{B}\left(V - V^{B0} - V^{A}\right)\right) \\ &= R + \frac{1}{2}\mu\left(\underline{\theta}\right)\left(\underline{\theta}\left(V - V^{Bn} - V^{A}\right) + \left(1-\underline{\theta}\right)\left(V - V^{B0} - V^{A}\right)\right), \end{aligned}$$

or

$$V^{A} = \frac{R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} + \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V - V^{B0} - \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q\left(\underline{\theta}\right)}\right).$$

Analogous to Equation 54

$$V^{S2} - V^{A} = \frac{u_2 + \frac{1}{2}\mu\left(\underline{\theta}\right)V}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.$$

This in turn implies that

$$V - V^{S2} = \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A} = \frac{u - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^{A}.$$

Turning to the value functions of mismatched owners, a mismatched seller has a value function given by

$$rV^{S1} = u - \chi + \frac{1}{2}\mu(\underline{\theta})\left(V - V^{S1} - \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})}\right),$$

which can be re-written as

$$V^{S1} = \frac{u - \chi}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})}V - \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})}\underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\theta)}.$$

For the value function of a deviating mismatched buyer, assuming that trade takes place when he meets a real-estate firm but not when he meets a mismatched seller, writes

$$rV^{B1} = u - \chi + \frac{1}{2}q(\underline{\theta})\,\underline{\theta}\Sigma_{AB1}.$$

Or

$$\left(r + \frac{1}{2}\mu(\underline{\theta})\right)V^{B1} = u - \chi + \frac{1}{2}\mu(\underline{\theta})\left(V^{S2} - V^{A}\right).$$

Consider the difference between the utilities from buying first compared to selling first. In the limit we consider, we have that

$$\left(r + \frac{1}{2}\mu(\underline{\theta})\right)\left(V^{B1} - V^{S1}\right) = \frac{1}{2}\mu(\underline{\theta})\left(V^{S2} - V^{A} - V + \underline{\theta}\frac{u_n - u_0}{\rho + \frac{1}{2}q(\underline{\theta})}\right).$$

Substituting for $V^{S2} - V^{A} - V$, we get that

$$V^{B1} - V^{S1} = \frac{\frac{1}{2}\mu(\underline{\theta})}{r + \frac{1}{2}\mu(\underline{\theta})}\left(\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})}\right).$$

Note that at $\underline{\theta} = 1$ (i.e. for $\kappa = 0$),

$$\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} = 0,$$

given Assumption B1. As $\underline{\theta}$ moves away from 1 toward 0 (κ increases), we have that $\frac{u_2 - u}{r + \frac{1}{2}\mu(\underline{\theta})} + \underline{\theta}\frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})}$ decreases, so $V^{B1} < V^{S1}$ for $\underline{\theta} < 1$. Therefore, it is not optimal for a mismatched owner to deviate and buy first in an equilibrium in which mismatched owners sell first and $\theta < 1$.

Step 2. We verify that our conjectures for the surpluses are correct. It is clear given our assumptions that $\Sigma_{S2B1} = V - V^{B1} > 0$ and $\Sigma_{S1B0} = V - V^{S1} > 0$. Also, $\Sigma_{ABn} \geq 0$. Next, we show that $\Sigma_{S1Bn} > 0$. In the limit we consider,

$$
\begin{aligned}
\Sigma_{S1Bn} &= V - V^{Bn} + V^{B0} - V^{S1} = V - V^{S1} - \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{\chi}{r + \frac{1}{2}\mu(\underline{\theta})} + \frac{\frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1) - r}{r + \frac{1}{2}\mu(\underline{\theta})} \frac{u_n - u_0}{r + \frac{1}{2}q(\underline{\theta})} \\
&= \frac{r(\chi + u_0 - u_n) + \frac{1}{2}q(\underline{\theta})\chi + \frac{1}{2}\mu(\underline{\theta})(\underline{\theta} - 1)(u_n - u_0)}{\left(r + \frac{1}{2}q(\underline{\theta})\right)\left(r + \frac{1}{2}\mu(\underline{\theta})\right)}.
\end{aligned}
$$

Therefore, at $\underline{\theta} = 1$, $\Sigma_{S1Bn} > 1$ if

$$
r(\chi + u_0 - u_n) + \frac{1}{2}\mu_0\chi > 0.
$$

Note that given Assumption B1, this is equivalent to

$$
r(u_2 - (u - \chi)) + \frac{1}{2}\mu_0\chi > 0,
$$

which holds by Assumption B2. Therefore, by continuity of the value functions with respect to θ, it follows that there is a $\kappa_1 > 0$, such that for $\kappa < \kappa_1$, $\Sigma_{S1Bn} > 0$. Next, we show that $\Sigma_{ABn} > 0$. To show, this suppose, toward a contraduction, that $\Sigma_{ABn} < 0$. Then

$$
rV^{Bn} + rV^{A} \leq u_n + \frac{1}{2}q(\underline{\theta})\Sigma_{ABn} + \frac{1}{2}\mu(\underline{\theta})\Sigma_{ABn},
$$

where the inequality comes from $\Sigma_{ABn} < 0 < \Sigma_{S1Bn}$ and $\Sigma_{ABn} < \Sigma_{AB0}$, since $V^{Bn} > V^{B0}$. Therefore,

$$
\Sigma_{ABn} \geq \frac{rV - u_n}{r + \frac{1}{2}q(\underline{\theta}) + \frac{1}{2}\mu(\underline{\theta})} > 0,
$$

so we arrive at a contradiction. $\Sigma_{ABn} > 0$ also implies that $\Sigma_{AB0} > 0$, since

$V^{Bn} > V^{B0}$. Next notice that

$$\Sigma_{S2Bn} = \Sigma_{ABn} + V - V^{S2} + V^A = \Sigma_{ABn} + \frac{rV - u_2}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} > 0.$$

Again, this also implies that $\Sigma_{S2B0} > 0$. Next, we show that $\Sigma_{AB1} > 0$. In the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2} - V^{B1} - V^A = V^{S2} - V^A - \frac{u-\chi}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - \frac{\frac{1}{2}\mu\left(\underline{\theta}\right)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}\left(V^{S2} - V^A\right) \\
&= \frac{r\left(V^{S2} - V^A\right) - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} = \frac{\frac{r}{r+\frac{1}{2}\mu(\underline{\theta})}u_2 + \frac{\frac{1}{2}\mu(\underline{\theta})}{r+\frac{1}{2}\mu(\underline{\theta})}u - (u-\chi)}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} \\
&= \frac{r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu\left(\underline{\theta}\right)\chi}{\left(r + \frac{1}{2}\mu\left(\underline{\theta}\right)\right)^2}.
\end{aligned}$$

At $\underline{\theta} = 1$, $\Sigma_{AB1} > 0$ if $r\left(u_2 - (u-\chi)\right) + \frac{1}{2}\mu_0\chi > 0$, which is our parametric Assumption B2. Therefore, by continuity of the value functions with respect to $\underline{\theta}$, it follows that there is a $\kappa_2 > 0$, such that for $\kappa < \kappa_2$, $\Sigma_{AB1} > 0$. Finally, in the limit we consider

$$\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)} - V^A \\
&= \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu\left(\underline{\theta}\right)}.
\end{aligned}$$

At $\underline{\theta} = 1$,

$$\begin{aligned}
\frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2}\mu_0} &= \frac{\frac{r}{r+\frac{1}{2}\mu_0}\left(u_0 - R\right) + \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}u - \frac{\frac{1}{2}\mu_0}{r+\frac{1}{2}\mu_0}rV^A - (u-\chi) + R}{r + \frac{1}{2}\mu_0} \\
&= \frac{ru_0 + \frac{1}{2}\mu_0 u - \frac{1}{2}\mu_0\left(rV^A - R\right) - \left(r + \frac{1}{2}\mu_0\right)(u-\chi)}{\left(r + \frac{1}{2}\mu_0\right)^2}.
\end{aligned}$$

Substituting for Σ_{AB1}, we get

$$\Sigma_{S1B1} = \frac{r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu\chi - \frac{1}{2}\mu_0\left(rV^A - R\right)}{\left(r + \frac{1}{2}\mu_0\right)^2}.$$

Therefore, a sufficient condition for $\Sigma_{S1B1} < 0$ at $\underline{\theta} = 1$ is

$$r\left(u_0 + u_2 - 2\left(u - \chi\right)\right) + \mu_0\chi \leq 0,$$

or

$$r\left(u_2 - u_0\right) \geq 2\left[r\left(u_2 - \left(u - \chi\right)\right) + \frac{1}{2}\mu_0\chi\right],$$

which is our parametric assumption B3. Again by continuity of the value functions with respect to $\underline{\theta}$, we have that there is a $\kappa_3 > 0$, s.t. for $\kappa < \kappa_3$, $\Sigma_{S1B1} < 0$. Taking $\underline{\kappa} = \min\{\kappa_1, \kappa_2, \kappa_3\}$, we have that for $\kappa < \underline{\kappa}$, there is a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$.

"Buy first" equilibrium existence.

We follow the same two steps to show the existence of a "Buy first" equilibrium with $\theta = \overline{\theta} > 1$. Again, we make the same conjectures on the different surpluses as in the case of the "Sell first" equilibrium. In the limit economy, the fraction of buyers who are new entrants is

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{B_n}{B} = \frac{1}{\overline{\theta}},$$

where $\overline{\theta} = 1 + \kappa$. Also,

$$\lim_{g\to 0, \gamma\to 0, \frac{g}{\gamma}=\kappa} \frac{A}{S} = \frac{1}{\overline{\theta}}$$

as well. Therefore, similarly to the value functions in the "Sell first" equilibrium, we have that

$$\left(r + \frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^A = R + \frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V - V^{Bn}\right) + \frac{\overline{\theta} - 1}{\overline{\theta}}\left(V^{S2} - V^{B1}\right)\right),$$

and

$$\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{S2}=u_2+\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)V^{A}+\frac{1}{2}\mu\left(\overline{\theta}\right)V.$$

Therefore, as in the "Sell first" equilibrium,

$$V-V^{S2}=\frac{rV-u_2}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}-V^{A}.$$

Also, as in the "Sell first" equilibrium,

$$V^{Bn}-V^{B0}=\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Turning to the value functions of a mismatched buyer, we have that

$$rV^{B1}=u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\left(V^{S2}-V^{B1}-V^{A}\right)+(1-\frac{1}{\overline{\theta}})\left(V-V^{B1}\right)\right),$$

For the value function of a deviating agent who chooses to sell first, we have that

$$rV^{S1}=u-\chi+\frac{1}{2}\mu\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\Sigma_{S1Bn}\right),$$

since $\Sigma_{S1B1}<0$. Then,

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)V^{S1}=u-\chi+\frac{1}{2}q\left(\overline{\theta}\right)V+\frac{1}{2}q\left(\overline{\theta}\right)\frac{u_0-u_n}{r+\frac{1}{2}q\left(\overline{\theta}\right)}.$$

Therefore, the difference between $V^{B1}-V^{S1}$ satisfies

$$\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)\left(V^{B1}-V^{S1}\right)=\frac{1}{2}q\left(\overline{\theta}\right)\left(\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\right).$$

At $\overline{\theta}=1$, we have that

$$\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)}=0,$$

by Assumption B1. As $\overline{\theta}$ increases, we have that $\frac{1}{\overline{\theta}}\frac{u_2-u}{r+\frac{1}{2}\mu(\overline{\theta})}+\frac{u_n-u_0}{r+\frac{1}{2}q(\overline{\theta})}$ increases, so $V^{B1}>V^{S1}$ for $\overline{\theta}>1$. Therefore, it is not optimal for a mismatched owner to deviate and sell first in an equilibrium in which mismatched owners buy first and $\theta>1$.

Finally, we verify that our conjectures for the surpluses are correct. As in the "Sell first" case, $\Sigma_{S1B0}>0$ and $\Sigma_{S2B1}>0$. Also, as in the "Sell first" case, in the limit we consider,

$$\begin{aligned}
\Sigma_{AB1} &= V^{S2}-V^{B1}-V^{A}=V^{S2}-V^{A}-V+V \\
&- \frac{u-\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)}-\frac{\frac{1}{2}q\left(\overline{\theta}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\left[\frac{1}{\overline{\theta}}\left(V^{S2}-V^{A}-V\right)+V\right] \\
&= \frac{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\right)}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_2-u}{r+\frac{1}{2}\mu\left(\overline{\theta}\right)}+\frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(u_2-(u-\chi)\right)+\frac{1}{2}\mu\left(\overline{\theta}\right)\chi+\frac{1}{2}q\left(\overline{\theta}\right)\frac{\overline{\theta}-1}{\overline{\theta}}\left(u_2-u\right)}{\left(r+\frac{1}{2}\mu\left(\overline{\theta}\right)\right)\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)}.
\end{aligned}$$

Note that at $\overline{\theta}=1$, Σ_{AB1} in the "Buy first" case is the same as the "Sell first" case. Therefore, there is a $\kappa_4>0$, such that for $\kappa<\kappa_4$ and $\overline{\theta}=1+\kappa$, $\Sigma_{AB1}>0$. Similarly,

$$\begin{aligned}
\Sigma_{S1Bn} &= V-V^{Bn}+V^{B0}-V^{S1}=V-V^{S1}-\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{\chi}{r+\frac{1}{2}q\left(\overline{\theta}\right)}-\frac{r}{r+\frac{1}{2}q\left(\overline{\theta}\right)}\frac{u_n-u_0}{r+\frac{1}{2}q\left(\overline{\theta}\right)} \\
&= \frac{r\left(\chi+u_0-u_n\right)+\frac{1}{2}q\left(\overline{\theta}\right)\chi}{\left(r+\frac{1}{2}q\left(\overline{\theta}\right)\right)^2},
\end{aligned}$$

which at $\overline{\theta}=1$ is again the same as for the "Sell first" case. Therefore, there is a $\kappa_5>0$, such that for $\kappa<\kappa_5$, $\Sigma_{S1Bn}>0$. A similar argument to the one for the "Sell first" case also confirms that $\Sigma_{ABn}>0$, $\Sigma_{AB0}>0$, $\Sigma_{S2Bn}>0$ and $\Sigma_{S2B0}>0$. Finally,

$$
\begin{aligned}
\Sigma_{S1B1} &= V^{S2} - V^{B1} + V^{B0} - V^{S1} \\
&= V^{S2} - V^{B1} + \frac{rV^{B0} - (u-\chi) + R + \frac{1}{2} q\left(\overline{\theta}\right)\left(\overline{\theta} - 1\right)\left(V^{S2} - V^{B1} - V^{A}\right)}{r + \frac{1}{2} q\left(\overline{\theta}\right)} - V^{A} \\
&= \left(1 + \frac{\frac{1}{2} q\left(\overline{\theta}\right)\left(\overline{\theta} - 1\right)}{r + \frac{1}{2} q\left(\overline{\theta}\right)}\right) \Sigma_{AB1} + \frac{rV^{B0} - (u-\chi) + R}{r + \frac{1}{2} q\left(\overline{\theta}\right)}.
\end{aligned}
$$

At $\overline{\theta} = 1$, showing that $\Sigma_{S1B1} < 0$ in the "Buy first" case therefore follows the "Sell first" case, so that $\Sigma_{S1B1} < 0$ for $\kappa < \kappa_6$, for some $\kappa_6 > 0$. Taking $\overline{\kappa} = \min\{\kappa_4, \kappa_5, \kappa_6\}$, we have that for $\kappa < \overline{\kappa}$, there is a "Buy first" equilibrium with a market tightness given by $\overline{\theta} = 1 + \kappa$. Finally, taking $\kappa^* = \min\{\overline{\kappa}, \underline{\kappa}\}$, we arrive at the desired result. □

E. A model with competitive search

We first define a competitive search equilibrium for the economy described in Section 5.3. Let $(\mathcal{P}, \Theta)$ denote the active market segments in the economy, i.e. segments that attract a positive measure of buyers and sellers. The following equations describe the steady state value functions of agents. For new entrants we have:

$$
\rho V^{Bn} = u_n - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta)\left(-p + V - V^{Bn}\right)\right\}. \tag{55}
$$

Similarly, for a real estate firm, we have

$$
\rho V^{A} = R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ \mu(\theta)\left(p - V^{A}\right)\right\}. \tag{56}
$$

For mismatched owners that buy first, we have

$$
\rho V^{B1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{ q(\theta)\left(-p + V^{S2} - V^{B1}\right)\right\}\right\}, \tag{57}
$$

where the value function takes into account the possibility that a mismatched buyer may be better off not searching. Similarly, if the mismatched owner sells first, we have

$$\rho V^{S1} = u - \chi + \max\left\{0, \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{\mu(\theta)\left(p + V^{B0} - V^{S1}\right)\right\}\right\}. \tag{58}$$

A double owner solves

$$\rho V^{S2} = u_2 + R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{\mu(\theta)\left(p + V - V^{S2}\right)\right\}, \tag{59}$$

while a forced renter solves

$$\rho V^{B0} = u_0 - R + \max_{(p,\theta)\in(\mathcal{P},\Theta)} \left\{q(\theta)\left(-p + V - V^{B0}\right)\right\}. \tag{60}$$

Finally, for a matched owner we have

$$\rho V = u + \gamma\left(\max\left\{V^{B1}, V^{S1}\right\} - V\right). \tag{61}$$

Next, we describe the steady state stock-flow conditions. Let

$$\left(p^{Bn}, \theta^{Bn}\right) \in \left(\mathcal{P}^{Bn}, \Theta^{Bn}\right) \equiv \arg\max_{(p,\theta)} \left\{q(\theta)\left(-p + V - V^{Bn}\right)\right\} \subset (\mathcal{P}, \Theta) \tag{62}$$

denote a market segment that maximizes the value of searching for a new entrant. We define (p^j, θ^j) and $(\mathcal{P}^j, \Theta^j)$ analogously for an agent type $j \in \{A, B1, S1, B0, S2\}$. For agents $j \in \{B1, S1\}$, we adopt the convention that $\Theta^j = \emptyset$ if they choose not to search.

We have the following stock-flow conditions

$$g = \left(\sum_{\theta\in\Theta} x^{Bn}(\theta)\, q(\theta) + g\right) B_n, \tag{63}$$

$$\sum_{\theta\in\Theta} x^{S1}(\theta)\, \mu(\theta)\, S_1 = \left(\sum_{\theta\in\Theta} x^{B0}(\theta)\, q(\theta) + g\right) B_0, \tag{64}$$

$$\gamma x_b O = \left(\sum_{\theta \in \Theta} x^{B1}(\theta) q(\theta) + g \right) B_1, \tag{65}$$

$$\gamma x_s O = \left(\sum_{\theta \in \Theta} x^{S1}(\theta) \mu(\theta) + g \right) S_1, \tag{66}$$

$$\sum_{\theta \in \Theta} x^{B1}(\theta) q(\theta) B_1 = \left(\sum_{\theta \in \Theta} x^{S2}(\theta) \mu(\theta) + g \right) S_2, \tag{67}$$

$$g(O + B_1 + S_1 + 2S_2) = \sum_{\theta \in \Theta} x^A(\theta) \mu(\theta) A, \tag{68}$$

$$x_b + x_s = 1, \tag{69}$$

with

$$\sum_{\theta \in \Theta} x^j(\theta) = 1 \ \forall j \in \{Bn, A, B0, S2\}, \tag{70}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$ and, if a mismatched buyer/seller chooses to search,

$$\sum_{\theta \in \Theta} x^j(\theta) = 1 \text{ for } j \in \{B1, S1\}, \tag{71}$$

with $x^j(\theta) = 0$ if $\theta \notin \Theta^j$. In the above expressions $\mathbf{x}^j(\theta) \geq 0$ is the vector of mixing probabilities over segments in Θ for an agent $j \in \{Bn, A, B1, S1, B0, S2\}$. Market tightnesses in each segment are given by

$$\theta = \frac{x^{Bn}(\theta) B_n + x^{B1}(\theta) B_1 + x^{B0}(\theta) B_0}{x^A(\theta) A + x^{S2}(\theta) S_2 + x^{S1}(\theta) S_1}, \tag{72}$$

where $x^j(\theta) = 0$ if $\theta \notin \Theta^j$.

Finally, we have the population constancy and housing ownership conditions

$$B_n + B_0 + B_1 + S_1 + S_2 + O = 1, \tag{73}$$

and

$$O + B_1 + S_1 + A + 2S_2 = 1. \tag{74}$$

Following Moen (1997), we additionally require that the active market segments $(\mathcal{P}, \Theta)$ are such that the equilibrium allocation is a "no-surplus allocation". Formally, we make the following requirement.

No-surplus allocation Let $\mathcal{B} \subset \{Bn, B1, B0\}$ and $\mathcal{S} \subset \{A, S1, S2\}$ denote the sets of *active* buyers and sellers in a steady state equilibrium, that is agents that have a strictly positive measure in steady state. Given the set of active segments $(\mathcal{P}, \Theta)$ and agents' steady state value functions $\left\{V^{Bn}, V^{B1}, V^{B0}, V^{A}, V^{S1}, V^{S2}\right\}$, there exists no pair $(p, \theta) \notin (\mathcal{P}, \Theta)$, such that $V^i(p, \theta) > V^i$, for some $i \in \{Bn, B1, B0\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathcal{S}$, or $V^i(p, \theta) > V^i$, for some $i \in \{A, S1, S2\}$, and $V^j(p, \theta) \geq V^j$ for some $j \in \mathcal{B}$, where $V^i(p, \theta)$ denotes the steady state value function of an agent that trades in segment (p, θ), for $i \in \{Bn, B1, B0, A, S1, S2\}$.

Informally, the no-surplus allocation condition requires that in equilibrium there are no agents that would be strictly better off from deviating and opening a new market segment that would be at least as attractive for some active agents (buyers or sellers) compared to their equilibrium values.

We can now define a symmetric steady state competitive search equilibrium of this economy as follows

Definition 2. A symmetric steady state competitive search equilibrium of this economy consists of a set of active market segments $(\mathcal{P}, \Theta)$, steady state value functions V^{Bn}, V^{B0}, V^{B1}, V^{S2}, V^{S1}, V, V^{A}, fractions of mismatched owners that choose to buy first and sell first, x_b, and x_s, aggregate stock variables, B_n, B_0, B_1, S_1, S_2, O, and A, distributions of agent types over active market segments $\{\mathbf{x}^j\}_{j \in \{Bn, A, B1, S1, B0, S2\}}$, and set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, such that

1. The value functions satisfy equations (55) - (61) and the mixing distributions $\{\mathbf{x}^j\}_j$ are consistent with the agents' optimization problems.

2. Mismatched owners choose to buy first or sell first, to maximize $\overline{V} =$

$\max\left\{V^{B1}, V^{S1}\right\}$ and the fractions x_b, and x_s reflect that choice, i.e.

$$x_b = \int_i I\left\{x_i = b\right\} di,$$

where $i \in [0,1]$ indexes the i-th mismatched owner, and similarly for x_s;

3. The aggregate stock variables B_n, B_0, B_1, S_1, S_2, O, and A, solve (63)-(68) and (73)-(74) given Θ, $\left\{\mathbf{x}^j\right\}_j$ and mismatched owners' optimal decisions, reflected in x_b and x_s.

4. Every $\theta \in \Theta$ satisfies equation (72) given B_n, B_0, B_1, S_1, S_2, O, A, and $\left\{\mathbf{x}^j\right\}_j$;

5. The set of active buyers and sellers, $\mathcal{B}$ and $\mathcal{S}$, is consistent with mismatched owners' optimal decisions;

6. $(\mathcal{P}, \Theta)$ and agents' steady state value functions satisfy the no-surplus allocation condition.

Before proceeding with the proof of Proposition 3, we provide a more informal discussion of equilibrium characterization. In the "Buy first" equilibrium, the buyers are mismatched owners and new entrants, while the sellers are real estate firms and double owners as in figure 6a, where blue indicates sellers and red buyers. The most patient buyer is the mismatched owner, while the most impatient sellers are the double owners. Hence these agents always transact. The least patient buyers are the new entrants, while the most patient sellers are the real estate firms. Hence submarkets for real estate firms and new entrants will always exist. In addition, a market for new entrants and double owners will also open.

Note that the match surplus Σ_{B1S2} between a mismatched buyer and a double owner is given by

$$\Sigma_{B1S2} = V + V^{S2} - V^{B1} - V^{S2} = V - V^{B1} > 0,$$

so that there is trade in this market. Now consider a mismatched owner that deviates and sells first. For a small χ, this seller will be more patient than both the real estate firms and the double owners, and will, therefore, transact with the most impatient buyers among the non-deviating buyers, the new entrants. It will then become a new buyer type – a forced renter – that is even more impatient than the new entrant because $u_n > u_0$. It will, therefore, transact with the most patient sellers, which are the real estate firms. Given that a forced renter is more impatient than a new entrant, real estate firms are willing to open a new submarket for the deviating agent. In particular, the value from being a forced renter, V^{B0}, maximizes his gain from search given the value of the real estate firm.

The match surplus between the deviating mismatched owner and the new entrant, Σ_{BnS1}, can be written as

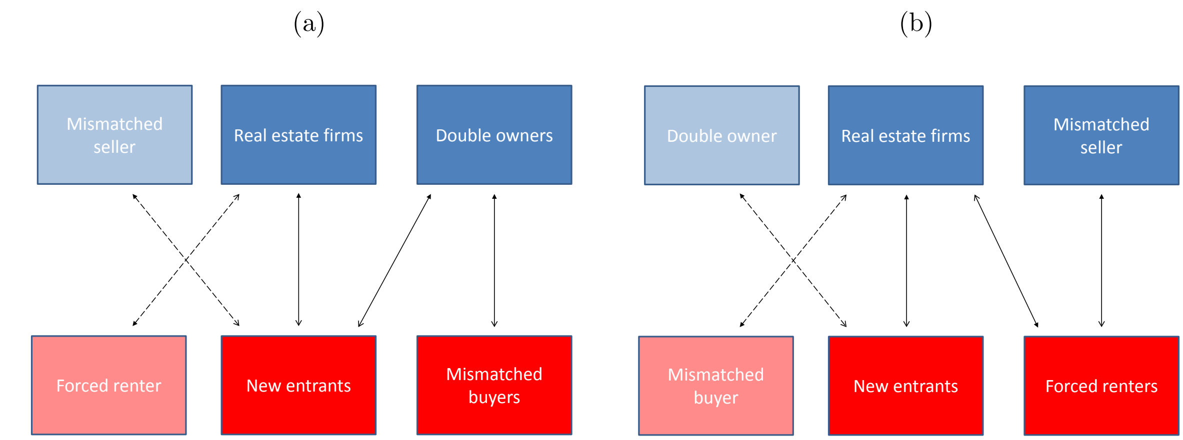
$$\Sigma_{BnS1} = V + V^{B0} - V^{Bn} - V^{S1}.$$

Note that also $\lim_{\chi \to 0} V^{S1} = \lim_{\chi \to 0} V$. Moreover, for $u_0 < u_n$, V^{B0} is strictly lower than V^{Bn}, also in the limit as $\chi \to 0$. As a result, the match surplus Σ_{BnS1} is negative for small values of χ, so that the mismatched owner cannot gain by deviating and the "Buy first" equilibrium exists.

In the "Sell first" equilibrium, the sellers are mismatched homeowners and real estate firms, while the buyers are new entrants and forced renters. The most patient buyers are the new entrants, and the most patient sellers – the mismatched owners. The active submarkets will be between new entrants and real estate firms , mismatched buyers and forced renters, and forced renters and real estate firms. The markets (together with a deviating agent) are illustrated in Figure 6b.

The asset values in the real-estate-new agents are as in the "Buy first" equilibrium, so that this market is active. For a real estate agent, the surplus of a transaction with a (more impatient) forced renter is even larger than with a new entrant, so that there are benefits of trade in this market as well. Hence, a forced renter obtains the same value V^{B0} as the (deviating) forced renter in

Figure 6: Equilibrium market segments (solid colors) and deviators (weaker colors) for “Buy first” (a) and “Sell first” (b) competitive search equilibria.



the “Buy first” equilibrium. The match surplus between a forced renter and a mismatched seller, Σ_{B0S1}, is given by

$$\Sigma_{B0S1} = V + V^{B0} - V^{S1} - V^{B0} = V - V^{S1} > 0.$$

Now, consider a mismatched agent that deviates and buys first. This buyer will be more patient than both new entrants and forced renters, and will thus transact with the real estate firm, the most impatient seller. He will then become a double owner, thus, becoming more impatient than the real estate firm, and will therefore transact with the new entrants. A new submarket will open up, and the expression for the asset value V^{S2} is the same as for the double owner in the “Buy first” equilibrium. However, for the same reasons as in the “Buy first” equilibrium, the match surplus between the mismatched buyer and the real estate firm, Σ_{B1A}, is negative for low values of u_2. It follows that the deviation is unprofitable. We conclude that the model still exhibits multiple equilibria, as stated in the following proposition.

Proof of Proposition 3

Consider first the "Buy first" equilibrium as described above. In this "Buy first" equilibrium there are three active market segments characterized by prices $p_1^{B1} > p_2^{B1} > p_3^{B1}$ and market tightnesses $\theta_1^{B1} < \theta_2^{B1} < \theta_3^{B1}$. New entrants trade with real estate agents in market 1 and with double owners in market 2, while the latter also trade with mismatched buyers in market 3. Let x^{Bn} denote the probability with which a new entrant visits segment $\left(p_1^{B1}, \theta_1^{B1}\right)$, and x^{S2} the probability with which a double owner visits segment $\left(p_2^{B1}, \theta_2^{B1}\right)$. The stock-flow conditions for this equilibrium are

$$B_n = \frac{g}{x^{Bn} q\left(\theta_1^{B1}\right) + \left(1 - x^{Bn}\right) q\left(\theta_2^{B1}\right) + g}, \tag{75}$$

$$A = \frac{g}{\mu\left(\theta_1^{B1}\right) + g}, \tag{76}$$

$$B_1 = \frac{\gamma O}{q\left(\theta_3^{B1}\right) + g}, \tag{77}$$

$$S_2 = \frac{q\left(\theta_3^{B1}\right) B_1}{x^{S2} \mu\left(\theta_2^{B1}\right) + \left(1 - x^{S2}\right) \mu\left(\theta_3^{B1}\right) + g}, \tag{78}$$

$$B_n + B_1 + S_2 + O = 1, \tag{79}$$

and

$$B_n = A + S_2. \tag{80}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{B1} = \frac{x^{Bn} B_n}{A}, \tag{81}$$

$$\theta_2^{B1} = \frac{\left(1 - x^{Bn}\right) B_n}{x^{S2} S_2}, \tag{82}$$

and

$$\theta_3^{B1} = \frac{B_1}{\left(1 - x^{S2}\right) S_2}. \tag{83}$$

Observe that (75), (76), (80), and (81) imply that $x^{Bn} < 1$, as otherwise, (75), (76) and (81) give

$$\theta_1^{B1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{B1}\right) + g}{q\left(\theta_1^{B1}\right) + g}, \tag{84}$$

which has a unique solution at $\theta_1^{B1} = 1$. However, this is inconsistent with (80).

Let Σ_{ij}, for $i \in \{B_n, B_0, B_1\}$ and $j \in \{A, S_1, S_2\}$ denote the match surplus from trading between a buyer i and seller j. The no-surplus allocation condition determines the equilibrium prices in each segment as a function of the steady state values of agents. Define

$$\begin{aligned}\overline{V}^{Bn} &= q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) \\ &= q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),\end{aligned}$$

$$\overline{V}^{A} = \mu\left(\theta_1^{B1}\right)\left(p_1^{B1} - V^{A}\right),$$

$$\overline{V}^{B1} = q\left(\theta_3^{B1}\right)\left(-p_3^{B1} + V^{S2} - V^{B1}\right),$$

and

$$\begin{aligned}\overline{V}^{S2} &= \mu\left(\theta_2^{B1}\right)\left(p_2^{B1} + V - V^{S2}\right) \\ &= \mu\left(\theta_3^{B1}\right)\left(p_3^{B1} + V - V^{S2}\right),\end{aligned}$$

as the maximized value of searching for each trader. The no-surplus allocation condition implies that

$$\left(p_1^{B1}, \theta_1^{B1}\right) = \arg\max_{p,\theta} \mu\left(\theta\right)\left(p - V^{A}\right),$$

$$\text{s.t. } q\left(\theta\right)\left(-p + V - V^{Bn}\right) \geq \overline{V}^{Bn}.$$

Denote the elasticity of the matching function with respect to buyers by α (which may depend on θ). Solving for p_1^{B1} and θ_1^{B1} gives the well-known Hosios rule (Hosios (1990)),

$$p_1^{B1} - V^{A} = (1-\alpha)\,\Sigma_{BnA},$$

or equivalently,

$$p_1^{B1} = (1-\alpha)\left(V - V^{Bn}\right) + \alpha V^A.$$

Therefore,

$$\overline{V}^{Bn} = \alpha q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2},$$

or

$$q\left(\theta_1^{B1}\right)\Sigma_{BnA} = q\left(\theta_2^{B1}\right)\Sigma_{BnS2}. \tag{85}$$

We have similar surplus sharing rules between the other trading pairs, which determine p_2^{B1} and p_3^{B1}. There is one more indifference condition for a double owner that relates θ_2^{B1} and θ_3^{B1}. Specifically,

$$\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \mu\left(\theta_3^{B1}\right)\Sigma_{B1S2}. \tag{86}$$

These surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{87}$$

$$\rho V^A = R + (1-\alpha)\mu\left(\theta_1^{B1}\right)\Sigma_{BnA}, \tag{88}$$

$$\rho V^{B1} = u - \chi + \alpha q\left(\theta_3^{B1}\right)\Sigma_{B1S2}, \tag{89}$$

$$\rho V^{S2} = u_2 + R + (1-\alpha)\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2}, \tag{90}$$

and

$$\rho V = u + \gamma\left(V^{B1} - V\right). \tag{91}$$

Finally, use V^{Bn} and V^{S2} from (87) and (90) to solve for

$$\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\rho + \alpha q\left(\theta_2^{B1}\right) + (1-\alpha)\mu\left(\theta_2^{B1}\right)}. \tag{92}$$

Similarly, using V^{Bn} and V^A from (87) and (56), combined with indifference condition (85), to solve for

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{B1}\right) + (1-\alpha)\,\mu\left(\theta_1^{B1}\right)}. \tag{93}$$

Solving for V^{B1} from equation (89), we get

$$V^{B1} = \frac{u-\chi}{\rho + \alpha q\left(\theta_3^{B1}\right)} + \frac{\alpha q\left(\theta_3^{B1}\right)}{\rho + \alpha q\left(\theta_3^{B1}\right)} V,$$

so

$$\Sigma_{B1S2} = V - V^{B1} = \frac{\rho V - (u-\chi)}{\rho + \alpha q\left(\theta_3^{B1}\right)}. \tag{94}$$

Therefore, equations (75)-(83), combined with the two indifference conditions (85) and (86), and the value function equations (87)-(91) with surpluses (92)-(94) jointly determine the equilibrium stocks of agents, market tightnesses, mixing probabilities x^{Bn} and x^{S2}, and active agent value functions in a "Buy first" equilibrium.

We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n. Note that $\Sigma^{S2B1} = V - V^{B1} > 0$ for any u_2, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{B1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0}\Sigma_{B1S2} = 0$. This in turn implies that $\lim_{\chi\to 0}\theta_3^{B1} = \infty$ and $\lim_{\chi\to 0} x^{S2} = 1$. To see this, suppose to the contrary that as $\chi \to 0$, θ_3^{B1} remains bounded and thus x^{S2} is strictly below one. Therefore, $\mu\left(\theta_3^{B1}\right)\Sigma_{B1S2} \to 0$, so indifference condition (86) implies that $\mu\left(\theta_2^{B1}\right)\Sigma_{BnS2} \to 0$. Given (92), this in turn means that $\theta_2^{B1} \to 0$ and thus $x^{Bn} \to 1$. However,

$$\lim_{\theta_2^{B1}\to 0} q\left(\theta_2^{B1}\right)\Sigma_{BnS2} = \frac{2\rho V - u_n - u_2}{\alpha},$$

which is inconsistent with $x^{Bn} \to 1$. To see this, remember from (84) that $\theta_1^{B1} \to 1$ as $x^{Bn} \to 1$. Because

$$\lim_{\theta_1^{B1}\to 1} q\left(\theta_1^{B1}\right)\Sigma_{BnA} = \frac{\rho V - u_n}{\frac{\rho}{q(1)} + 1} < \rho V - u_n < \frac{2\rho V - u_n - u_2}{\alpha},$$

in this case new entrants would be strictly better off participating in the second market segment. Thus, we arrive at a contradiction.

As $\theta_3^{B1} \to \infty$, $q\left(\theta_3^{B1}\right) \to 0$, and mismatched owners do not buy to become double owners: $S_2 \to 0$. Without trading partners in market 2, all new entrants visit market 1: $x^{Bn} \to 1$ and thus $\theta_1^{B1} \to \frac{B_n}{A} \to 1$. In this case, V^{Bn} from (87) is given by

$$\lim_{\chi \to 0} \rho V^{Bn} = u_n + \frac{\alpha q(1)}{\rho + q(1)}(u - u_n) - R, \tag{95}$$

which is strictly between 0 and ρV, as long as R is not too large. Similarly, using that $\mu(1) = q(1)$, V^A is given by

$$\lim_{\chi \to 0} \rho V^A = R + \frac{(1-\alpha) q(1)}{\rho + q(1)}(u - u_n), \tag{96}$$

which is also strictly between 0 and ρV if R is not too large. As a result,

$$\lim_{\chi \to 0} \left(V^A + V^{Bn}\right) = \frac{u_n}{\rho} + \frac{q(1)}{\rho + q(1)} \frac{u - u_n}{\rho},$$

which is strictly between 0 and V. By continuity, there exists a $\overline{\chi}_1 > 0$ such that for $\chi \in (0, \overline{\chi}_1)$, it is the case that $\Sigma_{BnA} > 0$, $x^{Bn} \in (0,1)$ and $\theta_1^{B1} \in (0,1)$, but also $\Sigma_{B1S2} > 0$.

With V^{Bn} as defined in (95) above, it follows that V^{S2} is uniquely determined as

$$\rho V^{S2} = \max_{p,\theta} \left\{u_2 + R + \mu(\theta)(V + p - V^{S2})\right\},$$

subject to $u_n - R + q(\theta)(V - p - V^{Bn}) = \rho V^{Bn}$. Note that V^{S2} goes to negative infinity for any $\chi > 0$ when u_2 does. To see this, suppose to the contrary that V^{S2} remains bounded when u_2 goes to negative infinity. Then $\mu(\theta)$ must go to infinity, and hence θ must go to infinity. But then $q(\theta)$ goes to zero, and for the new entrants to get their outside option, p must go to $-\infty$. In this case V^{S2} still goes to negative infinity, so that we arrive at a contradiction. Consequently, for any $\chi > 0$ there exists a $\overline{u}_2^{B1}$ such that for $u_2 < \overline{u}_2^{B1}$, V^{S2} is sufficiently low such that both $\Sigma_{B1A} = V^{S2} - V^A - V^{B1} < 0$ and $\Sigma_{BnS2} = 2V - V^{Bn} - V^{S2} > \Sigma_{BnA} > 0$. We can then conclude that there is trade in markets 1, 2, and 3, but that real estate agents and mismatched buyers do not open a fourth market.[29]

Furthermore, it follows that for any $u_2 < \overline{u}_2^{B1}$ there exists a $\overline{\chi}_2 > 0$ such that for $\chi \in (0, \overline{\chi}_2)$, it is the case that $\Sigma_{BnS2} > \Sigma_{B1S2}$. Given this ranking and the fact that $\Sigma_{BnA} < \Sigma_{BnS2}$, the ranking of tightnesses across segments then follows from the indifference conditions (85) and (86). Having established the ranking of tightnesses, the ranking of prices across segments immediately follows from the indifference conditions as well. Specifically, (85) implies that

$$q\left(\theta_1^{B1}\right)\left(-p_1^{B1} + V - V^{Bn}\right) = q\left(\theta_2^{B1}\right)\left(-p_2^{B1} + V - V^{Bn}\right),$$

or

$$\frac{q\left(\theta_1^{B1}\right)}{q\left(\theta_2^{B1}\right)} = \frac{-p_2^{B1} + V - V^{Bn}}{-p_1^{B1} + V - V^{Bn}}.$$

$\theta_1^{B1} < \theta_2^{B1}$ and $q(.)$ decreasing imply that $p_1^{B1} > p_2^{B1}$. Similarly, (86) implies that $p_2^{B1} > p_3^{B1}$. Finally, note that $\Sigma_{BnS2} > \Sigma_{B1S2}$ implies that $V - V^{Bn} > V^{S2} - V^{B1}$, so a new entrant is more impatient than a mismatched buyer in the sense that the direct utility gain from transacting is higher for a new entrant compared to a mismatched buyer.[30]

Consider now a mismatched owner that deviates and sells first, and upon trade becomes a forced renter. We allow both a mismatched seller and a forced renter to open new market segments with active agents as counterparties. First, observe that $V^{Bn} > V^{B0}$ for any $\chi > 0$, that is, a new entrant is always better off than a forced renter. This ranking comes from the assumption that $u_0 < u_n$ and from a revealed preference argument. Specifically, suppose to the contrary that $V^{B0} > V^{Bn}$. Suppose also that it is optimal for a forced renter to trade with a real estate firm (the argument for the case where the forced renter trades with a double owner is analogous). The no-surplus allocation

[29]For market 2 to be active, it is sufficient for u_2 to be low enough to ensure $\Sigma_{BnS2} > 0$, even if $\Sigma_{BnS2} < \Sigma_{BnA}$. However, $u_2 < \overline{u}_2^{B1}$ ensures the ranking of tightnesses and prices proven next.

[30]This also implies that a new entrant has steeper sloped indifference curves in the $\theta - p$ space, so he is willing to trade-off a higher price for the same decrease in market tightness compared to a mismatched buyer.

condition again implies that the Hosios condition holds, so

$$\rho V^{B0} = u_0 - R + \alpha q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right),$$

where $\tilde{\theta}$ is such that a real estate firm is indifferent between trading in this new segment and trading in the segment with a tightness of θ_1^{B1} and a price of p_1^{B1}. In contrast, we have that

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right).$$

Since $u_0 < u_n$ but $V^{B0} > V^{Bn}$, it follows that $q\left(\tilde{\theta}\right)\left(V - V^{B0} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$ and so $\tilde{\theta} < \theta_1^{B1}$. But then a new entrant is better off deviating and trading in the segment with tightness $\tilde{\theta}$, since $q\left(\tilde{\theta}\right)\left(V - V^{Bn} - V^{A}\right) > q\left(\theta_1^{B1}\right)\left(V - V^{Bn} - V^{A}\right)$. Furthermore, given that $V^{B0} > V^{Bn}$, $\Sigma_{BnA} > \Sigma_{B0A}$, so a real estate firm is in fact also strictly better off trading with a new entrant in the segment with tightness $\tilde{\theta}$. However, this is not consistent with $\left(p_1^{B1}, \theta_1^{B1}\right)$ not violating the no-surplus allocation condition. Therefore, in an equilibrium where $\left(p_1^{B1}, \theta_1^{B1}\right)$ are consistent with the no-surplus allocation, we must have $q\left(\tilde{\theta}\right) < q\left(\theta_1^{B1}\right)$. However, this means that $V^{B0} < V^{Bn}$, and we arrive at a contradiction.

We conclude that $V^{B0} > V^{Bn}$ and $\Sigma_{BnA} < \Sigma_{B0A}$, so that a forced renter is the most impatient of the buyers. The forced renter will therefore trade with a real estate agent, the most patient of the sellers. A new submarket opens up, and real estate firms flow into this submarket up to the point where they are indifferent between selling to the deviator and to a new agent. Now suppose the deviating mismatched owner sells to a new entrant. Then the match surplus reads

$$\Sigma_{BnS1} = V - V^{Bn} + V^{B0} - V^{S1} \leq V - V^{Bn} + V^{B0} - \frac{u - \chi}{\rho}.$$

Given that $V^{Bn} - V^{B0}$ is bounded away from zero for any $\chi > 0$, there exists a $\overline{\chi}_3 > 0$ such that for $\chi \in (0, \overline{\chi}_3)$, it is the case that $\Sigma_{BnS1} < 0$. Note, however, that $\Sigma_{BnS1} > \Sigma_{B1S1}$ for $\chi < \overline{\chi}_2$, since, as shown above, in that case $V - V^{Bn} > V^{S2} - V^{B1}$, meaning that a new entrant is more impatient than a mismatched buyer. Therefore, for $\chi < \min\{\overline{\chi}_1, \overline{\chi}_2, \overline{\chi}_3\}$, $\Sigma_{B1S1} < \Sigma_{BnS1} < 0$ and $\Sigma_{B1S2} > 0$. In that case a mismatched owner that deviates and sells first is better off not trading. However, not trading is dominated by buying first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from buying first in a "Buy first" equilibrium.

Constructing a "Sell first" equilibrium follows similar steps. In this equilibrium there are three active market segments characterized by prices $p_1^{S1} < p_2^{S1} < p_3^{S1}$ and market tightnesses $\theta_1^{S1} > \theta_2^{S1} > \theta_3^{S1}$. Real estate agents trade with new entrants in market 1 and with forced renters in market 2, while the latter also trade with mismatched sellers in market 3. Let x^A denote the probability with which a real estate firm visits segment $\left(p_1^{S1}, \theta_1^{S1}\right)$, and x^{B0} the probability with which a forced renter visits segment $\left(p_2^{S1}, \theta_2^{S1}\right)$. The stock-flow conditions in this case become

$$B_n = \frac{g}{q\left(\theta_1^{S1}\right) + g}, \tag{97}$$

$$A = \frac{g}{x^A \mu\left(\theta_1^{S1}\right) + \left(1 - x^A\right)\mu\left(\theta_2^{S1}\right) + g}, \tag{98}$$

$$S_1 = \frac{\gamma O}{\mu\left(\theta_3^{S1}\right) + g}, \tag{99}$$

$$B_0 = \frac{\mu\left(\theta_3^{S1}\right) S_1}{x^{B0} q\left(\theta_2^{S1}\right) + \left(1 - x^{B0}\right) q\left(\theta_3^{S1}\right) + g}, \tag{100}$$

$$B_n + B_0 + S_1 + O = 1, \tag{101}$$

and

$$B_n + B_0 = A. \tag{102}$$

The market tightnesses in each active segment satisfy

$$\theta_1^{S1} = \frac{B_n}{x^A A}, \tag{103}$$

$$\theta_2^{S1} = \frac{x^{B0} B_0}{(1 - x^A) A}. \quad (104)$$

and

$$\theta_3^{S1} = \frac{\left(1 - x^{B0}\right) B_0}{S_1}, \quad (105)$$

Similarly to before, observe that (97), (98), (102), and (105) imply that $x^A < 1$, as otherwise, (97), (98) and (105) give

$$\theta_1^{S1} = \frac{B_n}{A} = \frac{\mu\left(\theta_1^{S1}\right) + g}{q\left(\theta_1^{S1}\right) + g},$$

which has a unique solution at $\theta_1^{S1} = 1$. However, this is inconsistent with (102). As before, the no-surplus allocation implies that the match surpluses between trading pairs are split according to the Hosios rule. Consequently, there are two indifference conditions for real estate firms and forced renters given by

$$\mu\left(\theta_1^{S1}\right) \Sigma_{BnA} = \mu\left(\theta_2^{S1}\right) \Sigma_{B0A}, \quad (106)$$

and

$$q\left(\theta_2^{S1}\right) \Sigma_{B0A} = q\left(\theta_3^{S1}\right) \Sigma_{B0S1}, \quad (107)$$

respectively. In addition, the surplus sharing rules imply that the value functions of active agents satisfy the equations

$$\rho V^{Bn} = u_n - R + \alpha q\left(\theta_1^{S1}\right) \Sigma_{BnA}, \quad (108)$$

$$\rho V^{A} = R + (1 - \alpha) \mu\left(\theta_1^{S1}\right) \Sigma_{BnA}, \quad (109)$$

$$\rho V^{S1} = u - \chi + (1 - \alpha) q\left(\theta_3^{S1}\right) \Sigma_{B0S1}, \quad (110)$$

$$\rho V^{B0} = u_0 - R + \alpha\mu\left(\theta_3^{S1}\right) \Sigma_{B0S1}, \quad (111)$$

and

$$\rho V = u + \gamma\left(V^{S1} - V\right). \quad (112)$$

Finally, the above value functions allow us to solve for the surpluses as follows:

$$\Sigma_{BnA} = V - V^{Bn} - V^{A} = \frac{\rho V - u_n}{\rho + \alpha q\left(\theta_1^{S1}\right) + (1-\alpha)\,\mu\left(\theta_1^{S1}\right)}. \tag{113}$$

$$\Sigma_{B0S1} = V - V^{S1} = \frac{\rho V - (u-\chi)}{\rho + (1-\alpha)\,\mu\left(\theta_3^{S1}\right)}, \tag{114}$$

and

$$\Sigma_{B0A} = \frac{\rho V - u_0}{\rho + \alpha q\left(\theta_2^{S1}\right) + (1-\alpha)\,\mu\left(\theta_2^{S1}\right)}. \tag{115}$$

The stock-flow and market tightness equations (97)-(105), combined with the two indifference conditions (106) and (107), and value functions and surpluses (108)-(115) fully characterize the equilibrium stocks of agents, market tightnesses, mixing probabilities x^A and x^{B0}, and active agent value functions in a "Sell first" equilibrium. We now prove existence of this equilibrium when χ and u_2 are small, and u_0 is strictly smaller than u_n.

Note that $\Sigma_{B0S1} = V - V^{S1} > 0$ for any u_0, but that $\lim_{\chi\to 0} V = \lim_{\chi\to 0} V^{S1} = \frac{u}{\rho}$, so that $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. Then, a set of arguments similar to the case of the "Buy first" equilibrium shows that $\lim_{\chi\to 0} \theta_3^{B1} = 0$ and $\lim_{\chi\to 0} x^{B0} = 1$, so that $\lim_{\chi\to 0} \mu\left(\theta_3^{B1}\right) = 0$ and $\lim_{\chi\to 0} B_0 = 0$, implying that $\lim_{\chi\to 0} x^A = 1$ and $\lim_{\chi\to 0} \theta_1^{B1} = 1$. As a result, $\lim_{\chi\to 0} V^{Bn}$ and $\lim_{\chi\to 0} V^A$ are the same as in the "Buy first" equilibrium, and there exists a $\overline{\chi}_4 > 0$ such that for $\chi \in (0, \overline{\chi}_4)$, it is the case that $\Sigma_{BnA} > 0$, $x^A \in (0,1)$ and $\theta_1^{S1} > 1$ but remains bounded, while $\Sigma_{B0S1} > 0$. As a result, markets 1 and 3 are active.

Following the same arguments as in the "Buy first" equilibrium, it is then the case that $V^{Bn} > V^{B0}$, also as $\chi \to 0$, so that $0 < \Sigma_{BnA} < \Sigma_{B0A}$ and market 2 is active. Furthermore, there must exist a $\overline{\chi}_5 > 0$ such that for $\chi \in (0, \overline{\chi}_5)$, it is the case that $\Sigma_{B0A} > \Sigma_{B0S1}$, since $\lim_{\chi\to 0} \Sigma_{B0S1} = 0$. This ranking implies that $-V^A > V^{B0} - V^{S1}$, so that a real estate firm is more impatient than a mismatched seller in the sense that the direct utility gain from transacting is higher for a real estate firm compared to a mismatched seller. The ranking of tightnesses and prices across segments then follows from the indifference conditions (106) and (107), similar to the case of a "Buy first" equilibrium.

The fact that $V^{Bn} - V^{B0}$ is bounded away from zero also implies that there exists a exist a $\overline{\chi}_6 > 0$ such that for $\chi \in (0, \overline{\chi}_6)$, a mismatched owner and a new entrant will not open a fourth market, because $\lim_{\chi \to 0} V = \lim_{\chi \to 0} V^{S1} = \frac{u}{\rho}$ and thus $\Sigma_{BnS1} = V - V^{S1} + V^{B0} - V^{Bn} < 0$ for a sufficiently small χ.

Now consider a mismatched owner that deviates and buys first. Potential sellers are real estate firms and mismatched homeowners, and upon trade the deviator becomes a double owner, who can open up new market segments with new entrants and forced renters. Note that V^{S2} falls without bounds as u_2 does, because a deviating double owner has to offer new entrants or forced renters their market value, following a similar argument as in the "Buy first" equilibrium. Then there exists a $\overline{u}_2^{S1}$ such that for all $u_2 < \overline{u}_2^{S1}$ it is the case that $\Sigma_{B1A} = V^{S2} - V^A - V^m < 0$, so that a deviating mismatched owner does not buy from a real estate agent. Note, however, that $\Sigma_{B1S1} < \Sigma_{B1A} < 0$ for $\chi < \overline{\chi}_5$ since, as shown above, in that case $-V^A > V^{B0} - V^{S1}$, meaning that a new entrant is more impatient than a mismatched buyer. As a result, a mismatched owner that deviates and buys first is better off not trading. However, not trading is dominated by selling first since $V^{B1} > \frac{u-\chi}{\rho}$. Therefore, a mismatched owner is never better off deviating from selling first in a "Sell first" equilibrium.

Finally, setting $\overline{\chi} = \min\{\overline{\chi}_1, \overline{\chi}_2, \overline{\chi}_3, \overline{\chi}_4, \overline{\chi}_5, \overline{\chi}_6\}$ and $\overline{u}_2 = \min\left\{\overline{u}_2^{B1}, \overline{u}_2^{S1}\right\}$, we arrive at our result. □

F. Additional extensions

Simultaneous Entry as Buyer and Seller

We assume that a mismatched owner can allocate a fixed amount of time (normalized to 1 unit) to search in the housing market as a buyer or a seller. A mismatched owner that chooses to enter as a buyer or seller only allocates all of his time to one activity. Otherwise, a mismatched owner that enters as both a buyer and a seller can allocate a fraction $\phi \in (0, 1)$ of his time to searching as buyer, and searches the remaining $1 - \phi$ of his time as seller. For a given market tightness θ, the value function V^{SB} for a mismatched owner that enters as both buyer and seller satisfies the following equation in a steady state equilibrium:

$$\rho V^{SB} = u - \chi + (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} + \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}.$$

We then show the following

Proposition 6. *For $\theta \in \left(0, \tilde{\theta}\right)$, $V^{S1} > V^{SB}$, for any $\phi \in (0,1)$. Also, for $\theta \in \left(\tilde{\theta}, \infty\right)$, $V^{B1} > V^{SB}$, for any $\phi \in (0,1)$.*

Proof. To show the first part, suppose the opposite, so $V^{S1} \leq V^{SB}$. Then

$$\begin{aligned} \mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}. \end{aligned}$$

Under the assumption that $V^{S1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{B1} < V^{S1}$ for $\theta \in \left(0, \tilde{\theta}\right)$, it must then be the case that

$$\begin{aligned} \mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{B1}\right\}, \end{aligned}$$

which does not hold because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > 0$ for $\theta \in \left(0, \tilde{\theta}\right)$ by Assumption A2, and because $\mu(\theta)\left(p + V^{B0} - V^{S1}\right) > q(\theta)\left(-p + V^{S2} - V^{B1}\right)$ for $\theta \in \left(0, \tilde{\theta}\right)$, as in Lemma 1.

To show the second part, suppose the opposite, so $V^{B1} \leq V^{SB}$. Then

$$\begin{aligned} q(\theta)\max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{SB}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{SB}\right\}. \end{aligned}$$

Under the assumption that $V^{B1} \leq V^{SB}$, and since we know from Lemma 1 that $V^{S1} < V^{B1}$ for $\theta \in \left(\tilde{\theta}, \infty\right)$, it must then be the case that

$$\begin{aligned} q(\theta)\max\left\{0, p + V^{S2} - V^{B1}\right\} &\leq (1-\phi)\mu(\theta)\max\left\{0, p + V^{B0} - V^{S1}\right\} \\ &+ \phi q(\theta)\max\left\{0, -p + V^{S2} - V^{B1}\right\}, \end{aligned}$$

which does not hold because $q(\theta)\left(-p+V^{S2}-V^{B1}\right)>0$ for $\theta\in\left(\tilde{\theta},\infty\right)$ by Assumption A2, and because $\mu(\theta)\left(p+V^{B0}-V^{S1}\right)<q(\theta)\left(-p+V^{S2}-V^{B1}\right)$ for $\theta\in\left(\tilde{\theta},\infty\right)$, as in Lemma 1.[31] $\square$

Finally, note that under payoff symmetry (i.e. $\tilde{u}_0=\tilde{u}_2=c$) the possibility to enter as both buyer and seller while allocating each an equal amount of time can result in an equilibrium with a market tightness of $\theta=1$. Specifically, at $\theta=1$, $\mu(\theta)=q(\theta)=\mu(1)$. At these flow rates it can easily be seen that if $\tilde{u}_0=\tilde{u}_2=c$, then $V^{B1}=V^{S1}=V^{SB}$ for any ϕ. Finally, a tightness of $\theta=1$ can result from mismatched owners entering as buyers and sellers simultaneously and allocating each an equal amount of time (so $\phi=0.5$).

This is analogous to the equilibrium described in Proposition 1, with the only difference that now agents follow symmetric strategies compared to asymmetric strategies with one half of mismatched owners buying first and the other half selling first.

Homeowners compensated for their housing unit upon exit

Suppose that upon exit homeowners receive bids for their housing unit(s) from a set of competitive real estate firms. Therefore, given that the value of a housing unit to a real estate firm is $V^A(\theta)$, homeowners receive $V^A(\theta)$ for each housing unit that they own. Again, we consider a steady state equilibrium with a fixed market tightness θ. We define $\tilde{u}_0(\theta,g)\equiv u_0+\triangle-gV^A(\theta)$ and $\tilde{u}_2(\theta,g)=u_2-\triangle+gV^A(\theta)$. Note that $V^A(\theta)$ is (weakly) increasing in θ, so $\tilde{u}_2$ is increasing in θ and $\tilde{u}_0$ is decreasing in θ;

Given this definition, the difference between the values from buying first and selling first (assuming a mismatched owner transacts in both cases), $D(\theta)\equiv V^{B1}-V^{S1}$, can be written as

[31] Note also that for $\theta=0$ and $\theta\to\infty$, mismatched owners are indifferent between remaining mismatched and any search strategy, because $V^{B1}=V^{S1}=V^{SB}=\frac{u-\chi}{\rho}$, but that such tightnesses cannot occur in steady state by Lemma 2.

$$D(\theta) = \frac{\mu(\theta)}{(\rho + q(\theta))(\rho + \mu(\theta))} \left[\left(1 - \frac{1}{\theta}\right)(u - \chi - \tilde{u}_2(\theta, g)) - \tilde{u}_0(\theta, g) + \tilde{u}_2(\theta, g) \right].$$

Let $\tilde{\theta}$ be defined implicitly by

$$\tilde{\theta} \equiv \frac{u - \chi - \tilde{u}_2\left(\tilde{\theta}, g\right)}{u - \chi - \tilde{u}_0\left(\tilde{\theta}, g\right)},$$

whenever that equation has a solution.[32] Note that in the limit as $g \to 0$, assumption A1 will hold. Therefore, for g sufficiently close to zero, we will have that $u - \chi > \max\{\tilde{u}_0(\theta, g), \tilde{u}_2(\theta, g)\}$, for all $\theta \in \left[\underline{\theta}, \overline{\theta}\right]$, and so a version of Lemma 1 will hold in this case as well. Given this result one can then easily construct multiple steady state equilibria as in Proposition 1.

A fixed price as the outcome of take-it-or-leave-it offers under private information

In this section we show that a fixed price equal to the present discounted value of rental income can be microfounded as the outcome of bargaining under private information about types, with full bargaining power for buyers. Suppose therefore in this section that buyers make take-it-or-leave-it offers, but do not know the type of the seller. However, buyers do know the fractions of the types in the economy. Because of heterogeneity among sellers, their reservation prices vary. Matching is still random, so that buyers cannot direct their search to the seller type with the lowest reservation price but meet a particular seller type with a probability equal to their proportion in the population of sellers. The question is then whether buyers, upon meeting a seller, make an offer that only sellers with a low reservation price would accept (and thus trade only if

[32]Note that the above equation for $\tilde{\theta}$, whenever it has a solution, has a unique solution for any $g \geq 0$, since given the properties of $\tilde{u}_0$ and $\tilde{u}_2$, it follows that the right hand side of this expression is (weakly) decreasing in θ. Furthermore, the right hand side is strictly decreasing in g for any $\theta > 0$, so by the implicit function theorem, $\tilde{\theta}$ is decreasing in g.

they have met a seller of this type), or make an offer that all sellers would accept (and therefore trade for sure).

We consider the symmetric case with $\tilde{u}_0 = \tilde{u}_2 = c$ (which for $p = \frac{R}{\rho}$ amounts to $u_0 = u_2 = c$), so that $\tilde{\theta} = 1$. In addition, we maintain Assumptions A1 and A2 and assume that $u_n < u - \chi$, so that both mismatched owners and new entrants are strictly better off to enter the market. As in the model with symmetric Nash bargaining, we focus on steady state equilibria with value functions, market tightness θ, and the stocks of different agent types constant over time. Moreover, although results hold more generally, we again consider a limit economy with small flows where $g \to 0$ and $\gamma \to 0$ but the ratio $\frac{\gamma}{g} = \kappa$ is kept constant in the limit. Remember that in this case $\overline{\theta} \to 1 + \kappa$ and $\underline{\theta} \to \frac{1}{1+\kappa}$. We will show that under these conditions both in a "Buy first" and in a "Sell first" equilibrium no buyer has an incentive to deviate from targeting both types of sellers by demanding a lower price than the unique prevailing price $p = \frac{R}{\rho}$.

Still denoting the value of a matched owner that remains passive upon mismatch by $\tilde{V}$, note first that at $\tilde{\theta} = 1$ Assumption A2 can be simplified to

$$\frac{u-\chi}{\rho} < \frac{u-\chi}{\rho+\mu_0} + \frac{\mu_0}{(\rho+\mu_0)^2}c + \frac{{\mu_0}^2}{(\rho+\mu_0)^2}\tilde{V},$$

$$\Leftrightarrow \frac{u-\chi}{\rho} < \frac{c}{\rho+\mu_0} + \frac{\mu_0}{\rho+\mu_0}\tilde{V},$$

$$\Leftrightarrow 0 < \rho\left(c - (u-\chi)\right) + \mu_0\left(\rho\tilde{V} - (u-\chi)\right),$$

which, for future reference, is not greater than $\rho\left(c - (u-\chi)\right) + \mu_0\left(\rho V - (u-\chi)\right)$.

Under the unique price to be proven, the value functions are given by equations (5)-(8) and (28)-(30), given θ and R. We first show that in an equilibrium in which mismatched owners "Buy first", buyers have no incentive to demand a lower price than $p = \frac{R}{\rho}$. In such an equilibrium there are two types of sellers: double owners and real estate agents. As before, the lowest price that a real estate agent would be willing to accept is $p^A = V^A = \frac{R}{\rho}$. The lowest price that a double owner would be willing to accept is $p^{S2} = V^{S2} - V$.

Substituting these prices in the value functions, in an equilibrium with price dispersion $p^{S2} < p^A$, since

$$\rho\left(V^{S2} - V - V^A\right) = u_2 + R + \mu\left(\theta\right)\left(p^{S2} + V - V^{S2}\right) - \rho V - R - \mu\left(\theta\right)\left(p^A - V^A\right),$$

$$\Leftrightarrow \rho\left(V^{S2} - V - V^A\right) = u_2 - \rho V < 0,$$

$$\Leftrightarrow V^{S2} - V < V^A.$$

For that reason, under full information buyers would like to buy from a double owner, but the question is whether under private information they will make an offer that only double owners would accept. Note that for any $p \geq p^{S2}$ double owners are willing to sell, while for $p < p^{S2}$ they are not. As a result, since buying a house is preferred to being passive, among all possible deviations no offer is more profitable than demanding $V^{S2} - V$. The proof can therefore be restricted to this deviating offer. Note also that a deviating mismatched seller has zero mass, so that its presence doesn't affect the take-it-or-leave-it offers that buyers make.

First considering new entrants, for them to demand p^A it must be the case that

$$V - V^{Bn} - p^A \geq \frac{S_2}{S}\left(V - V^{Bn} - p^{S2}\right).$$

Substituting prices and using that $S = S_2 + A$ yields

$$\frac{A}{S}\left(V - V^{Bn} - \frac{R}{\rho}\right) \geq \frac{S_2}{S}\left(V - V^{S2} + \frac{R}{\rho}\right). \tag{116}$$

From the value functions we have that

$$\rho\left(V - V^{Bn} - \frac{R}{\rho}\right) = \rho V - u_n + R - q(\theta)\left(V - V^{Bn} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow (\rho + q(\theta)) \left(V - V^{Bn} - \frac{R}{\rho} \right) = \rho V - u_n,$$

and

$$\rho \left(V - V^{S2} + \frac{R}{\rho} \right) = \rho V - u_2 - R - \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right) + R,$$

$$\Leftrightarrow (\rho + \mu(\theta)) \left(V - V^{S2} + \frac{R}{\rho} \right) = \rho V - u_2. \tag{117}$$

Moreover, in the limit we consider, we know from the section on Nash bargaining that $\frac{A}{S} = \frac{1}{\overline{\theta}}$ and $\frac{S_2}{S} = \frac{\overline{\theta}-1}{\overline{\theta}}$, so that (116) amounts to

$$\frac{1}{\overline{\theta}} \left(\rho + \mu\left(\overline{\theta}\right) \right) (\rho V - u_n) \geq \frac{\overline{\theta} - 1}{\overline{\theta}} \left(\rho + q\left(\overline{\theta}\right) \right) (\rho V - u_2).$$

where both sides are positive, but where the right-hand side can be made arbitrarily close to zero by moving closer to $\overline{\theta} = 1$. Therefore, it follows that there is a $\kappa_7 > 0$, such that for $\kappa < \kappa_7$, new entrants in a "Buy first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a selller. Substituting u_0 for u_n, the same condition holds for a deviating mismatched seller, and then becomes a forced renter. Therefore, there is a $\kappa_8 > 0$, such that for $\kappa < \kappa_8$, forced renters in a "Buy first" equilibrium make the same offer.

For mismatched owners that buy first to demand p^A it must be the case that

$$V^{S2} - V^{B1} - p^A \geq \frac{S_2}{S} \left(V^{S2} - V^{B1} - p^{S2} \right),$$

$$\Leftrightarrow \frac{A}{S} \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) \geq \frac{S_2}{S} \left(V - V^{S2} + \frac{R}{\rho} \right). \tag{118}$$

Rearranging the value functions yields

$$\rho \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) = u_2 + R + \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right) - R - (u - \chi) - q(\theta) \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right),$$

$$\Leftrightarrow (\rho + q(\theta)) \left(V^{S2} - V^{B1} - \frac{R}{\rho} \right) = u_2 - (u - \chi) + \mu(\theta) \left(V - V^{S2} + \frac{R}{\rho} \right). \tag{119}$$

Substituting the steady state fractions and (119) into (118), in the limit we consider we have that

$$\frac{1}{\overline{\theta}}\left(u_2-(u-\chi)+\mu\left(\overline{\theta}\right)\left(\frac{R}{\rho}+V-V^{S2}\right)\right)\geq\frac{\overline{\theta}-1}{\overline{\theta}}\left(\rho+q\left(\overline{\theta}\right)\right)\left(V-V^{S2}+\frac{R}{\rho}\right),$$

$$u_2-(u-\chi)\geq\left[\left(\overline{\theta}-1\right)\left(\rho+q\left(\overline{\theta}\right)\right)-\mu\left(\overline{\theta}\right)\right]\left(V-V^{S2}+\frac{R}{\rho}\right).$$

Substituting (117) yields

$$\left(\rho+\mu\left(\overline{\theta}\right)\right)\left(u_2-(u-\chi)\right)\geq\left[\left(\overline{\theta}-1\right)\left(\rho+q\left(\overline{\theta}\right)\right)-\mu\left(\overline{\theta}\right)\right]\left(\rho V-u_2\right),$$

$$\Leftrightarrow\rho\left(u_2-(u-\chi)\right)+\mu\left(\overline{\theta}\right)\left(\rho V-(u-\chi)\right)\geq\left(\overline{\theta}-1\right)\left(\rho+q\left(\overline{\theta}\right)\right)\left(\rho V-u_2\right).$$

The left-hand side is positive for any $\overline{\theta}\geq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_9>0$, such that for $\kappa<\kappa_9$, mismatched owners that buy first in a "Buy first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a seller. Taking $\overline{\kappa}'=\min\{\kappa_7,\kappa_8,\kappa_9\}$, we have that for $\kappa<\overline{\kappa}'$, all buyers demand $p=\frac{R}{\rho}$ upon meeting a seller in a "Buy first" equilibrium with a market tightness given by $\overline{\theta}=1+\kappa$.

Secondly, we show that in an equilibrium in which mismatched owners sell first, buyers have no incentive to demand a lower price than $p=\frac{R}{\rho}$. In such an equilibrium there are two types of sellers: mismatched owners that sell first, and real estate agents. The lowest price that a real estate agent would be willing to accept is still $p^A=V^A=\frac{R}{\rho}$. The lowest price that a mismatched owner would be willing to accept is $p^{S1}=V^{S1}-V^{B0}$. It must be the case that $V^{B0}-V^{S1}+p^A\geq 0$, because mismatched owners don't remain passive by Assumption A2. It follows that $p^{S1}\leq p^A$, so that with full information buyers would like to buy from a mismatched owner. Again the question is whether under private information buyers will make an offer that only mismatched owners would accept. Similar to the "Buy first" equilibrium, the proof can be restricted to the deviation of demanding $V^{S1}-V^{B0}$.

First considering forced renters, for them to demand p^A it must be the case that

$$V - V^{B0} - p^A \geq \frac{S_1}{S}\left(V - V^{B0} - p^{S1}\right),$$

$$\Leftrightarrow \frac{A}{S}\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right). \tag{120}$$

Rearranging the value functions yields

$$\rho\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0 + R - q(\theta)\left(V - V^{B0} - \frac{R}{\rho}\right) - R,$$

$$\Leftrightarrow (\rho + q(\theta))\left(V - V^{B0} - \frac{R}{\rho}\right) = \rho V - u_0, \tag{121}$$

and

$$\rho\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - R + q\left(\theta\right)\left(-\frac{R}{\rho} + V - V^{B0}\right) - (u - \chi) - \mu\left(\theta\right)\left(\frac{R}{\rho} + V^{B0} - V^{S1}\right) + R,$$

$$\Leftrightarrow (\rho + \mu(\theta))\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right) = u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{B0} - \frac{R}{\rho}\right). \tag{122}$$

Substituting (122), (120) therefore amounts to

$$\frac{A}{S}(\rho + \mu(\theta))\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi) + q\left(\theta\right)\left(V - V^{B0} - \frac{R}{\rho}\right)\right),$$

$$\Leftrightarrow \left[\frac{A}{S}(\rho + \mu(\theta)) - \frac{S_1}{S}q(\theta)\right]\left(V - V^{B0} - \frac{R}{\rho}\right) \geq \frac{S_1}{S}\left(u_0 - (u - \chi)\right).$$

Substituting (121) yields

$$\left[\frac{A}{S}(\rho + \mu(\theta)) - \frac{S_1}{S}q(\theta)\right](\rho V - u_0) \geq \frac{S_1}{S}(\rho + q(\theta))\left(u_0 - (u - \chi)\right),$$

$$\Leftrightarrow \frac{A}{S}(\rho + \mu(\theta))(\rho V - u_0) \geq \frac{S_1}{S}\rho\left(u_0 - (u - \chi)\right) + \frac{S_1}{S}q(\theta)\left(\rho V - (u - \chi)\right).$$

From the section on Nash bargaining we know that $\frac{A}{S} = \underline{\theta}$ and $\frac{S_1}{S} = 1 - \underline{\theta}$. Substituting these steady state fractions, we have that

$$\underline{\theta}\left(\rho+\mu\left(\underline{\theta}\right)\right)\left(\rho V-u_{0}\right)\geq\left(1-\underline{\theta}\right)\left[\rho\left(u_{0}-\left(u-\chi\right)\right)+q\left(\underline{\theta}\right)\left(\rho V-\left(u-\chi\right)\right).\right]$$

Again, by moving towards $\underline{\theta}=1$ the right-hand side can be made arbitrarily close to zero while the left-hand side remains positive. Therefore, there exists a $\kappa_{10}>0$, such that for $\kappa<\kappa_{10}$, forced renters in a "Sell first" equilibrium demand $p=\frac{R}{\rho}$ upon meeting a seller. Substituting u_n for u_0 the same condition holds for a new entrant, so that there is a $\kappa_{11}>0$, such that for $\kappa<\kappa_{11}$, new entrants make the same offer.

Finally, for a deviating mismatched buyer to demand p^A it must be the case that

$$V^{S2}-V^{B1}-p^{A}\geq\frac{S_{1}}{S}\left(V^{S2}-V^{B1}-p^{S1}\right),$$

$$\Leftrightarrow\frac{A}{S}\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)\geq\frac{S_{1}}{S}\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right). \qquad (123)$$

Rearranging the value functions yields

$$\rho\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_{2}+R+\mu\left(\theta\right)\left(\frac{R}{\rho}+V-V^{S2}\right)-u-\chi+q\left(\theta\right)\left(-\frac{R}{\rho}+V^{S2}-V^{B1}\right)-R,$$

$$\Leftrightarrow\left(\rho+q(\theta)+\mu\left(\theta\right)\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right)=u_{2}-\left(u-\chi\right)+\mu\left(\theta\right)\left(V-V^{B1}\right),$$

with

$$\mu\left(\theta\right)\left(V-V^{B1}\right)=\mu\left(\theta\right)\left(V-\frac{u-\chi}{\rho}\right)-\mu\left(\theta\right)q\left(\theta\right)\left(V^{S2}-V^{B1}-\frac{R}{\rho}\right).$$

From (122) we know that

$$\left(\rho+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_{0}-(u-\chi)+q\left(\theta\right)\left(V-V^{B0}-\frac{R}{\rho}+V^{S1}-V^{S1}\right),$$

$$\Leftrightarrow\left(\rho+q(\theta)+\mu(\theta)\right)\left(V^{B0}-V^{S1}+\frac{R}{\rho}\right)=u_{0}-\left(u-\chi\right)+q\left(\theta\right)\left(V-V^{S1}\right),$$

with

$$q(\theta)\left(V - V^{S1}\right) = q(\theta)\left(V - \frac{u-\chi}{\rho}\right) - \mu(\theta)\, q(\theta)\left(V^{B0} - V^{S1} + \frac{R}{\rho}\right).$$

Therefore, (123) simply amounts to

$$\frac{A}{S}\left(u_2 - (u-\chi) + \mu(\theta)\left(V - \frac{u-\chi}{\rho}\right)\right) \geq \frac{S_1}{S}\left(u_0 - (u-\chi) + q(\theta)\left(V - \frac{u-\chi}{\rho}\right)\right).$$

Substituting the steady state fractions, we have that

$$\underline{\theta}\left(u_2 - (u-\chi) + \mu(\underline{\theta})\left(V - \frac{u-\chi}{\rho}\right)\right) \geq (1-\underline{\theta})\left(u_0 - (u-\chi) + q(\underline{\theta})\left(V - \frac{u-\chi}{\rho}\right)\right).$$

The left-hand side is positive for any $0 < \underline{\theta} \leq 1$ by Assumption A2. Moving $\overline{\theta}$ towards 1 can make the right-hand side arbitrarily close to zero, so that there exists a $\kappa_{12} > 0$, such that for $\kappa < \kappa_{12}$, deviating mismatched owners that buy first in a "Sell first" equilibrium demand $p = \frac{R}{\rho}$ upon meeting a seller. Taking $\underline{\kappa}' = \min\{\kappa_{10}, \kappa_{11}, \kappa_{12}\}$, we have that for $\kappa < \underline{\kappa}'$, all buyers demand $p = \frac{R}{\rho}$ upon meeting a seller in a "Sell first" equilibrium with a market tightness given by $\underline{\theta} = \frac{1}{1+\kappa}$. Finally, taking $\kappa' = \min\{\overline{\kappa}', \underline{\kappa}'\}$, we have that both in a "Buy first" and in a "Sell first" equilibrium, the take-it-or-leave-it offer that buyers make is equal to $p = \frac{R}{\rho}$.